

Semestre : 3
Unité d'enseignement : UEF
Matière : Transfert de chaleur et de masse
VHS: 45h (cours: 1h30, TD: 1h30)
Crédits : 3
Coefficient : 1

Objectifs de l'enseignement:

Cette matière comporte deux parties ; permet à l'étudiant d'apprendre et d'assimiler les différents modes de transfert de chaleur et les lois qui les gouvernent, la seconde partie traite et explique le phénomène de diffusion et donne les lois qui le gouvernent.

Connaissances préalables recommandées:

Mathématiques

Contenu de la matière :

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Chapitre 1. Généralité sur les transferts de chaleur. (01 semaine)

Introduction. Définitions ; chaleur, champs de température, gradient de température, flux.

Chapitre 2. Transferts de chaleur par conduction en régime permanent (01 semaine)

L'équation de la Chaleur. Transfert de chaleur unidirectionnel. Transfert de chaleur multidirectionnel.

Chapitre 3. Transferts de chaleur par conduction en régime variable (02 semaines)

Conduction unidirectionnelle en régime variable. Conduction multidirectionnelle en régime variable.

Chapitre 4. Transferts de chaleur par convection (02 semaines)

Rappels sur l'analyse dimensionnelle. Convection sans changement d'état. Convection avec changement d'état

Chapitre 5 Transferts de chaleur par rayonnement (01 semaine)

Lois du rayonnement. Rayonnement réciproque de plusieurs surfaces

PARTIE B : Transfert de masse

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1ere loi de Fick

2eme loi de Fick , Coefficient de diffusion

Chapitre 3 : Théorie phénoménologique de la diffusion *(01 semaine)*

Chapitre 4 : Diffusion dans les métaux et alliages en l'absence de gradients chimiques *(01 semaine)*

Chapitre 5 : La diffusion superficielle *(01 semaine)*

Chapitre 6 : Application de la diffusion *(02 semaine)*

Homogénéisation, cémentation, soudage et brasage, oxydation des métaux, frittage.

Mode d'évaluation : Contrôle continue 40% ; Examen 60%

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This subject is divided into two parts: the first enables the student to learn and assimilate the different modes of heat transfer and the laws that govern them, while the second deals with and explains the phenomenon of diffusion and the laws that govern it.

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[*Recommended prior knowledge*](#)

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[PART A: Heat transfer](#)

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(01 semaine)

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Introduction. Définitions ; chaleur, champs de température, gradient de température, flux.

Introduction. Definitions; heat, temperature fields, temperature gradient, flux.

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The heat equation. Unidirectional heat transfer. Multidirectional heat transfer.

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Chapter 3: Heat transfer by conduction in variable regime (02 weeks)

Conduction unidirectionnelle en régime variable. Conduction multidirectionnelle en régime variable.

Unidirectional conduction at variable regime. Multidirectional conduction variable regime .

Chapitre 4. Transferts de chaleur par convection (02 semaines)

Chapter 4. Convection Heat Transfer (01 week)

Rappels sur l'analyse dimensionnelle. Convection sans changement d'état. Convection avec changement d'état

Dimensional analysis reminders. Convection without phase change. Convection with phase change

Chapitre 5 Transferts de chaleur par rayonnement (01 semaine)

Chapter 5 Radiant heat transfer (01 week)

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Homogénéisation, cémentation, soudage et brasage, oxidation of metals, sentering .

Chapter 6: Application of diffusion (02 weeks)

Homogenization, Cementation, Welding and brazing (soudage et brasage), sentering

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MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

UNIVERSITY MOHAMED SEDDIK BENYAHIA OF JIJEL

FACULTÉ DES SCIENCES ET DE LA TECHNOLOGIE

PROCESS ENGINEERING DEPARTMENT



**Lecture Notes
on**

Heat and Mass Transfer

By **Nabil MAHAMADIOUA**

Professor at University Mohammed Seddik Benyahia of Jijel

This lecture notes on "Heat and mass transfer" was written for second year students as part of a professional master degree in Metallurgical Engineering in the Sciences and Technology field in the Process Engineering department of the faculty of Science and Technology from Mohamed Seddik Benyahia University of Jijel.

It is a course of the fundamental unit of semester 3 and whose semester hourly volume is 45 hours divided into 1h30 of lessons and 1h30 of tutorial per week, for 15 weeks.

General introduction and objectives

Heat transfer is a fundamental engineering topic that involves the movement of thermal energy caused by temperature variations. Conduction (through materials), convection (fluid motion), and radiation (electromagnetic waves) are the three primary modes. These principles are used by engineers to manage temperature, enhance energy efficiency, and maintain safety in a wide range of applications, from electronic equipment to industrial processes. Understanding heat transfer is critical for maximizing system performance, selecting materials, and managing thermal systems.

PART A :

HEAT TRANSFER

Chapter

CHAPTER 1: HEAT TRANSFER IN GENERAL

Content :

1. Introduction.

2. Definitions;

- a. heat,
- b. temperature
- c. fields,
- d. temperature gradient,
- e. flux.

I-1. Introduction.

Heat transfer is the **process of energy exchange** through temperature **differences**, typically occurring in three main ways: **conduction** (direct contact), **convection** (fluid movement), and **radiation** (electromagnetic waves).

