2. CALCULATING THE PERFORMANCE OF HEAT EXCHANGERS

2.1. Assumptions

In all the calculations discussed, the following assumptions are made:

- Steady state: all coefficients and variables are constant over time.
- No phase changes during heat transfer for the fluids used.
- No heat loss: The heat exchange surface is the surface separating the two fluids.
- Temperatures are one-dimensional and vary only in one direction.

2.2. Operating modes

We consider tubular heat exchangers, formed by two concentric tubes, through which flow the two fluids, hot and cold, respectively defined by the following quantities:

- T_{hin} and T_{cin}: Input temperatures [K].
- T_{hout} and T_{cout}: Output temperatures [K].
- $\dot{m}_h$ et $\dot{m}_c$: Mass flow rates [kg / S].

Two types of flow are possible:

- Parallel or co-current flow.
- Opposite or counter-current flow.

2.3. Logarithmic mean temperature difference method LMTD

2.3.1. The parallel flow heat exchangers

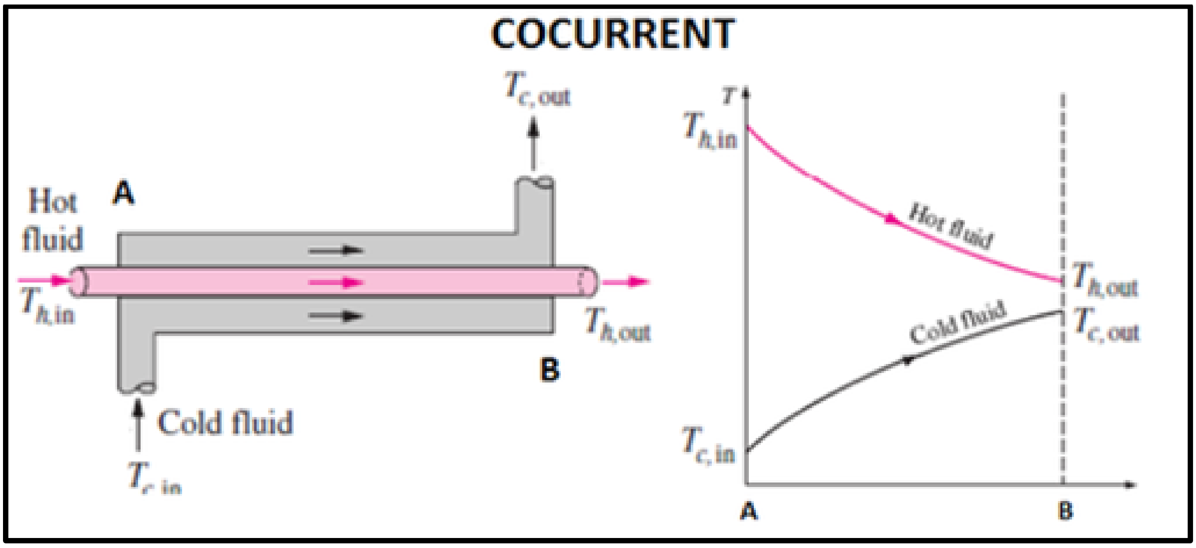

Figure 2.1: Temperature distribution - Parallel flow heat exchanger

The heat flux exchanged between the two fluids through surface element ds can be written as follows:

$$d\Phi = k\,(T_h - T_c)\,ds \quad \text{et} \quad d\Phi = -\,\dot{m}_h C_{ph}\,dT_h = \dot{m}_c C_{pc}\,dT_c$$

$$d\Phi = k\,(T_h - T_c)\,ds \quad \text{et} \quad d\Phi = -\,C_h\,dT_h = C_c\,dT_c$$

$$dT_h - dT_c = d(T_h - T_c) = -\,[(1/\dot{m}_h C_{ph}) + (1/\dot{m}_c C_{pc})]\,d\Phi \qquad (1)$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) kS = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) kS \qquad (2)$$

By integrating, we obtain:

$$\log\frac{(T_{h\,out} - T_{c\,out})}{(T_{h\,in} - T_{c\,in})} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) kS \qquad (3)$$

However, the total flow exchanged can also be expressed as a function of the input and output temperatures of the fluids:

$$\Phi = k\,\frac{(T_{h\,out} - T_{c\,out}) - (T_{h\,in} - T_{c\,in})}{\log\frac{(T_{h\,out} - T_{c\,out})}{(T_{h\,in} - T_{c\,in})}}\,S \qquad (4)$$

