*University of Jijel - Faculty of exact sciences and computer science - Mathematics department*

*Series No.* 01 *2023/2024*

**Exercise 01:** Prove that the following identities hold for all non-zero natural numbers n:

**1)** $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

**2)** $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}$.

**Exercise 02:**

**1)** Prove that for any real number $x \geq -1$ and any natural number n, we have the following **Bernoulli's inequality**:

$$(1+x)^n \geq 1 + nx.$$

**2)** By applying **Bernoulli's inequality**, first with $x = \frac{2}{n}$ and then with $x = \frac{-2}{3n}$ prove that :

$$1 + \frac{2}{n} \geq 3^{\frac{1}{n}} \geq 1 + \frac{2}{3n-2} \quad for\ n = 1,2,3,\dots .$$

**Exercise 03:**

1) Find the integer part of the following numbers:

$-9.1, \quad 3.8, \quad 0.1, \quad \pi, \quad e, \quad 11, \quad -3, \quad -4.6$ .

2) Prove that:

a) $\forall x \in \mathbb{R}: [x+1] = [x] + 1.$

b) $\forall x, y \in \mathbb{R}_+ : [x][y] \leq [xy].$

c) $\forall x \in \mathbb{R}, \forall n \in \mathbb{N}^*: \left[\frac{[nx]}{n}\right] = [x].$

d) $\forall x, y \in \mathbb{R}: [x] + [y] \leq [x+y] \leq [x] + [y] + 1.$

**Exercise 04:** Solve the following inequalities:

$a)\ |x-2| \leq |x+1|, \quad b)\ \sqrt{4x-3} > x, \quad c)\ \frac{x-1}{x^2+4} < \frac{x+1}{x^2-4},$

$d)\ |x+1| + |x-1| < 4, \quad e)\ |17 - 2x^4| \leq 15, \quad f)\ |2x^2 - 13| < 5$ .

**Exercise 05:** Rewrite the following inequalities :

1) Without the absolute value sign:

$$a)\ |x-1| \leq 2, \qquad b)\ |x+5| > 3.$$

2) By using the absolute value sign:

$$a) -3 < x < 0, \qquad b)\ 1 < x+2 < 4, \qquad c) -2 \leq\ x-1 \leq 5.$$

**Exercise 06:**

1) Prove that the following inequalities hold for all real numbers $x$ and $y$ :
   a) $|x+y| \leq |x| + |y|$.
   a) $\big||x| - |y|\big| \leq |x-y|$.
2) Deduce that :

$$\forall x, y, z \in \mathbb{R}: |x-z| \leq |x-y| + |y-z|.$$

3) Use the trigonometric inequality to prove that :

$$a)\ |x| \leq 1 \Rightarrow |3+x^3| \leq 4, \qquad b)\ |y| < 1\ \Rightarrow |3-y| > 2.$$

4) Are the reverse implications of a) and b) true? (Justify that).

**Exercise 07:** Prove that for any two real numbers $x$ and $y$, we have:

$$\min\{x, y\} = \frac{1}{2}(x+y-|x-y|)$$

and

$$\max\{x, y\} = \frac{1}{2}(x+y+|x-y|).$$

**Exercise 08:** Prove that $\sqrt{3}$ is an irrational number.