$$Q = \int d^3x\, j^0 = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \int d^3q \int \frac{d^3x}{(2\pi)^3}$$

$$\left\{ a^+(k)\, a(q)\, e^{i(k-q)x} - b(k)\, b^+(q)\, e^{-i(k-q)x} \right\} \quad -(\beta)^\circ$$

On sait bien que:

$$\int d^3x\, e^{\pm i(q-k)x} = \int d^3x\, e^{\pm i(q_0-k_0)x^0}\, e^{\mp i(\vec{q}-\vec{k})\vec{x}}.$$

$$= e^{\pm i(q_0-k_0)x^0} \int d^3x\, e^{\mp i(\vec{q}-\vec{k})\vec{x}}.$$

$$= e^{\pm i(q_0-k_0)x^0} (2\pi)^3\, \delta(\vec{q}-\vec{k}).$$

$$\Rightarrow \int \frac{d^3x}{(2\pi)^3}\, e^{\pm i(q-k)x} = \delta(\vec{q}-\vec{k})\, e^{\pm i(q_0-k_0)x^0} \quad -(\alpha)^\circ$$

on remplace $(\alpha)^\circ$ dans $(\beta)^\circ$, on trouve:

$$Q = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \int d^3q \left\{ a^+(k)\, a(q)\, \delta(\vec{q}-\vec{k}) - b(k)\, b^+(q)\, \delta(\vec{q}-\vec{k}) \right\}.$$

$$Q = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left[a^+(k)\, a(k) - b(k)\, b^+(k) \right].$$

$$Q = \int d^3k \left[\frac{1}{(2\pi)^3 2\omega_k} a^+(k)\, a(k) - \frac{1}{(2\pi)^3 2\omega_k} b(k)\, b^+(k) \right]$$

$$b(k)\, b^+(k) = \overbrace{b^+(k)\, b(k) + \alpha\, \delta(\vec{0})}$$

$$:Q: = \int d^3k \left[N_a(k) - N_b(k) \right]$$

$$:Q: = N_a - N_b.$$

On a: $Q|0\rangle = 0$.

$[Q, a^+(k)] = a^+(k)$.

$[Q, b^+(k)] = -b^+(k)$.

Démonstration :

$$\bullet [Q, a^+(k)] = \int \frac{d^3q}{(2\pi)^3 2\omega_q} \Big\{ [a^+(q)a(q), a^+(k)] - [b^+(q)b(q), a^+(k)] \Big\}$$

$$= \int \frac{d^3q}{(2\pi)^3 2\omega_q} \Big\{ a^+(q)[a(q), a^+(k)] + \underbrace{[a^+(q), a^+(k)]}_{=0} a(q) - b^+(q)\underbrace{[b(q), a^+(k)]}_{=0} - \underbrace{[b(q), a^+(k)]}_{=0} b^+(q) \Big\}.$$

$$= \int \frac{d^3q}{(2\pi)^3 2\omega_q} (2\pi)^3 2\omega_k \, \delta(\vec{k} - \vec{q}) \, a^+(q).$$

$$[Q, a^+(k)] = a^+(k)$$

$$\bullet [Q, b^+(k)] = \int \frac{d^3q}{(2\pi)^3 2\omega_q} \Big\{ [a^+(q)a(q), b^+(k)] - [b^+(q)b(q), b^+(k)] \Big\}.$$

$$= \int \frac{d^3q}{(2\pi)^3 2\omega_q} \Big\{ a^+(q)\underbrace{[a(q), b^+(k)]}_{=0} + \underbrace{[a^+(q), b^+(k)]}_{=0} a(q) - b^+(q)[b(q), b^+(k)] - \underbrace{[b^+(q), b^+(k)]}_{=0} b(q) \Big\}.$$

$$= \int \frac{d^3q}{(2\pi)^3 2\omega_q} \Big(- b^+(q)(2\pi)^3 2\omega_k \, \delta(\vec{k} - \vec{q}) \Big)$$

$$[Q, b^+(k)] = -b^+(k)$$

$a^+(k)$ augmente Q de 1.

