Solution TD N° 03

Exo 1 : Triangle de puissance

$v(t) = 150 \sin(\omega t + 10°) \longrightarrow \overline{V} = \left(\frac{150}{\sqrt{2}}\right)^{\angle 10°}$ V

$i(t) = 5 \sin(\omega t - 50°) \longrightarrow \overline{I} = \left(\frac{5}{\sqrt{2}}\right)^{\angle -50°}$ I

La puissance complexe est :

$\overline{S} = \overline{V} \cdot \overline{I}^* = \left(\frac{150}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}}\right)^{\angle 10+50} = 375^{\angle 60°}$ VA

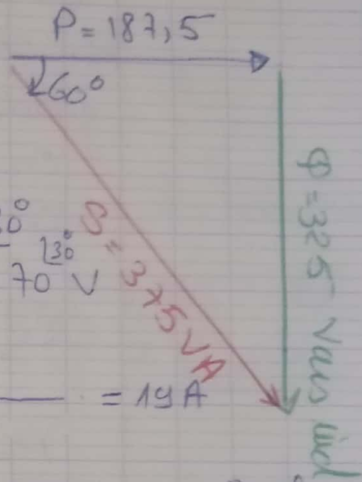
$\overline{S} = P + jQ = 187,5 + j\,325$ VA

$P = \mathrm{Re}(\overline{S}) = 187,5$ W.

$Q = \mathrm{Im}(\overline{S}) = 325$ Vars inductifs

$S = |\overline{S}| = 375$ VA

$F.P = \cos 60° = 0,5$ inductif



Exo 2

$V(t) = 99 \sin(6000t + 30°) \rightarrow \overline{V} = \left(\frac{99}{\sqrt{2}}\right)^{\angle 30°} = 70^{\angle 30°}$ V

La puissance active est

$P = VI\cos\theta \Rightarrow I = \frac{P}{V\cos\theta} = \frac{940}{70 \cdot} = 19$ A

Le courant est en avance de $\arccos(0,707) = 45°$ sur la tension $\Rightarrow \overline{I} = 19^{\angle 75°}$ A.

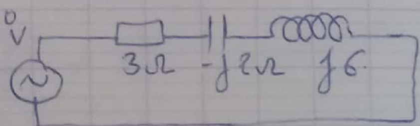
L'impédance du circuit est : $\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{70^{\angle 30}}{19^{\angle 75}} = 3,68^{\angle -45°}$ Ω

$\overline{Z} = 2,6 - j\,2,6$ Ω

Comme $\overline{Z} = R - jX_C \;/\; X_C = \frac{1}{\omega C}$

$R = 2,6\,\Omega$ et $C = \frac{1}{\omega X_C} = \frac{1}{6000 \cdot 2,6} = 64,1\,\mu F$.

Exo 3



$\overline{Z} = 3 + 6j - 2j = 3 + 4j$

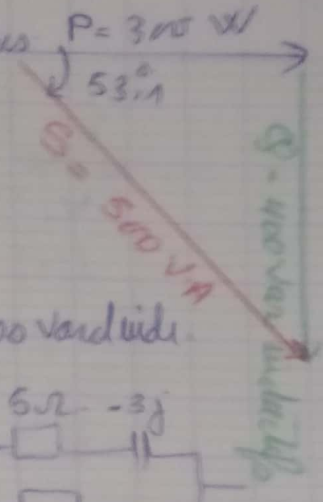
$\overline{Z} = 5^{\angle 53,1}$ Ω

$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{50^{\angle -90°}}{5^{\angle 53,1°}} = 10^{\angle -143,1}$ A

La puissance apparente $\overline{S} = \overline{V} \cdot \overline{I}^* = 50^{\angle -90} \cdot 10^{\angle 143,1} = 500^{\angle 53,1°}$ VA