2.3.2. The Counter-current flow heat exchangers

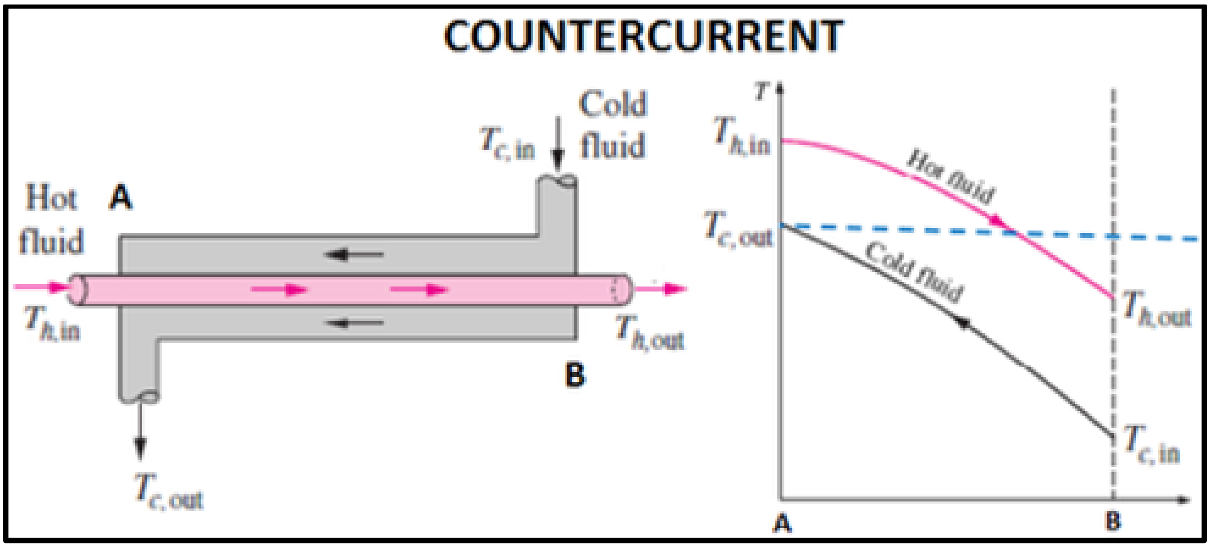

Figure 2.2: Temperature distribution – Counter current flow heat exchanger

$d\Phi = - \dot{m}_h C_{ph} dT_h = - \dot{m}_c C_{pc} dT_c$ $\quad\quad$ $d\Phi = - C_h dT_h = - C_c dT_c$ (5)

$$\Phi = k [(T_{h\,in} - T_{c\,out}) / (T_{h\,out} - T_{c\,in})] / Log [(T_{h\,in} - T_{c\,out}) / (T_{h\,out} - T_{c\,in})] S \quad (6)$$

2.3.3. Generalisation

$$\Phi = k\, S\, \frac{\Delta T_2 - \Delta T_1}{Log \frac{\Delta T_2}{\Delta T_1}} \quad (7)$$

- Parallel flow heat exchanger:

$$\Delta T_1 = (T_{hin} - T_{cin}) \quad\quad \Delta T_2 = (T_{hout} - T_{cout})$$

- Counter current flow heat exchanger:

$$\Delta T_1 = (T_{hin} - T_{cout}) \quad \Delta T_2 = (T_{hout} - T_{cin})$$

The heat output of a tubular heat exchanger is given by the following general relationship:

$$\Phi = k\, S\, \Delta T_{LMTD} \quad (8)$$

$$\Delta T_{LMTD} = \frac{[(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})]}{Log\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]} \quad (9)$$

LMTD is called the logarithmic mean temperature difference between the two fluids.

The above expression means that the thermal power exchanged is proportional to the exchange surface and the logarithmic mean temperature difference.

The proportionality coefficient is the overall (global) exchange coefficient k.

2.3.4. Global heat transfer coefficient

It is also necessary to know the global exchange coefficient defined by the relationship:

$$\mathbf{k} = \frac{1}{\frac{1}{h_h} + \frac{e}{\lambda} + \frac{1}{h_c}}$$

The convection in the hot fluid is governed by a convection coefficient $\mathbf{h_h}$, which defines a convective thermal resistance: $\mathbf{1/ h_h S}$

The convection in the cold fluid is governed by a convection coefficient $\mathbf{h_c}$, which defines a convective thermal resistance: $\mathbf{1/\ h_c\ S}$

The conduction through a solid wall of thickness **e** and thermal conductivity λ is given by the conductive thermal resistance $\mathbf{e/\lambda S}$.

To correctly account for the phenomena in a real heat exchanger. Exchange surfaces $\mathbf{S_h}$ and $\mathbf{S_c}$ must be introduced, and the global transfer coefficient must be related either to the unit of exchange surface on the hot side $\mathbf{k_h}$, or to the unit of exchange surface on the cold side $\mathbf{k_c}$.