I-2. Definitions

a. Thermal energy, heat and temperature

- In general, the meanings of the terms : **thermal energy**, **heat** and **temperature** are confused.
- They have very **distinct concepts**.
- They are physical quantities **closely linked** to **hot** and **cold** sensations.

i. Thermal energy

- It's the **energy** that a substance possesses as a result of the **agitation** of its **constituent** elements: atoms or molecules.
- It depends on :
 - o The **quantity** of constituent particles;
 - o The **degree** of their thermal agitation;
(Thermal **agitation** is the **extremely** rapid movement of nanoscale particles in all directions);
 - o As the **number** of particles increases, so does thermal energy.
 - o Similarly, as the thermal agitation of particles increases, so does the thermal energy;
 - o The SI unit of thermal energy is the **Joule** (J).

ii. Temperature ;

- Temperature is a **macroscopic** scalar thermodynamic quantity.
- On an atomic scale, it reflects **the degree of agitation of the particles** (atoms or molecules) making up the system;
- As agitation increases, so does temperature;
- Temperature **does not depend** on the quantity of agitated particles;
- That's why temperature is an **intensive** quantity;
- Temperature can be seen as the **measurable manifestation of stored thermal energy**;
- Temperature is generally denoted by **T**.
- The **unit** of temperature :
 - is the "Kelvin" in the International System (SI). It is denoted by K.
 - is also Celsius or Fahrenheit.

iii. Heat

- The meaning of **heat** is always **attached** to the **transfer of thermal energy**, which propagates from a **warmer** point of substance (i.e. at a higher temperature) to a **colder** one (i.e. at a lower temperature).
- Heat is the quantity of **thermal energy exchanged** between two bodies or two regions of a substance due to the difference in temperature. It is usually denoted by Q and measured in joules.

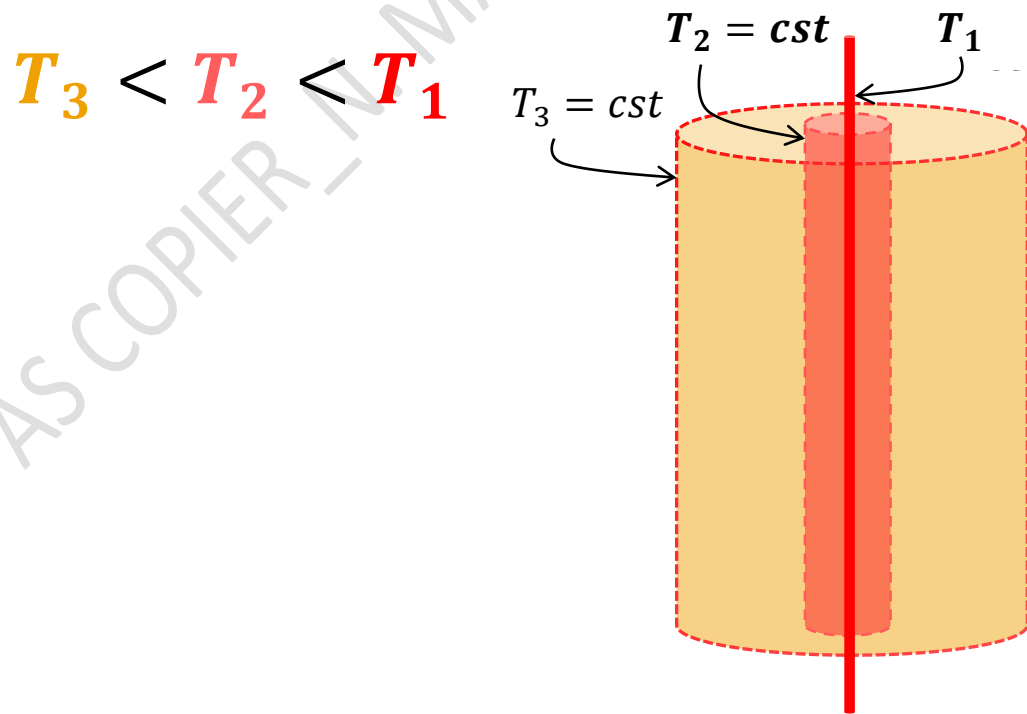
b. Temperature field

- Temperature fields **refer** to the spatial **distribution** of **temperatures** within a specific area or medium.
- They **describe** how temperatures **vary across** that **region**, often depicted in maps, graphs, or mathematical equations to represent the temperature distribution.
- Understanding temperature fields is **crucial** in **various fields**, such as physics, engineering, meteorology, and environmental science.
- The temperature field is the **set** of **temperature** values associated with points on a substance.
- In a **Cartesian** coordinate system, it is a function of x , y and z . It is therefore denoted $T=T(x,y,z)$. Since temperature generally changes with time, it is also a function of time (t). So $T=T(x,y,z,t)$.
- Depending on the dependence of T on t , we distinguish two **regimes**:
 - When the T field is independent of t : the regime is said to be **permanent** or **stationary**;
 - When the T field changes with time t : the regime is said to be **variable** or **unsteady**.

c. Isothermal surfaces and temperature gradients

i. Isothermal surfaces

- A surface is said to be **isothermal** if the **mean temperature** value is the **same at every point** on the surface.



ii. Temperature gradient

- Consider a temperature field that depends on a single variable x . It is then denoted : $T = T(x)$;
- Its derivative gives the magnitude of the change in temperature value with respect to a small change in x ;
- The temperature **gradient** is the **natural generalization** of this change to the rest of the **coordinates** of the same system: y and z ;

... $\overrightarrow{\text{grad}}(T)$

- If temperature is a **multivariate** function $T(x,y,z)$, then for small changes in these variables, the **gradient** characterizes the extent of temperature variability in the **vicinity** of a point M with **coordinates** (x,y,z) ;
- **Mathematically**, it's a **vector function**, called the gradient of T and denoted $\overrightarrow{\text{grad}}(T)$;