$\overline{S} = 300 + j\,400$ VA

$\overline{S} = P + jQ \Rightarrow P = 300\,W,\ Q = 400$ Vars



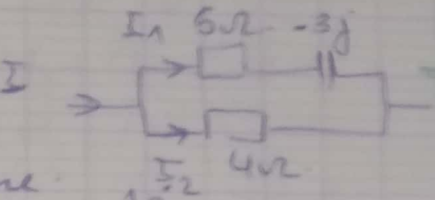
$\cos\varepsilon = 0,6 \rightarrow \varepsilon = 53,1$ inductif

2eme méthode

$P = RI^2 = 3\ 10^2 = 300\ W$

$Q = Q_{6j} + Q_{-2j} = 6 \cdot 10^2 - 2\ 10^2 = 400$ Var inductif

Exo 4 $\quad I_T = 30A$



On suppose que la phase du courant I est choisit comme grandeur de reference $\Rightarrow I = 30^{\angle 0°}\ A$

On a un diviseur du courant

$\overline{I}_1 = 30^{\angle 0} \cdot \left(\frac{5-3j}{9-3j}\right) = 18,45^{\angle -12,55°}\ A$

$\overline{I}_2 = 30^{\angle 0}\left(\frac{4}{9-3j}\right) = 12,7^{\angle 18,43°}\ A$

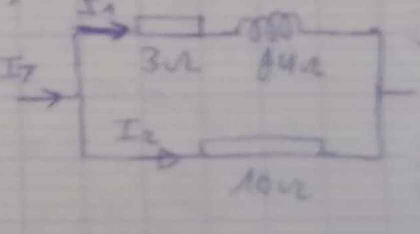
$P = P_4 + P_5 = R_4 I_2^2 + R_5 I_1^2 = 4(18,45)^2 + 5(12,7)^2 = 2165\ W$

$Q = X I_1^2 = 3(12,7)^2 = 483$ Vars capacitif

$\overline{S} = P - jQ = 2165 - j483 = 2210^{\angle -12,6°} \Rightarrow S = 2210VA$

$F.P = \cos\varphi = \frac{P}{S} = \frac{2165}{2210} = 0,98$ capacitif

E5 $\quad P = 1100\ W$



$\overline{I}_1 = \frac{\overline{V}}{\overline{Z}_1} = \frac{\overline{V}}{3+4j} = \frac{\overline{V}}{5^{\angle 53,1}}$

$\overline{I}_2 = \frac{\overline{V}}{\overline{Z}_2} = \frac{\overline{V}}{10}$

Le rapport des modules des courants est $\frac{I_1}{I_2} = \frac{V/5}{V/10} = \frac{2}{1}$

$\frac{P_3}{P_{10}} = \frac{R_3 I_1^2}{R_{10} I_2^2} = \frac{3}{10}\left(\frac{2}{1}\right)^2 = \frac{6}{5}$

$P_T = P_3 + P_{10} \Rightarrow \frac{P_T}{P_{10}} = \frac{P_3}{P_{10}} + 1$

On remplace $\frac{P_3}{P_{10}}$ par $\frac{6}{5}$ $\quad \frac{1100}{P_{10}} = \frac{6}{5} + 1$

$\Rightarrow P_{10} = 1100\left(\frac{5}{11}\right) = 500\ W$

$P_3 = 1100 - 500 = 600\ W$

Suite exo 5

$P_3 = R I_1^2 \Rightarrow I_1 = \sqrt{\frac{P_3}{3}} = \sqrt{\frac{600}{3}} = 14,14$ A

Si on pose $\bar{V} = V\angle 0°$

$\bar{I}_1 = 14,14\angle -53,1 = 8,48 - j\,11,31$ A

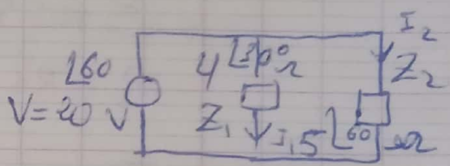
$\cancel{\bar{I}_2 = \bar{I}_1 / 2} =$

$I_2 = \frac{I_1}{2} = 7,07 \Rightarrow \bar{I}_2 = 7,07\angle 0°$ A

$I_T = \bar{I}_1 + \bar{I}_2 = 15,55 - j\,11,31 = 19,25\angle -36°$ A

Exo6 : Triangle de puissance pour chacune des branches

pour la branche 1 :



$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{20\angle 60}{4\angle 30°} = 5\angle 30°$ A

$\bar{S}_1 = \bar{V}_1\,\bar{I}_1^* = 20\angle 60 \times 5\angle -30 = 100\angle 30 = 86,6 + j\,50$ VA

$P_1 = 86,6$ W $\quad Q = 50$ Var inductif, $S_1 = 100$ VA.