In addition, after a certain period of operation, the heat exchanger walls become covered with a film of dirt. These deposits of scale and dirt have a low thermal conductivity compared with that of metal, and therefore constitute additional $\mathbf{R_{eh}}$ and $\mathbf{R_{ec}}$ thermal resistances that resist heat exchange.

Thus:

$$\mathbf{k}_h = \frac{1}{\frac{1}{h_h} + Re_h + \frac{e}{\lambda}\frac{S_h}{S_m} + \left(R_{ec} + \frac{1}{h_c}\right)\frac{S_h}{S_c}}$$

$$\mathbf{k}_c = \frac{1}{\frac{1}{h_c} + R_{ec} + \frac{e}{\lambda}\frac{S_c}{S_m} + \left(R_{eh} + \frac{1}{h_h}\right)\frac{S_c}{S_h}}$$

$\mathbf{S_c}$ is the cold-side heat exchange surface [m^2].

$\mathbf{S_h}$ is the hot-side heat exchange surface [m^2].

$\mathbf{S_m}$ is the mean exchange surface [m^2].

$\mathbf{R_{eh}}$ et $\mathbf{R_{ec}}$ are the resistances per unit area of the dirt films deposited on the hot and cold sides of the exchange surface [(m^2.°C)/W].

$\mathbf{k_h}$ et $\mathbf{k_c}$ are expressed in W/(m^2.°C)

Table 2.1: Order of magnitude values of global heat-transfer coefficients *K*

Configuration	Typical value *K* [W/(m^2·K)]	Typical range *K* [W/(m^2·K)]
Gas-to-gas heat exchanger at normal pressure	20	5..50
Gas-to-gas heat exchanger at high pressure	200	50..500
Liquid-to-gas or gas-to-liquid heat exchanger	50	10..100
Liquid-to-liquid tubular heat exchanger	1000	200..2000
Liquid-to-liquid plate heat exchanger	2500	500..5000
Condenser, to a gas	50	10..100
Condenser, to a liquid	3000	500..6000
Vaporiser, to a gas	50	10..100

Vaporiser, to a liquid	5000	500..10 000
Vaporiser, to a condensing gas	3000	600..6000

2.3.5. Fouling resistances

Fouling resistance values are derived from comparative measurements between commissioning conditions and operation over time.

The range of variation is: 1.10^{-4} and 70.10^{-4} [m^2 .°C/W].

Table 2.2 : Some fouling resistance values

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

(*Source:* Tubular Exchange Manufacturers Association.)

Fluid	R_f, $m^2 \cdot$ °C/W
Distilled water, sea-water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

2.4. Number of transfer unit method:

When the inlet or outlet temperatures of the fluid streams are not known, a trial-and-error procedure could be applied for using the LMTD method in the thermal analysis of heat exchangers. To avoid a trial-and-error procedure, the method of the number of transfer units (NTU) based on the concept of heat exchanger effectiveness may be used.

2.4.1. Definition of heat exchanger efficiency

The efficiency of a heat exchanger is the ratio of the thermal power actually exchanged to the maximum exchange power theoretically possible, with the same fluid inlet conditions in the exchanger.

$$\varepsilon = \frac{\Phi_{real}}{\Phi_{max}}$$

- $C_c > C_h$: The hot fluid controls the transfer:

$$\Phi_{real} = C_h(T_{hin} - T_{hout}) = C_c(T_{cout} - T_{cin})$$

$$\Phi_{max} = C_h(T_{hin} - T_{cin})$$

So the Cooling efficiency is given by:

$$\varepsilon \frac{(T_{hin} - T_{hout})}{(T_{hin} - T_{cin})}$$

- $C_c < C_h$ The cold fluid controls the transfer:

$$\Phi_{real} = C_h(T_{hin} - T_{hout}) = C_c(T_{cout} - T_{cin})$$

$$\Phi_{max} = C_c(T_{hin} - T_{cin})$$

So the Heating efficiency is given by:

$$\varepsilon = \frac{(T_{cout} - T_{cin})}{(T_{hin} - T_{cin})}$$

2.4.2. Efficiency calculation

- **The parallel flow heat exchanger (co current)** $C_c > C_h$

$$\varepsilon = \frac{1 - exp\left(-\left(\frac{1}{C_c} + \frac{1}{C_h}\right) kS\right)}{\left(1 + \frac{C_h}{C_c}\right)} \qquad S \to \infty \qquad \varepsilon = \frac{C_c}{(C_h + C_c)}$$

- **The Counter-current flow heat exchanger** $C_c > C_h$

$$\varepsilon = \frac{1 - exp\left(-\left(\frac{1}{C_c} + \frac{1}{C_h}\right) kS\right)}{\left(1 - \frac{C_h}{C_c}\left(exp\left(-\left(\frac{1}{C_c} + \frac{1}{C_h}\right) kS\right)\right)\right)} \qquad S \to \infty \qquad \varepsilon = 1$$

2.5. NTU approach

The number of transfer units, NTU, is the dimensionless ratio:

$$NTU = \frac{k\,S}{C_{min}}$$

The NTU represents the exchanger's capacity.