$b^+(k)$ diminue Q de 1.

Alors :

* $a^+(k)$ crée une particule de charge +1 d'impulsion $\vec{k}$
  et $a(k)$ la détruit.
* $b^+(k)$ crée une particule de charge -1 d'impulsion $\vec{k}$
  et $b(k)$ la détruit.
* $\varphi(x)$ contient $a(k)$ et $b(k)$ sa charge est alors -1
  $[Q, \varphi(x)] = -\varphi(x)$, on dit qu'il détruit une particule
  ou crée une anti-particule.
* $\varphi^+(x)$ contient $a^+(k)$ et $b(k)$ sa charge est alors +1
  $[Q, \varphi^+(x)] = +\varphi^+(x)$, on dit qu'il crée une particule
  et détruit une anti-particule.

Démonstration :

$$[Q, \varphi(x)] = \int d^3k \frac{1}{(2\pi)^3 2\omega_k} \int d^3q \frac{1}{(2\pi)^3 2\omega_q}$$

$$\left[a^+(k)a(k) - b^+(k)b(k),\ a(q) e^{-iqx} + b^+(q) e^{iqx} \right].$$

$$= \int d^3k \frac{1}{(2\pi)^3 2\omega_k} \int d^3q \frac{1}{(2\pi)^3 2\omega_q}$$

$$\times \Big\{ [a^+(k)a(k), a(q)] e^{-iqx} + [a^+(k)a(k), b^+(q)] e^{iqx}$$

$$- [b^+(k)b(k), a(q)] e^{-iqx} - [b^+(k)b(k), b^+(q)] e^{iqx} \Big\}$$

$$[Q,\varphi(x)] = \int \frac{d^3k}{(2\pi)^3\, 2\omega_k} \int \frac{d^3q}{(2\pi)^3\, 2\omega_q}$$

$$\Big\{ a^+(k)\,\underbrace{[a(k), a(q)]}_{0=}\, e^{-iqx} + [a^+(k), a(q)]\, a(k)\, e^{-iqx}$$

$$+ a^+(k)\,\underbrace{[a(k), b^+(q)]}_{0=}\, e^{iqx} + \underbrace{[a^+(k), b^+(q)]}_{0=}\, a(k)\, e^{iqx}$$

$$- b^+(k)\,\underbrace{[b(k), a(q)]}_{0=}\, e^{-iqx} - \underbrace{[b^+(k), a(q)]}_{0=}\, b(k)\, e^{-iqx}$$

$$- b^+(k)\,[b(k), b^+(q)]\, e^{iqx} - \underbrace{[b^+(k), b^+(q)]}_{0=}\, b(k)\, e^{iqx} \Big\}.$$

$$[Q,\varphi(x)] = -\int \frac{d^3k}{(2\pi)^3\, 2\omega_k} \int \frac{d^3q}{(2\pi)^3\, 2\omega_q} \Big\{ [a(q), a^+(k)]\, a(k)\, e^{-iqx}$$

$$+ b^+(k)\,[b(k), b^+(q)]\, e^{iqx} \Big\}.$$

$$= -\int \frac{d^3k}{(2\pi)^3\, 2\omega_k} \int \frac{d^3q}{(2\pi)^3\, 2\omega_q} \Big\{ (2\pi)^3\, 2\omega_k\, \delta(\vec{k}-\vec{q})\, a(k)\, e^{-iqx}$$

$$+ b^+(k)\,(2\pi)^3\, 2\omega_k\, \delta(\vec{k}-\vec{q})\, e^{iqx} \Big\}.$$

$$[Q,\varphi(x)] = -\int \frac{d^3k}{(2\pi)^3\, 2\omega_k} \left[a(k)\, e^{-ikx} + b^+(k)\, e^{ikx} \right]$$

$$[Q,\varphi(x)] = -\varphi(x)$$

$k\lambda = k\nu\lambda$ =

Tenseur - impulsion énergie:

on sait que le Tenseur impulsion - énergie de ce champ est:

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial^\nu \varphi - g^{\mu\nu} \mathcal{L}.$$

1) Montrons que l'impulsion - énergie est donné par,

$$P^\mu = :\int dk\, k^\mu N(k):$$

Nous avons:

$$P^\mu = \int d^3x.\ T^{0\mu}$$

$$T^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha \varphi \partial^\alpha \varphi - \frac{1}{2} m^2 \varphi^2\right).$$

$$T^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi - \frac{1}{2} g^{\mu\nu} \partial_\alpha \varphi \partial^\alpha \varphi + \frac{1}{2} g^{\mu\nu} m^2 \varphi^2.$$

$$T^{0\nu} = \partial^0 \varphi \partial^\nu \varphi - \frac{1}{2} g^{0\nu} \partial_\alpha \varphi \partial^\alpha \varphi + \frac{1}{2} g^{0\nu} m^2 \varphi^2.$$

$$\boxed{T^{0\nu} = g^{\nu\alpha} \partial^0 \varphi \partial_\alpha \varphi - \frac{1}{2} g^{0\nu} \partial_\alpha \varphi \partial^\alpha \varphi + \frac{1}{2} g^{0\nu} m^2 \varphi^2.} \quad -(*)$$

$$\varphi(x) = \int \frac{dk}{(2\pi)^3 2\omega_k} \left[a(k)\, e^{-ikx} + a^{+}(k)\, e^{ikx} \right].$$

$$\frac{\partial \varphi(x)}{\partial t} = \int \frac{dk}{(2\pi)^3 2\omega_k} \left[-i\omega_k\, a(k)\, e^{-ikx} + i\omega_k\, a^{+}(k)\, e^{ikx} \right].$$

$$\boxed{\partial^0 \varphi = (-i) \int \frac{dk\ \omega_k}{(2\pi)^3 2\omega_k} \left[a(k)\, e^{-ikx} - a^{+}(k)\, e^{ikx} \right]}$$

$$\partial_\alpha \varphi = \int \frac{dk}{(2\pi)^3 2\omega_k} \left[a(k) \frac{\partial}{\partial x^\alpha} e^{-ik_\alpha x^\alpha} + a^{+}(k) \frac{\partial}{\partial x^\alpha} e^{ik_\alpha x^\alpha} \right].$$

$$= \int \frac{dk}{(2\pi)^3 2\omega_k} \left[-ik_\alpha\, a(k)\, e^{-ikx} + ik_\alpha\, a^{+}(k)\, e^{ikx} \right].$$

$$\boxed{\partial_\alpha \varphi = (-i) \int \frac{dk\ k_\alpha}{(2\pi)^3 2\omega_k} \left[a(k)\, e^{-ikx} - a^{+}(k)\, e^{ikx} \right]}.$$

$$\varphi^2 = \int \frac{dk}{(2\pi)^3 2\omega_k} \int \frac{dk'}{(2\pi)^3 2\omega_{k'}} \left[a(k)\, e^{-ikx} + a^{+}(k)\, e^{ikx} \right] \times \left[a(k')\, e^{-ik'x} + a^{+}(k')\, e^{ik'x} \right].$$

$$\varphi^2 = \int \frac{dk}{(2\pi)^3 2\omega_k} \int \frac{dk'}{(2\pi)^3 2\omega_{k'}} \left\{ a(k) a(k')\, e^{-i(k+k')x} + a^{+}(k) a(k')\, e^{i(k-k')x} + a(k) a^{+}(k')\, e^{-i(k-k')x} + a^{+}(k) a^{+}(k')\, e^{i(k+k')x} \right\}.$$