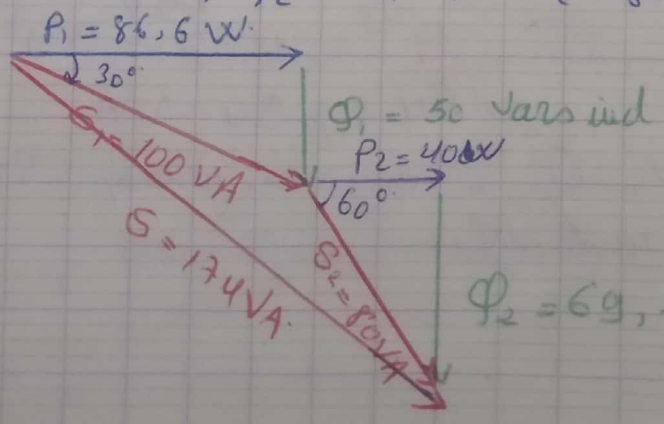
$FP_1 = 0,86$ inductif.

pour branche 2

$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{20\angle 60°}{5\angle 60°} = 4\angle 0°$ A

$S_2 = \bar{V}_2 . \bar{I}_2^* = 20\angle 60 . 4\angle 0° = 80\angle 60° = 40 + j\,69,2$ VA

$P_2 = 40$ W, $Q_2 = 69,2$ Var $S_2 = 80$ VA $FP_2 = \cos 60 = 0,5$ ind



$P_T = P_1 + P_2 = 86,6 + 40 = 126,6$ W

$Q_T = Q_1 + Q_2 = 50 + 69,2 = 119,2$ Var inductif

$\bar{S}_T = \bar{P}_T + j\bar{Q}_T = 126,6 + j\,119,2$

$S_T = 174$ VA

Ex 7 : $\eta = 85\%$ $\quad P_{sortie} = 1492\,W$ $\quad F.P = 0,8$ inductif.

$$\eta = \frac{P_{sortie}}{P_{fournie}} \Rightarrow \qquad P_{fournie} \rightarrow \boxed{\text{Moteur}} \rightarrow P_{sortie}$$

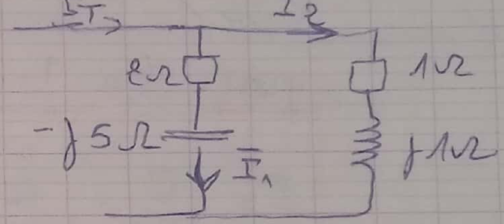
$$P_{fournie} = \frac{P_{sortie}}{\eta} = \frac{1492}{0,85} = 1755\ W$$

$$P = S\cos\varepsilon \Rightarrow S = \frac{P}{\cos\varepsilon} = \frac{1755}{0,8} = 2190\ V.A$$

$$\varepsilon = \arccos(0,8) = 36,9^\circ$$

$$Q = S\sin\varepsilon = 2190 \cdot \sin(36,9^\circ) = 1315 \text{ var inductifs}$$

Ex n° 8 : $P_2 = 20\ W$



$$P_2 = R_2 \cdot I_1^2 \Rightarrow I_1 = \sqrt{\frac{P_2}{R_2}} = 3,16A$$

$$\bar{Z}_1 = 2 - 5j = 5,38^{\angle -68,2^\circ}$$

On pren $\bar{V} = V^{\angle 0^\circ}$

$$\bar{V} = \bar{Z}_1 \cdot \bar{I}_1 \Rightarrow V = Z \cdot I_1 = 5,38 \cdot 3,16 = 17\,V$$

$$\bar{V} = 17^{\angle 0^\circ}, \quad \bar{I}_1 = 3,16^{\angle +68,12^\circ} = 1,173 + j2,934\ A$$

