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Department of Mathematics Asymptotic Analysis
Master 2 : Functional Analysis 2025 November

Tutorial number 2

Exercice 01 :

Obtain the asymptotic expansion of the given sequence to the requested precision.

- ⋆ $u_n = \ln(n+1)$ with respect to $\delta_n = \frac{1}{n^2}$.
- ⋆ $u_n = \sqrt{n+1} - \sqrt{n-1}$ with respect to $\delta_n = \frac{1}{n^2}$
- ⋆ $u_n = \sqrt{n+\sqrt{n}} - \sqrt{n}$ with respect to $\delta_n = frac1n$
- ⋆ $u_n = \left(1+\frac{1}{n}\right)^n$ with respect to $\delta_n = \frac{1}{n^2}$

Exercice 02 : Let $n \geq 1$

1. Show that the equation $\tan x = x$ has a unique solution x_n in the interval $]n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}[$.
2. What relation connects x_n and $\arctan(x_n)$?
3. Show that $x_n = n\pi + \frac{\pi}{2} + o(1)$.
4. By writing $x_n = n\pi + \frac{\pi}{2} + \varepsilon_n$ and using the result of Question 2, deduce that

$$x_n = n\pi + \frac{\pi}{2} - \frac{1}{n\pi} + \frac{1}{2n^2\pi} + o\left(\frac{1}{n^2}\right).$$

Exercice 03 :

1. Let $f : [0,1] \to \mathbb{R}$ be a $\mathcal{C}^\infty$ function. Determine an asymptotic expansion, up to the precision n^{-r} (where $r \in \mathbb{N}$), of the sequence $(u_n)_{n\in\mathbb{N}}$ defined by

$$\forall n \in \mathbb{N}, \quad u_n = \int_0^1 t^n f(t)\,\mathrm{d}t.$$

2. Answer the same question for the sequence $(u_n)_{n\in\mathbb{N}}$ defined by

$$\forall n \in \mathbb{N}^*, \quad u_n = \cosh(n)^{1/n}.$$

Exercice 04 :

1. Compute the first two nonzero terms of the asymptotic expansion of the function

$$f(x) = \frac{1}{e^x - 1} - \frac{1}{x},$$

with respect to the sequence $\delta_n(x) = x^n$ in the neighborhood of $x = 0$.

2. Answer the same question for the function

$$g(x) = \int_x^{+\infty} t^{-a} e^{-t}\,\mathrm{d}t, \quad a > 0,\ x > 0,$$

with respect to the sequence $\delta_n(x) = x^{-n}$ in the neighborhood of $x = +\infty$.

Exercice 05 : Exercice 06(page2)