- Its expression is:

- In Cartesian coordinates ::

$$\overrightarrow{\text{grad}}(T) = \frac{\partial T(x, y, z)}{\partial x} \vec{i} + \frac{\partial T(x, y, z)}{\partial y} \vec{j} + \frac{\partial T(x, y, z)}{\partial z} \vec{k} \quad (1)$$

- In cylindrical coordinates:

$$\overrightarrow{\text{grad}}(T) = \frac{\partial T(r, \theta, z)}{\partial r} \overrightarrow{U_r} + \frac{1}{r} \frac{\partial T(r, \theta, z)}{\partial \theta} \overrightarrow{U_\theta} + \frac{\partial T(r, \theta, z)}{\partial z} \vec{k} \quad (2)$$

- In spherical coordinates:

$$\overrightarrow{\text{grad}}(T) = \frac{\partial T(r, \theta, \varphi)}{\partial r} \overrightarrow{U_r} + \frac{1}{r} \frac{\partial T(r, \theta, \varphi)}{\partial \theta} \overrightarrow{U_\theta} + \frac{1}{r \sin(\theta)} \frac{\partial T(r, \theta, \varphi)}{\partial \varphi} \overrightarrow{U_\varphi} \quad (3)$$

Note:

The **direction** of the vector $\vec{\text{grad}}(T)$ indicates the direction in which the change in T is "**fastest**".

d. Heat transfer

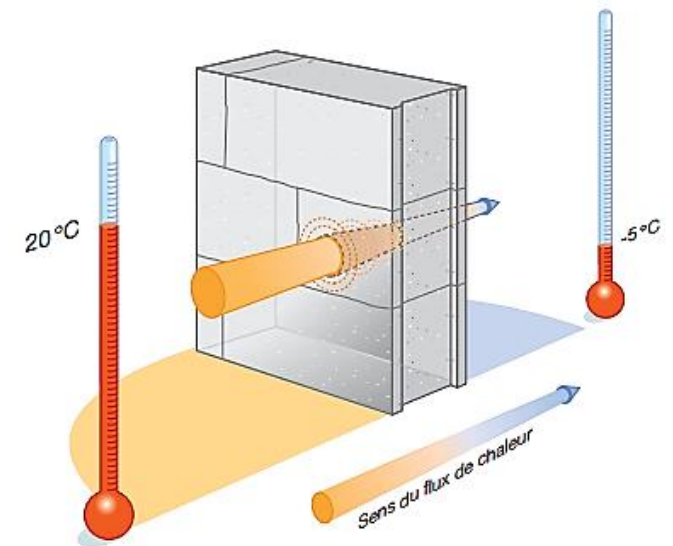
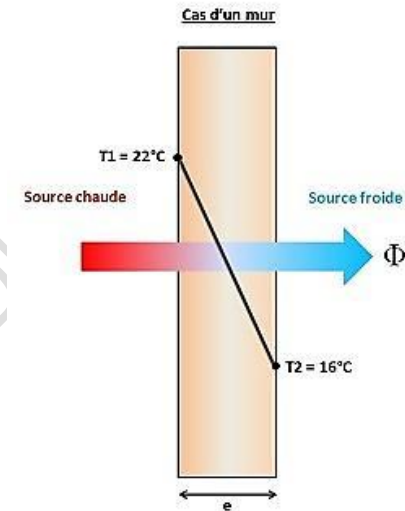
- Heat transfer is the transfer of thermal energy between two systems or between two regions within a substance.
- It sets up out of thermodynamic equilibrium from a higher-temperature region to a lower-temperature region.
- It is found in:
 - o industrial applications such as mechanical motors, electric motors and electronic components;
 - o domestic applications, such as air conditioning;
 - o nature, such as weather-related studies.
- There are four basic modes of heat transfer:
 - 1) **conduction** ;
 - 2) **convection** ;
 - 3) **radiation** ;
 - 4) **advection**.

e. Heat flux (thermal flux)

- Thermal flow, also known as heat flow;
- It is noted ϕ ;
- It is :
 - The exchanged power of thermal energy;
 - The amount of heat exchanged in a unit of time across a surface.
- It is expressed in Watts (W), and is given by:

$$\phi = \frac{\delta Q}{dt} = \dot{Q} \quad (4)$$

δQ : the quantity of exchanged heat.



f. Heat flux density

- It is noted ϕ ;
- It is :
 - heat flow per unit area.
 - quantity of heat exchanged per unit time and per unit area.
- is expressed in watts per square meter. (W/m^2) ;

- it is given, in the case of a surface perpendicular to it, by:

$$\phi = \frac{d\phi}{dS} = \frac{\delta Q}{dS \cdot dt} \quad (5)$$

- In the case where the flux is not perpendicular to the surface, the elementary flux is given by:

$$d\phi = \vec{\phi} \cdot \vec{n} \cdot dS = \phi \cdot S \cdot \cos(\theta) \quad (6)$$

- The total flux through a surface S is given by the equation (8):

$$\phi = \int_S d\phi = \int_S \vec{\phi} \cdot \vec{n} \cdot dS \quad (7)$$

g. Heat capacity

i. Heat capacity or thermal capacity : C

- Heat capacity is formerly known as calorific capacity.;
- The heat capacity of a system is the heat that must be transferred to increase its temperature by one Kelvin..
- It is a scalar quantity that reflects the system's capacity to absorb or "give back (rendre)" thermal energy by changing its temperature.
- It is an extensive quantity expressed in Joules per Kelvin ($J.K^{-1}$);
- It is denoted C.
- It is expressed by :

$$C = \lim_{\Delta T} \frac{\Delta Q}{\Delta T} = \frac{dQ}{dT} ; C = Q/T$$

Q is the quantity of heat supplied (fournie) to the system.