Let's consider the case of a simple tubular exchanger operating as counter-current and let's assume that the hot fluid controls the transfer: $C_c > C_h$ ($C_{min}=C_h$):

$$\varepsilon = \frac{(T_{hin} - T_{hout})}{(T_{hin} - T_{cin})}$$

Let's put: $Z = \frac{C_h}{C_c} < 1$ and $\Delta T_{max} = T_{hin} - T_{hout}$

$$NTU = \frac{1}{1-Z}\; Log\left[\frac{(1 - Z\varepsilon)}{(1-\varepsilon)}\right]$$

Co current	Counter current
$NTU_{max} = -\frac{Log[1-(1+Z)\varepsilon]}{1+Z}$	$NTU_{max} = \frac{1}{Z-1}\; Log\left[\frac{(\varepsilon - 1)}{(Z\varepsilon - 1)}\right]$
$\varepsilon = \frac{1 - exp[-NTU_{max}(1+Z)]}{1+Z}$	$\varepsilon = \frac{1 - exp[-NTU_{max}(1-Z)]}{1 - Z exp[-NTU_{max}(1-Z)]}$

$$NTU_{max} = \frac{kS}{C_{min}} \qquad Z = \frac{C_{min}}{C_{max}}$$

- **Particular cases:**
  - **All types of exchangers: If Z=0**

$$\varepsilon = 1 - exp[-NTU_{max}] \quad NTU_{max} = -\,Log[1-\varepsilon]$$

  - **For counter current exchanger: If Z=1**

$$\varepsilon = \frac{NTU_{max}}{NTU_{max}+1} \quad NTU_{max} = \frac{\varepsilon}{1-\varepsilon}$$

2.6. Calculating a heat exchange

2.6.1. 1st Case: Known output temperatures

Once the overall transfer coefficient k has been calculated, one of the two methods can be used to calculate S.

- **LMTD Method**:

✓ Calculate the flux: $\boldsymbol{\phi = \dot{m}_h\, C_{ph}(T_{hin} - T_{hout}) = \dot{m}_c\, C_{pc}(T_{cout} - T_{cin})}$

✓ Calculate LMTD : $\boldsymbol{\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{Log\frac{\Delta T_2}{\Delta T_1}}}$

✓ Deduce : $\boldsymbol{S = \frac{\phi}{k\,.\Delta T_{LMTD}}}$

- **NTU Method:**

✓ Calculate ε and $\boldsymbol{Z = \frac{C_{min}}{C_{max}}}$

✓ Calculate **NTU**$_{\mathbf{max}}$:

$$\boldsymbol{NTU_{max} = -\frac{Log[1-(1+Z)\varepsilon]}{1+Z}} \qquad \boldsymbol{NTU_{max} = \frac{1}{Z-1}\, Log\left[\frac{(\varepsilon-1)}{(Z\varepsilon-1)}\right]}$$

✓ Deduce : $\boldsymbol{S = NTU_{max}\left(\frac{C_{min}}{k}\right)}$

2.6.2. 2nd Case: Unknown output temperatures

- **LMTD method**:

The application of the LMTD method requires the numerical solution of the equations:

$$\begin{cases} \boldsymbol{\dot{m}_h\, C_{ph}(T_{hin} - T_{hout}) = k\;\; S\;\; \Delta T_{LMTD}} \\ \boldsymbol{\dot{m}_h\, C_{ph}(T_{hin} - T_{hout}) = \dot{m}_c\, C_{pc}(T_{cout} - T_{cin})} \end{cases}$$

- **NTU method**:

✓ Calculate $\boldsymbol{NTU_{max} = \frac{kS}{C_{min}}}$ and $\boldsymbol{Z = \frac{C_{min}}{C_{max}}}$

✓ Calculate ε :

$$\boldsymbol{\varepsilon = \frac{1-exp[-NTU_{max}(1+Z)]}{1+Z}} \qquad \boldsymbol{\varepsilon = \frac{1-exp[-NTU_{max}(1-Z)]}{1-Zexp[-NTU_{max}(1-Z)]}}$$

✓ Deduce one of the unknown output temperatures $\mathbf{T_{hout}}$ or $\mathbf{T_{cout}}$

✓ Determine the second unknown output temperature by:

$$\boldsymbol{\dot{m}_h\, C_{ph}(T_{hin} - T_{hout}) = \dot{m}_c\, C_{pc}(T_{cout} - T_{cin})}$$

In this case, this method is the preferred choice.