(a)

$$\partial_\alpha \varphi\, \partial^\alpha \varphi = (-i)^2 \int \frac{dk \quad k_\alpha}{(2\pi)^3 2\omega k} \left[a(k)\, e^{-ikx} - a^+(k)\, e^{ikx}\right].$$

$$\times \int \frac{dk' \quad k'^\alpha}{(2\pi)^3 2\omega k'} \left[a(k')\, e^{-ik'x} - a^+(k')\, e^{ik'x}\right].$$

$$\boxed{\begin{aligned}\partial_\alpha \varphi \partial^\alpha \varphi = \int \frac{dk}{(2\pi)^3 2\omega k} \int \frac{dk'}{(2\pi)^3 2\omega k'} k_\alpha k'^\alpha \Big[& a(k) a^+(k')\, e^{-i(k-k')x} \\ & + a^+(k) a(k')\, e^{i(k-k')x} - a(k) a(k')\, e^{-i(k+k')x} \\ & - a^+(k) a^+(k')\, e^{i(k+k')x} \Big].\end{aligned}}$$

(b)

$$\partial^0 \varphi\, \partial_\alpha \varphi = (-i)^2 \int \frac{dk \quad \omega k}{(2\pi)^3 2\omega k} \left[a(k)\, e^{-ikx} - a^+(k)\, e^{ikx}\right].$$

$$\times \int \frac{dk' \quad k'_\alpha}{(2\pi)^3 2\omega k'} \left[a(k')\, e^{-ik'x} - a^+(k')\, e^{ik'x}\right].$$

$$\boxed{\begin{aligned}\partial^0 \varphi \partial_\alpha \varphi = \int \frac{dk}{(2\pi)^3 2\omega k} \int \frac{dk'}{(2\pi)^3 2\omega k'} \omega_k k'_\alpha \Big\{ & a(k) a^+(k')\, e^{-i(k-k')x} \\ & + a^+(k) a(k')\, e^{i(k-k')x} - a(k) a(k')\, e^{-i(k+k')x} \\ & - a^+(k) a^+(k')\, e^{i(k+k')x} \Big\}.\end{aligned}}$$

(c).

on remplace (a), (b) et (c) dans (*)

$$T^{0\nu} = \int \frac{dk}{(2\pi)^3 2\omega_k} \int \frac{dk'}{(2\pi)^3 (2\omega_{k'})} \Big\{ \omega_k k'^\nu a(k) a^+(k') e^{-i(k-k')x}$$

$$+ \omega_k k'^\nu a^+(k) a(k') e^{i(k-k')x} - \omega_k k'^\nu a(k) a(k') e^{-i(k+k')x}$$

$$- \omega_k k'^\nu a^+(k) a^+(k') e^{i(k+k')x} + \frac{1}{2} k_\alpha k'^\alpha a(k) a(k') e^{-i(k+k')x} g^{0\nu}$$

$$+ \frac{1}{2} g^{0\nu} k_\alpha k'^\alpha a^+(k) a^+(k') e^{i(k+k')x} - \frac{1}{2} a(k) a^+(k') e^{-i(k-k')x} g^{0\nu} k_\alpha k'^\alpha$$

$$- \frac{1}{2} a^+(k) a(k') e^{i(k-k')x} g^{0\nu} k_\alpha k'^\alpha + \frac{1}{2} g^{0\nu} m^2 a(k) a(k') e^{-i(k+k')x}$$

$$+ \frac{1}{2} g^{0\nu} m^2 a^+(k) a(k') e^{i(k-k')x} + \frac{1}{2} g^{0\nu} m^2 a(k) a^+(k') e^{-i(k-k')x}$$

$$+ \frac{m^2}{2} g^{0\nu} a^+(k) a^+(k') e^{i(k+k')x} \Big\}.$$

$$T^{0\nu} = \frac{1}{2} \int \frac{dk}{(2\pi)^3 2\omega_k} \int \frac{dk'}{(2\pi)^3 2\omega_{k'}} \Big\{ \left[2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha + g^{0\nu} m^2 \right] a(k) a^+(k') e^{-i(k-k')x}$$

$$+ \left[2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha + m^2 g^{0\nu} \right] a^+(k) a(k') e^{i(k-k')x}$$

$$- \left[2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2 \right] a(k) a(k') e^{-i(k+k')x}$$

$$- \left[2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2 \right] a^+(k) a^+(k') e^{i(k+k')x} \Big\}.$$