ii. Specific heat capacity Capacité thermique massique (spécifique) : C_s

- It's the heat capacity per unit mass.
- it is so defined as the quantity of heat (J) absorbed per unit mass (kg) of the material when its temperature increases 1 K (or 1 °C),
- its units are $J/(kg \text{ } ^\circ C \text{ or } (J \cdot K^{-1} \cdot kg^{-1}))$

$$C_s = C/m = \frac{Q}{m \cdot \Delta T}$$

iii. Volumetric heat capacit (Capacité thermique volumique)

- It is the amount of energy that must be added, in the form of heat, to one unit of volume of the material in order to cause an increase of one unit in its temperature.
- Its unit is Joule per Kelvin per cubic meter ($\text{J} \cdot \text{m}^{-3} \cdot \text{K}^{-1}$).

$$C_V = \frac{C}{V} = \frac{Q}{V \cdot \Delta T}$$

V : System volume

iv. Molar heat capacity (Capacité thermique molaire)

- It is the amount of energy that must be added, in the form of heat, to one mole of the substance in order to cause an increase of one unit in its temperature.

$$C_n = \frac{C}{n} = \frac{Q}{n \cdot \Delta T}$$

n : number of moles of the system

h. Stored thermal energy (Energie thermique stockée)

- Stored thermal energy is the amount of thermal energy stored in a substance. Stored thermal energy refers to the energy that has been absorbed or released by a substance as a result of a change in its temperature.
- When a substance absorbs heat, its thermal energy increases, and when it releases heat, its thermal energy decreases.
- This stored thermal energy can be used for various purposes, such as heating, cooling, or generating power.
- It depends directly on the amount of matter in the system, its temperature rise and the body's heat capacity;
- Its expression is :

$$\phi_{st} = \rho \cdot V \cdot C \cdot \frac{\partial T}{\partial t}$$

ϕ_{st} : Stored heat flow (W)

ρ : Volumic mass (kg m^{-3})

C : Heat of mass ($\text{J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$)

i. Generated thermal energy (Energie thermique générée)

- The generated thermal energy is the energy converted from chemical, electrical, mechanical or nuclear energy into thermal energy..
- It is given by :

$$\phi_g = \dot{q} \cdot V \quad (8)$$

ϕ_g : Generated heat flux (W)

V : Volume (m^3)

$\dot{q}$: Volumetric density of thermal energy ($W \cdot m^{-3}$)

I-3. Phenomena, modes and flows Transfer

a. Transfer phenomena

i. Definition of the transfer phenomenon (Définition du phénomène de transfert)

- A transfer phenomenon, also known as a transport phenomenon, is one that ensures the transport of a physical quantity via its constituents: atoms or molecules (Romulus, 1993)..
- The origin of all transfer phenomena is the inhomogeneity of an intensive quantity..
- It has a spontaneous tendency to unify intensive quantities in nature..
- Given the nature of the constituents of the system under study and these conditions, we distinguish three transfer phenomena: **heat, matter and momentum transfer**..

ii. Heat transfer (Thermal)

- Heat transfer is also known as thermal transfer.
- The quantity transported is thermal energy (in joules) and the intensive quantity at its origin is temperature..
- This phenomenon takes place between two regions of a substance or between two systems with different temperatures..
- Il s'effectue de la température la plus élevée vers la température la plus basse.
- Temperature difference: the driving force of heat transfer (la *force motrice* du transfert thermique).

iii. Mass transfer

- During this transfer, the quantity transported is matter (in kg) and the intensive quantity at its origin is the mass concentration.
- This phenomenon occurs between two regions of the same system with different mass concentrations.
- It always moves from the region with the highest concentration to the one with the lowest.
- The difference in the concentration of matter is also called the driving force of matter transfer (la force motrice du transfert de matière).

iv. Transfer of momentum (Transfert de quantité de mouvement)

- During this transfer, the quantity transported is momentum (en kg. m. s^{-1}).
- The quantity at the origin of this phenomenon is velocity.
- This phenomenon takes place between two regions of a substance with different velocities..
- It always moves from the region with the higher velocity to the region with the lower velocity.
- The difference in velocity is also known as the driving force behind the transfer of momentum.

b. The three modes of heat transfer (Les trois modes de transfert thermique)

There are three modes of heat transfer: conduction, convection and radiation..

i. Conduction

- Thermal conduction is a diffusion phenomenon that allows the propagation of thermal energy (heat) within a solid substance out of thermal equilibrium.
- The most energetic particles communicate their energies by atomic collisions on a microscopic scale, without any macroscopic movement of matter (molecules or atoms).
- Similarly, thermal conduction can take place in liquids and gases at rest. However, it is usually accompanied by another phenomenon, giving rise to what is known as convection.

