Tutorials 01:
Formal Systems

Exercise 01

Write algorithms (iterative then recursive) to solve the following problems:

- Calculate the power a^n (a is real and n is an integer).
- Find the largest element in an array.
- Calculate the scalar product of two vectors A and B (of the same size n).
- Select a sub-list from a list by giving a start and end index.
- Insert a given integer into an ordered list to produce a new ordered list.

Exercise 02

Let be the following formal system:

- Alphabet = {p, q, –}
- Well-formed formulas = {–* p –* q –*}
- Axioms = {pq}
- Rules :
  - x → – x–
  - xpy → xp–y–

Are the following formulas theorems?

- – – p – q – – –
- – p – – q –
- – p – – q – – –
- – – – – – – p – – – q – – – – – – – – – –

Exercise 03

Let be the following formal system:

- Alphabet = {a, b, c}
- Well-formed formulas = $\{a^n bc^m / n,m \geq 0\}$
- Axioms = $\{a^{2i} bc^{2i} / i \geq 0\}$
- Rules (only one rule):

$$a^n bc^m \ , a^{n'} bc^{m'} \rightarrow a^{n+n'} bc^m$$

Are the following formulas theorems?

- a^2bc^2
- a^6bc^2
- $a^{10}b$
- a^4bc^6

Define the set of theorems of this formal system.

Exercise 04

Define a formal system whose theorems are all strings composed of a and b that contain only one b.