Nous avons : $\displaystyle\int \frac{d^3x}{(2\pi)^3} e^{i(k-k')x} = \delta(\vec{k}-\vec{k}')\, e^{i(\omega_{k'}-\omega_k)t}.$

Scanné avec CamScanner

Alors,

$$P^\nu = \int d^3x\, T^{0\nu} = \frac{1}{2}\int \frac{dk}{(2\pi)^3 2\omega_k}\int \frac{dk'}{2\omega_{k'}}$$

$$\Big\{ [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha + g^{0\nu} m^2]\, a(k)a^+(k') \int \frac{d^3x}{(2\pi)^3}\, e^{-i(k-k')x}$$

$$+ [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha + g^{0\nu} m^2]\, a^+(k)\, a(k') \int \frac{d^3x}{(2\pi)^3}\, e^{i(k-k')x}$$

$$- [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2]\, a(k)a(k') \int \frac{d^3x}{(2\pi)^3}\, e^{-i(k+k')x}$$

$$- [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2]\, a^+(k)a^+(k') \int \frac{d^3x}{(2\pi)^3}\, e^{i(k+k')x}$$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3 2\omega_k}\int \frac{dk'}{2\omega_{k'}} \Big\{ [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha + g^{0\nu} m^2]\, a(k)a^+(k')\, \delta(\vec{k}-\vec{k}')\, e^{i(\omega_k - \omega_{k'})t}$$

$$+ [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha + g^{0\nu} m^2]\, a^+(k)a(k')\, \delta(\vec{k}-\vec{k}')\, e^{i(\omega_{k'}-\omega_k)t}$$

$$- [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2]\, a(k)a(k')\, \delta(\vec{k}+\vec{k}')\, e^{i(\omega_k+\omega_{k'})t}$$

$$- [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2]\, a^+(k)a^+(k')\, \delta(\vec{k}+\vec{k}')\, e^{i(\omega_k+\omega_{k'})t} \Big\}.$$

$$- [2\omega_k k'^\nu - g^{0\nu} k_\alpha k'^\alpha - g^{0\nu} m^2]$$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3 (2\omega_k)}\, \frac{1}{2\omega_k} \Big\{ g^{0\nu}[2\omega_k k_0 - k_\alpha k^\alpha + m^2]_{\vec{k}=\vec{k}'}\, a(k)a^+(k)$$

$$+ g^{0\nu}[2\omega_k k_0 - k_\alpha k^\alpha + m^2]_{\vec{k}=\vec{k}'}\, a^+(k)a(k)$$

$$- g^{0\nu}[2\omega_k k_0 + k_\alpha k^\alpha - m^2]_{\vec{k}=\vec{k}'}\, a(k)a(k)\, e^{2i\omega_k t}$$

$$- g^{0\nu}[2\omega_k k_0 + k_\alpha k^\alpha - m^2]_{\vec{k}=-\vec{k}'}\, a^+(k)a^+(k)\, e^{2i\omega_k t} \Big\}$$

$$\begin{cases} [2\omega_k^2 - k^2 + m^2]_{\vec{k}=\vec{k}'} = 2\omega_k k_0 \\ [2\omega_k^2 + k^2 - m^2]_{\vec{k}=-\vec{k}'} = 0. \end{cases}$$

$k_0 = \omega_k$

$k_\alpha k^\alpha = k^2 = m^2$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3 2\omega_k}\frac{1}{\cancel{2\omega_k}}\left\{ 2g^{0\nu}\cancel{\omega_k}k^0\, a(k)a^+(k) + 2g^{0\nu}\omega_k k^0\, a^+(k)a(k)\right\}.$$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3(2\omega_k)} K^\nu \left\{ a(k)a^+(k) + a^+(k)a(k)\right\}.$$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3(2\omega_k)} k^\nu \left\{ a(k)a^+(k) - a^+(k)a(k) + 2a^+(k)a(k)\right\}$$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3(2\omega_k)} k^\nu \left\{ 2a^+(k)a(k) + [a(k), a^+(k)]\right\}.$$