Remarks :

1. Heat transfer by conduction in a vacuum **does not exist** because of the absence of a molecular carrier.
2. A good thermal conductor is a material that transfers the **greatest** possible quantity of thermal energy;
3. A good thermal insulator is a material that transfers the **smallest** possible amount of thermal energy;

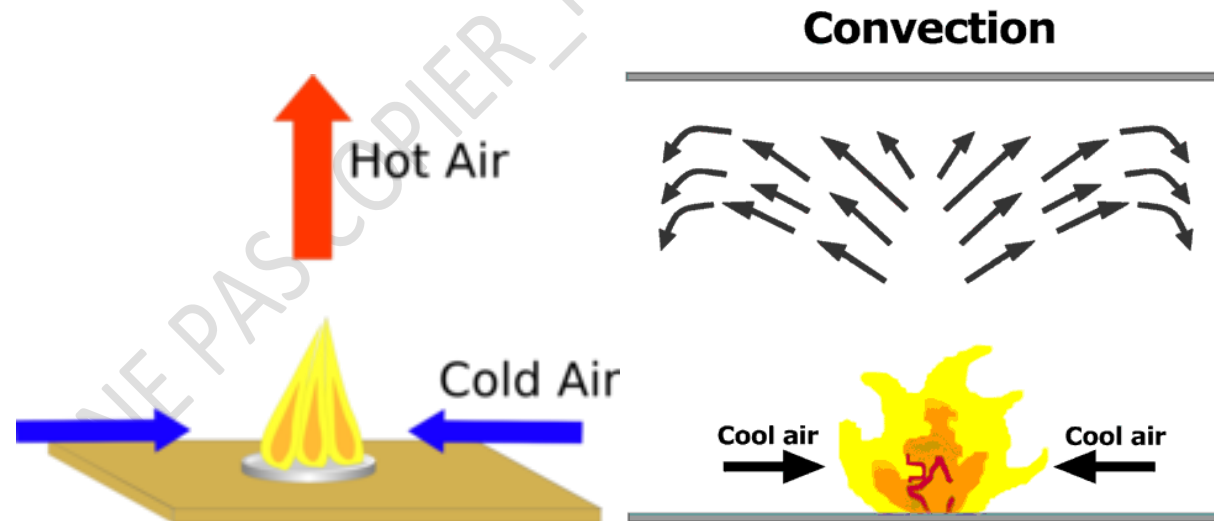
ii.The convection

- Thermal convection is a form of heat transfer in which, in addition to conduction, thermal energy is transported by the macroscopic movement of the system's constituent molecules.
- This transfer mode takes place in liquid or gaseous substances, or in a medium made up of a solid and a fluid..
- Depending on the nature of the forces under which the fluid moves, there are three types of convection:

- **Natural convection (free convection)**

Natural convection is a mechanism of heat transportation in which the fluid motion is not generated by an external source.

Example : Hot air rising above a fire (l'air chaud s'élevant au-dessus d'un feu)



- **Forced convection**

Forced convection is convection in which the fluid, the element transporting thermal energy, "necessarily" moves under an external action, independent of temperature differences, such as that of a pump, a fan(ventilateur), etc.

- **Mixed (combination) convection**

This is the convection that combines both natural and forced convection in a single transfer phenomenon.

iii. Radiation

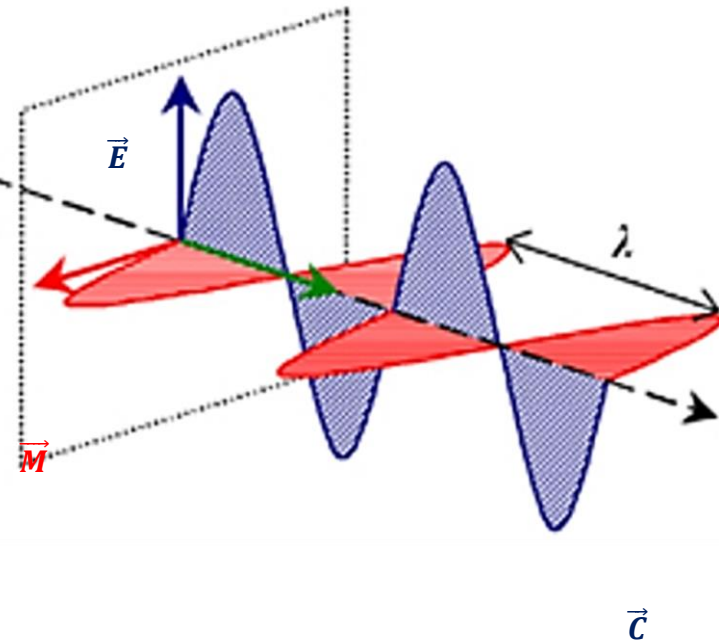
- Thermal radiation is a form of heat transfer that takes place by means of electromagnetic radiation in a vacuum, in a transparent medium or in a semi-transparent medium ;
- Radiation does not require any material support to carry thermal energy.
- It is established between two bodies separated by a medium allowing the propagation of radiation based on two phenomena: emission and absorption.
- Emission is the conversion of particle thermal agitation energy into radiative energy, and absorption is the reverse conversion.
- The passage of an electron from one energy level to a lower energy level produces radiation emission, according to Planck.
- According to wave mechanics, electromagnetic radiation is made up of two fields: magnetic and electric; both are perpendicular to the direction of propagation of the radiation.
- It is also defined by its wavelength or frequency.

Figure I-2 Composition du rayonnement électromagnétique.

