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Master 2 : Functional Analysis

Asymptotic Analysis

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Tutorial number 3

Exercise 01 :

Determine an asymptotic expansion for the following integrals when $x \to \infty$

$$I(x) = \int_0^{10} \frac{e^{-xt}}{\sqrt{1+t}} dt$$

$$J(x) = \int_0^{\infty} \frac{e^{-xt}}{\sqrt{1+t^2}} dt$$

$$K(x) = \int_0^{\frac{\pi}{2}} e^{-x\sin^2 t} dt$$

$$L(x) = \int_0^{\infty} \ln(1+t^2) e^{-xt} dt$$

$$M(x) = \int_0^{\infty} e^{-x\sinh^2 t} dt$$

Exercise 02 : Stirling's formula gives an asymptotic equivalent of the factorial as n tends towards infinity :

$$\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = 1$$

This formula is written in the following form :

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Recall that the factorial is a special case of the Euler function Γ, when the argument n is an integer. To do this, find an estimate of $\Gamma(x+1)$ for x large.

$$\Gamma(x+1) = \int_0^{+\infty} e^{-t} t^x \, dt$$

Exercise 03 :

1. Consider the modified Bessel function of the second kind

$$K_\nu(x) = \int_0^\infty e^{-x\cosh t}\cosh(\nu t)\mathbf{d}t$$

Determine an asymptotic expansion of the Bessel function for $x \to \infty$.

2. Same question for the Bessel function of order m, $m \in \mathbb{N}$ defined by :

$$J_m(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{ix\sin t}e^{-imt}\mathbf{d}t$$

3. Same question for the Airy function defined by :

$$A_i(x) = \int_{\mathbb{R}} e^{ixt+i\frac{t^3}{3}}\mathbf{d}t$$

Exercise 04 :

1. Determine an asymptotic expansion for the following integrals as the real parameter x tends towards $+\infty$.

$$I(x) := \int_{\mathbb{R}} \frac{e^{ixt}}{1+t^2}dt$$

$$J(x) := \int_0^\infty \cos(xt^2 - t)dt.$$

2. Show that

$$\int_{-1}^\infty \frac{e^{it}}{(t^3+3t-2i)^x}dt \sim \frac{2e i^x\sqrt{\pi}}{4^x\sqrt{3x}} \qquad (x \to \infty)$$

3. Let $\alpha \in]0,\infty[$.Determine an asymptotic expansion for

$$F_\alpha(x) := \int_{\mathbb{R}} \frac{e^{t-xt^2}}{1+x^\alpha t^2}dt.$$

We will distinguish the cases $\alpha < 1$, $\alpha = 1$ and $\alpha > 1$.