Nous avons :

$$[a(k), a^+(k')] = (2\pi)^3 2\omega_k\, \delta(k-k')$$

alors :

$$[a(k), a^+(k)] = (2\pi)^3 2\omega_k\, \delta(\vec{0}).$$

$$P^\nu = \frac{1}{2}\int \frac{dk}{(2\pi)^3(2\omega_k)} k^\nu \left\{ 2a^+(k)a(k) + (2\pi)^3 2\omega_k\, \delta(\vec{0})\right\}.$$

$$P^\nu = \int \frac{dk\; k^\nu}{(2\pi)^3(2\omega_k)} a^+(k)a(k) + \frac{1}{2}\int dk\, k^\nu\, \delta(\vec{0}).$$

le deuxième terme est infini, on doit l'éliminer en utilisant la notion du produit normale.

$$P^\nu = :\int \frac{dk\quad k^\nu}{(2\pi)^3(2\omega_k)} a^+(k)a(k): = \int dk\, k^\nu N(k)$$

$$P^i = \int d^3k \; k^i \left[N_a(k) + N_b(k) \right]$$

on a :

$$\begin{cases} [P^i, a(k)] = -k^i a(k). \\ [P^i, b(k)] = -k^i b(k). \\ [P^i, a^+(k)] = k^i a^+(k) \\ [P^i, b^+(k)] = k^i b(k) \end{cases}$$

Démonstration :

$$[P^i, a(k)] = \int d^3q \; q^i \left[N_a(q) + N_b(q), \; a(k) \right].$$

$$= \int d^3q \, q^i \left\{ [N_a(q), a(k)] + [N_b(q), a(k)] \right\}$$

$$= \int \frac{d^3q \, q^i}{(2\pi)^3 2\omega_q} \left\{ [a^+(q) a(q), a(k)] + [b^+(q) b(q), a(k)] \right\}$$

$$= \int \frac{d^3q \, q^i}{(2\pi)^3 2\omega_q} \left\{ a^+(q) \underbrace{[a(q), a(k)]}_{=0} + [a^+(q), a(k)] a(q) + b^+(q) \underbrace{[b(q), a(k)]}_{=0} + \underbrace{[b^+(q), a(k)]}_{=0} b(q) \right\}.$$

$$= \int \frac{d^3q \, q^i}{(2\pi)^3 2\omega_q} \left(- [a(k), a^+(q)] \, a(q) \right).$$

$$= - \int \frac{d^3q}{(2\pi)^3 2\omega_q} \, q^i \, (2\pi)^3 \, 2\omega_k \, \delta(\vec{k} - \vec{q}) \, a(q).$$

$$[P^i, a(k)] = -k^i \, a(k)$$

$$[P^i, b(k)] = \int d^3q \quad q^i \{[N_a(q) + N_b(q) , b(k)]$$

$$= \int \frac{d^3q}{(2\pi)^3 2\omega_q} q^i \left\{ [a^+(q)a(q), b(k)] + [b^+(q)\, b(q), b(k)] \right\}$$

$$= \int \frac{d^3q}{(2\pi)^3 2\omega_q} q^i \{ a^+(q) \overbrace{[a(q), b(k)]}^{=0} + \overbrace{[a^+(q), b(k)]}^{=0} a(q)$$

$$+ b^+(q) \underbrace{[b(q), b(k)]}_{=0} + [b^+(q), b(k)] b(q) \}.$$

$$= - \int \frac{d^3q \; q^i}{(2\pi)^3 2\omega_q} [b(k), b^+(q)] \, b(q).$$

$$= - \int \frac{d^3q \; q^i}{(2\pi)^3 2\omega_q} (2\pi)^3 2\omega_k \, \delta(\vec{k} - \vec{q}\,) b(q).$$