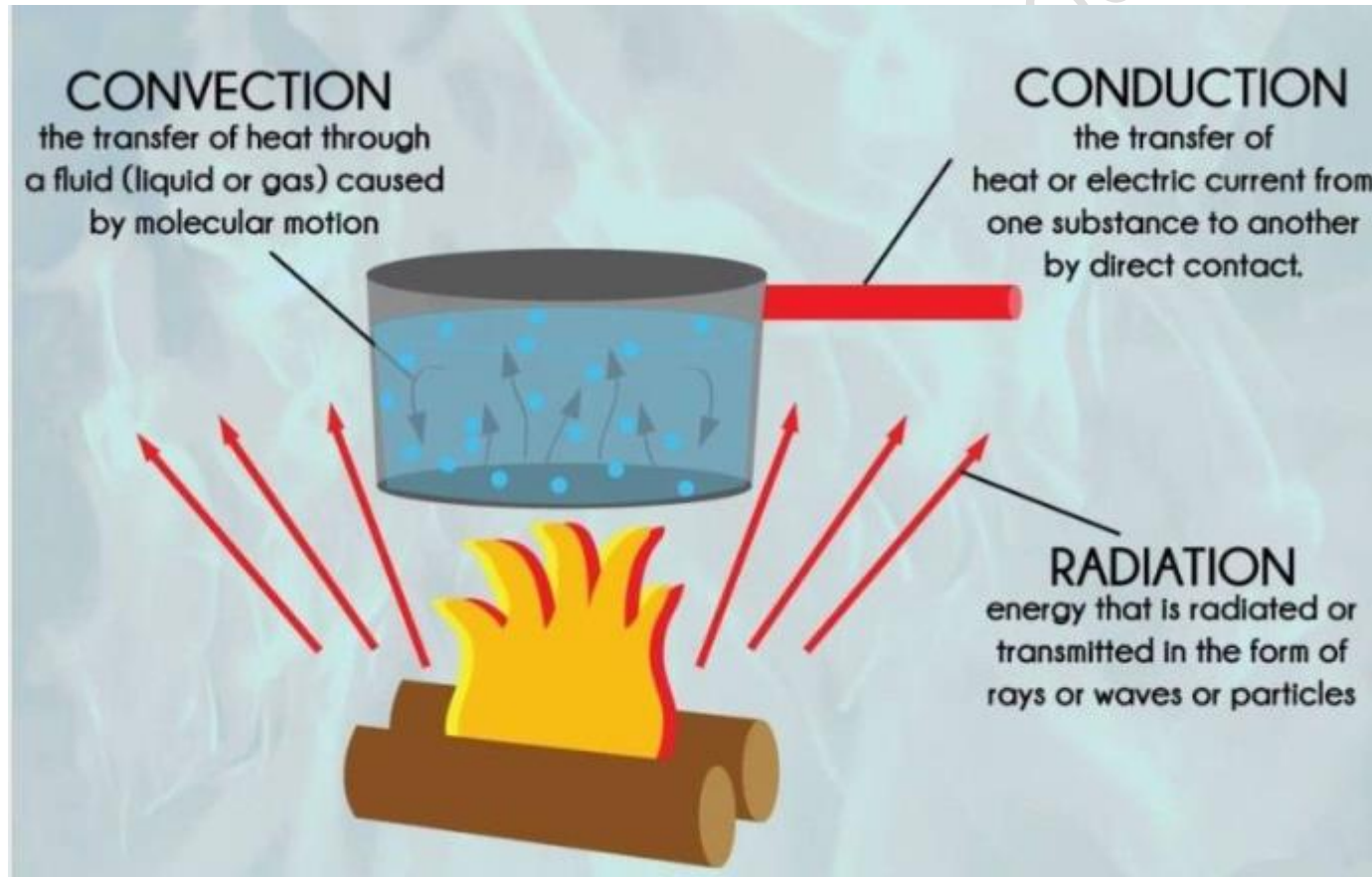
$\vec{E}$: Champ électrique

$\vec{M}$: Champ magnétique

$\vec{C}$: Vecteur vitesse de propagation



Note: The three heat transfer modes can be combined in the same situation.



c. Heat flow expressions

i. Fourier's law - Conductive exchange flow (Loi de Fourier – Flux d'échange par conduction)

- Fourier's law relates heat flux density to temperature gradient by:

$$\vec{\phi} = -\lambda \overrightarrow{\text{grad}T} = -\lambda \vec{\nabla}T \quad (9)$$

λ : Thermal conductivity (watts per meter-kelvin : $\text{W.m}^{-1}\text{K}^{-1}$)

$\vec{\phi}$: Heat flux density (W.m^{-2})

The thermal conductivity of a material is a scalar physical quantity which designates its material's ability to transmit thermal energy or its capacity to isolate it without macroscopic displacement of the material. In general, it varies according to temperature, the nature of the material and the coordinates of the local point where it is measured.

The "-" sign indicates that heat flows in the opposite direction to the temperature gradient.



In the case of a single direction x and through an elementary surface dS_x , for an infinitesimal duration dt ; Fourier's law gives the elementary heat quantity dQ :

$$dQ_x = -\lambda_x \frac{\delta T}{\delta x} dS_x dt \quad (10)$$

And the heat flow through dS_x :

$$d\phi = \frac{dQ_x}{dt} = -\lambda_x \frac{\delta T}{\delta x} dS_x \quad (11)$$

The thermal conductivity of some substances :

$$\lambda_{\text{laine de verre}} = 0.04 \text{ W/m/K}$$

$$\lambda_{\text{air}} = 0.026 \text{ W/m/K} \text{ (stationary air is a very good insulator)}$$