$$I_2 = \frac{\bar{V}}{Z_2} = \frac{17^{\angle 0}}{\sqrt{2}^{\angle 45}} = 12^{\angle -45^\circ} A = 8,485 - j8,485\ A$$

$$I_T = \bar{I}_1 + \bar{I}_2 = 11,1^{\angle -29,8^\circ} A$$

$$\bar{S}_T = \bar{V} \cdot \bar{I}_T^* = 17^{\angle 0} \cdot 11,1^{\angle 29,8} = 189^{\angle 29,8^\circ}$$

$$= 164 + j94\ VA$$

$P_T = 164\ W$, $Q_T = 94$ Vars inductifs $\quad S_T = 189\ VA$

$F_P = 0,86$ inductifs

Ex n° 04.

Charge 1 : $S_1 = 250$ VA FP = 0,5 inductif

$P_1 = S_1 \cos\varphi_1 = 250 \cdot 0{,}5 = 125$ W

$\varphi_1 = \arccos 0{,}5 = 60° \Rightarrow \sin\varphi_1 = 0{,}86$

$Q_1 = S_1 \sin\varphi_1 = 250 \times 0{,}86 = 216$ vars inductifs

Charge 2 : $P_2 = 180$ W , FP = 0,8 capacitif

$$S_2 = \frac{P_2}{\cos\varphi_2} = \frac{180}{0{,}8} = 225 \text{ VA}$$

$\varphi_2 = \arccos 0{,}8 = 36{,}9°$

$Q_2 = 225 \sin 36{,}9 = 135$ vars capacitifs

Charge 3 : $S_3 = 300$ VA . $Q_3 = 100$ vars inductif

$$\varphi_3 = \arcsin\left(\frac{Q_3}{S_3}\right) = \arcsin\left(\frac{100}{300}\right) = 19{,}5°$$

$P_3 = S_3 \cos 19{,}5 = 283$ W

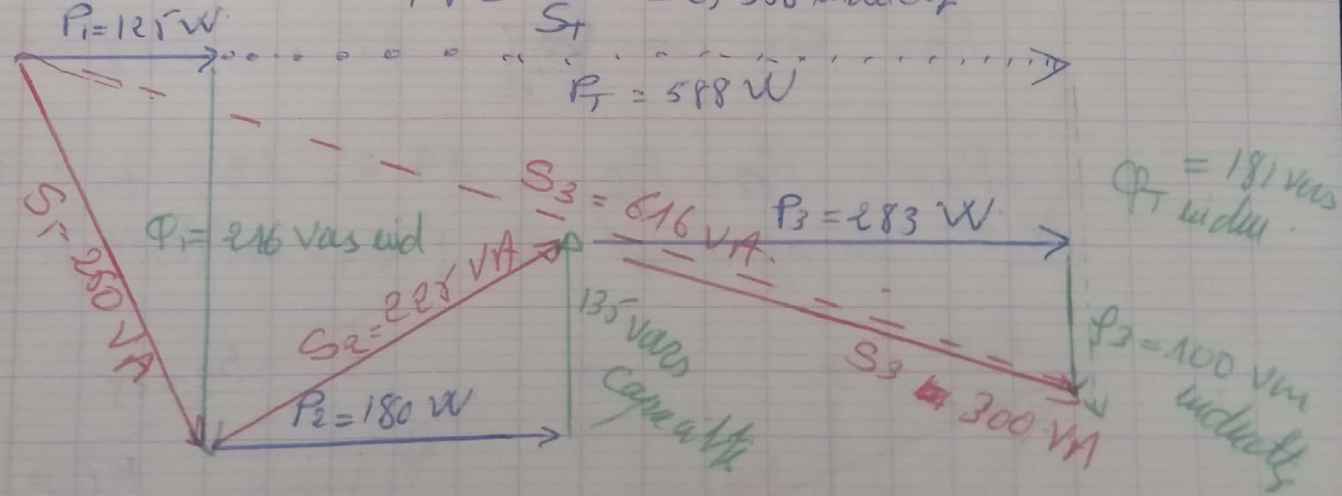
finalement $P_T = P_1 + P_2 + P_3 = 588$ W