$$[P^i, b(k)] = - k^i \, b(k)$$

$$[P^i, a^+(k)] = \int d^3q \; q^i \{ [N_a(q) + N_b(q) , a^+(k)]$$

$$= \int \frac{d^3q \; q^i}{(2\pi)^3 2\omega_q} \{ [a^+(q)a(q), a^+(k)] + [b^+(q)\, b(q), a^+(k)$$

$$= \int \frac{d^3q}{(2\pi)^3} \frac{q^i}{2\omega_q} \{ a^+(q) [a(q), a^+(k)] + \overbrace{[a^+(q), a^+(k)]}^{=0} a(q$$

$$+ b^+(q) \underbrace{[b(q), a^+(k)]}_{=0} + \underbrace{[b^+(q), a^+(k)]}_{=0} b($$

$$= \int \frac{d^3q \quad q^i}{(2\pi)^3 2\omega_q} a^+(q) (2\pi)^3 2\omega_k \, \delta(\vec{k} - \vec{q}\,)$$

$$[P^i, a^+(k)] = k^i \; a^+(k)$$

$$[P^i, b^+(k)] = \int d^3q\, q^i \{ [N_a(q) + N_b(q), b^+(k)] \}$$

$$= \int \frac{d^3q\, q^i}{(2\pi)^3 2\omega_q} \{ [a^+(q)a(q), b^+(k)] + [b^+(q)b(q), b^+(k)] \}$$

$$= \int \frac{d^3q\, q^i}{(2\pi)^3 2\omega_q} \{ a^+(q) \underbrace{[a(q), b^+(k)]}_{=0} + \underbrace{[a^+(q), b^+(k)]}_{=0} a(q)$$

$$+ b^+(q)[b(q), b^+(k)] + \underbrace{[b^+(q), b^+(k)]}_{=0} b(q) \}.$$

$$= \int \frac{d^3q\, q^i}{(2\pi)^3 2\omega_q} \; b^+(q) (2\pi)^3 2\omega_q \; \delta(k - q).$$

$$\boxed{[P^i, b^+(k)] = k^i\, b^+(k)}$$

$$[P^i, \varphi(x)] = -i\partial^i \varphi(x)$$

$H \longrightarrow :H:$ $\qquad$ $[H, \varphi(x)] = -i\partial^0 \varphi(x)$

$H = \int d^3x\, \omega_k [N_a(k) + N_b(k)]$

L'espace de Fock se construit à partir de $|0\rangle$ défini par :

$P^i|0\rangle = H|0\rangle = 0$. Impulsion-énergie de $|0\rangle$ nulle.

$N_a|0\rangle = N_b|0\rangle$ pas de particules :

En agissant par : les $a^+(k)$ et $b^+(k)$ un état à n particules

et m antiparticules. $a^+(k_1) \ldots a^+(k_n) b^+(q_1) \ldots b^+(q_m)|0\rangle$.

C'est un état totalement symétrique et ses particules vérifient la statistique de Bose-Einstein.

Démonstration :

$$[P_i , \varphi(x)] = -i\, \partial_i \varphi(x) .$$

on a : $P_i = -i\, \partial_i$.

$$[-i\, \partial_i , \varphi(x)] K(x) = (-i\, \partial_i \varphi(x)) K(x) + i\, \varphi(x) (\partial_i K(x))$$

$$= (-i\, \partial_i \varphi(x)) K(x) \cancel{- i\, \varphi(x)\, \partial_i K(x)}$$

$$\cancel{+ i\, \varphi(x)\, \partial_i K(x)}$$

$$[P_i , \varphi(x)] K(x) = -i (\partial_i \varphi(x)) K(x) \quad \forall K(x)$$

$$\Rightarrow \quad \boxed{[P_i , \varphi(x)] = -i\, \partial_i \varphi(x)}$$