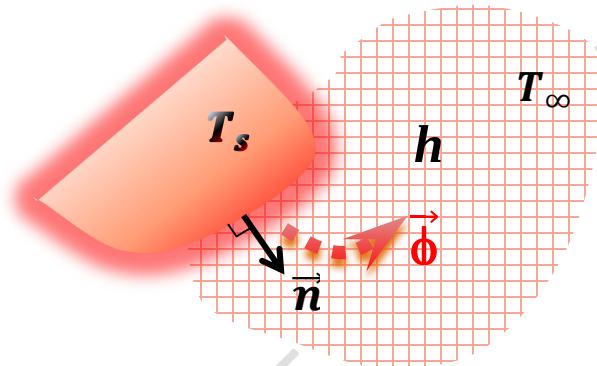
$$\lambda_{\text{verre}} = 1.2 \text{ W/m/K}$$

$$\lambda_{\text{cuivre}} = 390 \text{ W/m/K}$$

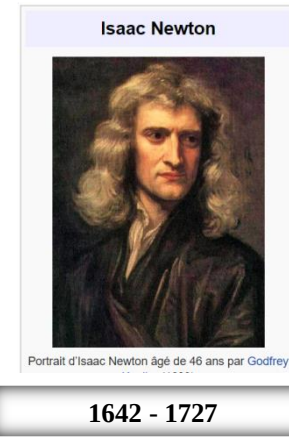
ii. Newton's Law - Convective exchange flow (Loi de Newton – Flux d'échange par convection)

Given a solid of temperature T_s exposed to a moving fluid of different temperature T_∞ , the amount of heat transported from (or to) the solid by the fluid is given by Newton's law (Isaac Newton 1642-1727):

$$\vec{\phi} = h S (T_s - T_\infty) \vec{n}$$



h : Convective exchange coefficient (>0) ($W.m^{-2}K^{-1}$)



iii. Stefan's Law - Radiation exchange flow (Loi de Stefan – Flux d'échange par Rayonnement)

Stefan's law, also known as Stefan-Boltzmann's law, gives the relationship between the temperature of black body and its temperature.:

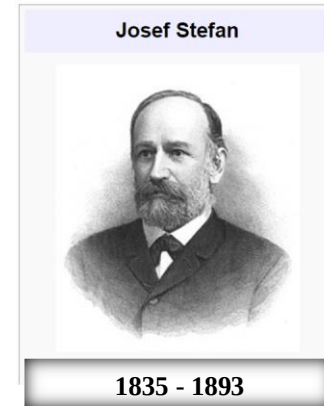
$$\phi = \sigma S T_s^4$$

ϕ : Heat flux transmitted by radiation

σ : Stephan-Boltzmann constant ($5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$)

T : Black-body temperature

S : Body surface area (m^2)



a

For a partially opaque body heated to a temperature T , ϕ is given by:

$$\phi = \varepsilon \sigma S T_s^4$$

ε : Surface emission factor

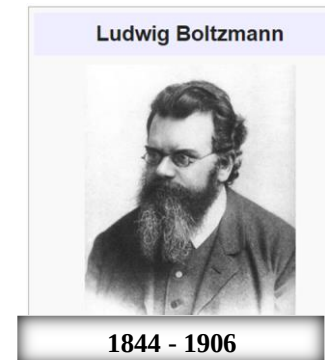
The heat transfer flux between an opaque or partially opaque body and the surrounding environment is given by:

$$\phi = \varepsilon \sigma S (T_s^4 - T_\infty^4)$$

T_s : Solid surface temperature (K)

T_∞ : Temperature of the environment surrounding the surface (K)

S : Area of surrounding solid/fluid surface



Mechanisms of Heat Transfer - Convection

$$q = h\Delta T$$

where

q is the local heat flux density [W.m^{-2}]

h is the heat transfer coefficient [$\text{W.m}^{-2}.\text{K}$]

ΔT is the temperature difference [K]

$$q = -k\nabla T$$

where

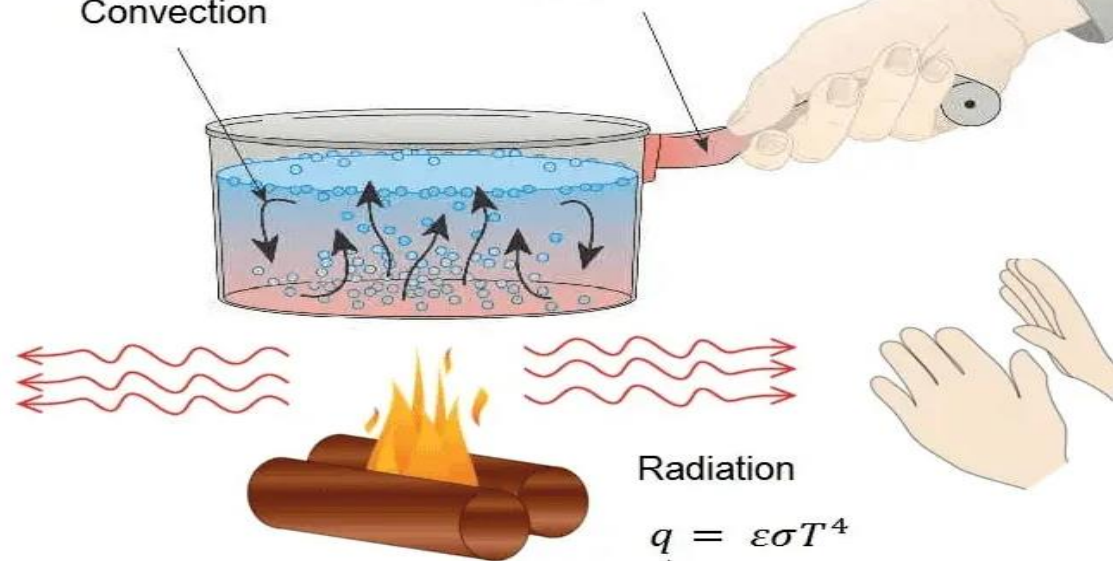
q is the local heat flux density [W.m^{-2}]

k is the materials conductivity [$\text{W.m}^{-1}.\text{K}^{-1}$]