$Q_T = Q_1 - Q_2 + Q_3 = 216 - 135 + 100 = 181$ vars inductif

$\overline{S_T} = P_T + jS_T = 588 + j181 = 616^{\angle 17{,}1°}$ VA

$S_T = 616$ VA

$$FP = \frac{P_T}{S_T} = 0{,}958 \text{ inductif}$$



Exer 10

$S_n = 25$ KVA $P_{ch} = 12$ KW. $FP = 0,6$ inductif

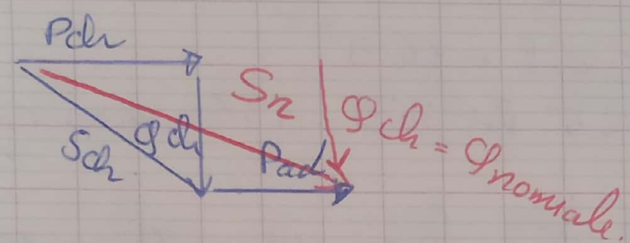
1°/ Le taux de charge du transformateur

$$P_{ch} = S_{ch} \times FP \Rightarrow S_{charge} = \frac{P_{ch}}{FP} = \frac{12}{0,6} = 20 \text{ KVA}$$

$S_n \longrightarrow 100\%$

$S_{ch} \longrightarrow \tau_{ch}$ } $\tau_{ch} = \frac{20.10^3 . 100}{25} = 80\%$

2°/



La charge additionnelle possède un $FP_{ad} = 1$

$\Rightarrow Q_{ad} = 0$ Var

$\Rightarrow Q_{ch}$ = reste inchangeable

$Q_{ch} = Q_n = S \sin 53,1 = 16$ kVars inductif

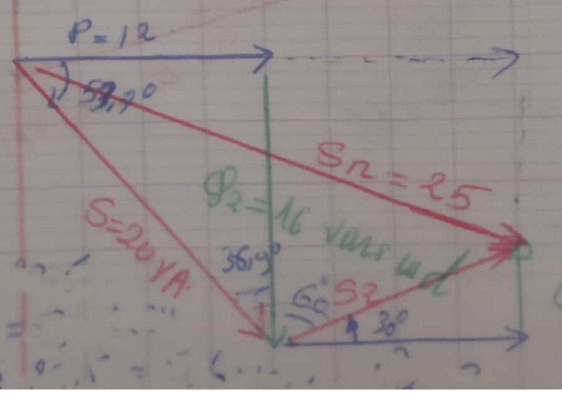
$FP_n = ?$ $Q_n = S_n \sin \theta_n \Rightarrow \theta_n = \arcsin\left(\frac{16}{25}\right)$

$\theta_n = 39,8°$

$P_n = S_n \cos \theta_n = 25 . \cos 39,8 = 19,2$ KW

$P_{ad} = P_n - P_{ch} = 19,2 - 12 = 7,2$ KW

10.3 En ajoutant S_2 ($\theta_2 = \arccos(0,86) = 30°$



Suite à l'exo 10.

$$P_2 = S_2 \cos 30 = \frac{\sqrt{3}}{2} S_2$$

$$Q_2 = S_2 \sin 30 = \frac{1}{2} S_2$$

$$S_n^2 = (P_{ch} + P_2)^2 + (Q_{ch} - Q_2)^2$$

$$25^2 = \left(12 + \frac{\sqrt{3}}{2} S_2\right)^2 + \left(16 - \frac{1}{2} S_2\right)^2$$

$$25^{-2} = 12^2 + 16^2 + (12\sqrt{3} - 16) S_2 + \left(\frac{3}{4} + \frac{1}{4}\right) S$$

la résoultion de cette eq du seconde

Ordre donne $\Rightarrow S_2 = 12,8$ kVA

$$P_2 = 12,8 \cdot \frac{\sqrt{3}}{2} = 11,1 \text{ kW}$$

$Q_2 = 6,4$ vars capacitif