∇T is the temperature gradient [K.m^{-1}]

Convection

Conduction



Radiation

$$q = \varepsilon\sigma T^4$$

where

q is the power radiated from an object [W.m^{-2}]

σ is the Stefan-Boltzmann constant [$\text{W.m}^{-2}.\text{K}^{-4}$]

ε is the emissivity of the surface of a material [-]

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PROCESS ENGINEERING DEPARTMENT



**Lecture Notes
on**

Heat and Mass Transfer

By ***Nabil MAHAMADIOUA***

Professor at University Mohammed Seddik Benyahia of Jijel

This lecture notes on "Heat and mass transfer" was written for second year students as part of a professional master degree in Metallurgical Engineering in the Sciences and Technology field in the Process Engineering department of the faculty of Science and Technology from Mohamed Seddik Benyahia University of Jijel.

It is a course of the fundamental unit of semester 3 and whose semester hourly volume is 45 hours divided into 1h30 of lessons and 1h30 of tutorial per week, for 15 weeks.

PART A :

HEAT TRANSFER

Chapter 02

CHAPTER 1: STEADY-STATE HEAT CONDUCTION

Content :

- II. 1. Heat equation: energy balance.**
- II. 2. One-Dimensional heat Conduction;**
- II. 3. Multidimensional heat conduction**
- II. 4. Multidimensional heat conduction in
cylindrical and spherical coordinates systems**
- II. 5. Heat Conduction : Steady state (permanent)
vs. transient (variable) regime**

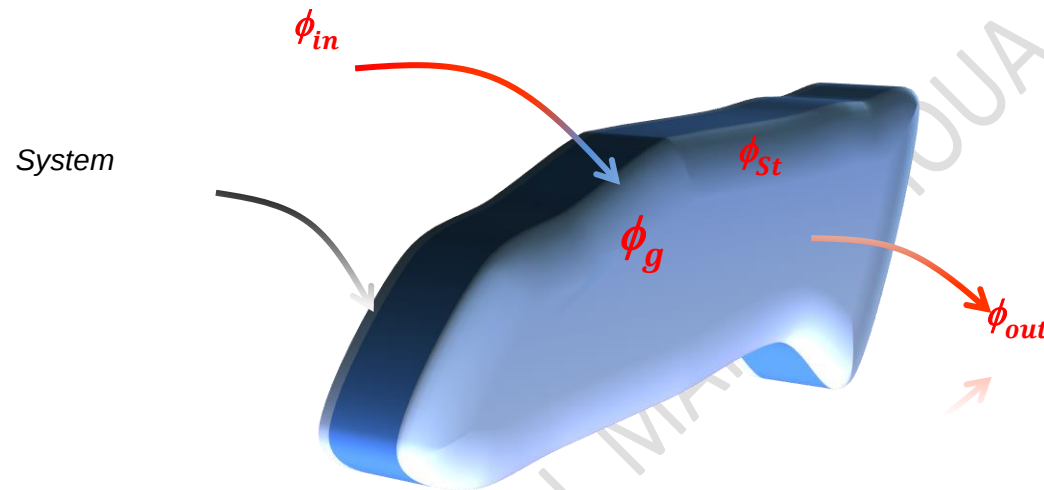
PART A :
HEAT TRANSFER

CHAPTER 2 :
STEADY-STATE HEAT CONDUCTION

II. 1. Heat equation: energy balance (L'équation de la Chaleur : bilan d'énergie)

Establishing the energy balance of a system comes down to applying the first principle of thermodynamics:

"During of any transformation of a closed system, the variation in its energy is equal to the quantity of energy exchanged with the external environment, by thermal transfer (heat) and mechanical transfer (work)."



$$[\text{input Energy}] + [\text{generated Energy}] = [\text{output Energy}] + [\text{accumulated (stored) Energy}].$$

Figure 1 Thermal system and energy balance

Applying the first principle of thermodynamics, the energy balance is

Accumulated energy = energy input - energy output + energy generated.

$$\phi_{St} = \phi_{In} - \phi_{Out} + \phi_g$$

$$\dot{q}.V = \phi_E - \phi_S + \rho.V.C.\frac{\partial T}{\partial t}$$

ϕ_{In} : Input heat flux (W);

$\dot{q}$: heat volumetric density ($W.m^{-3}$);

ϕ_{Out} : Out heat flux (W);

ρ : Density ($kg.m^{-3}$);

ϕ_g : Generated heat flux (W);

C : Heat capacity ($J.K^{-1}.kg^{-1}$);

ϕ_{St} : Stored heat flux (W);

V : Volume (m^3).

II. 2. One-Dimensional heat Conduction (Transfert de chaleur unidirectionnel (par conduction))

Let's consider the case of heat diffusion in a single direction. This type of transfer is called *unidirectional* or *one-dimensional*. We develop the heat equation by considering a flat plate of area S and thickness e (or a flat wall).

If, for example, we take the direction in which heat propagates to be x , and if we denote by ϕ_x et ϕ_{x+dx} the heat passing through the same surface S per unit time located at x and $x + dx$, then the heat equation becomes (see figure II-2)

$$\phi_{St} = \phi_x - \phi_{x+dx} + \phi_g$$

The amount of heat entering per unit time is given by Fourier's law in a single x direction is :

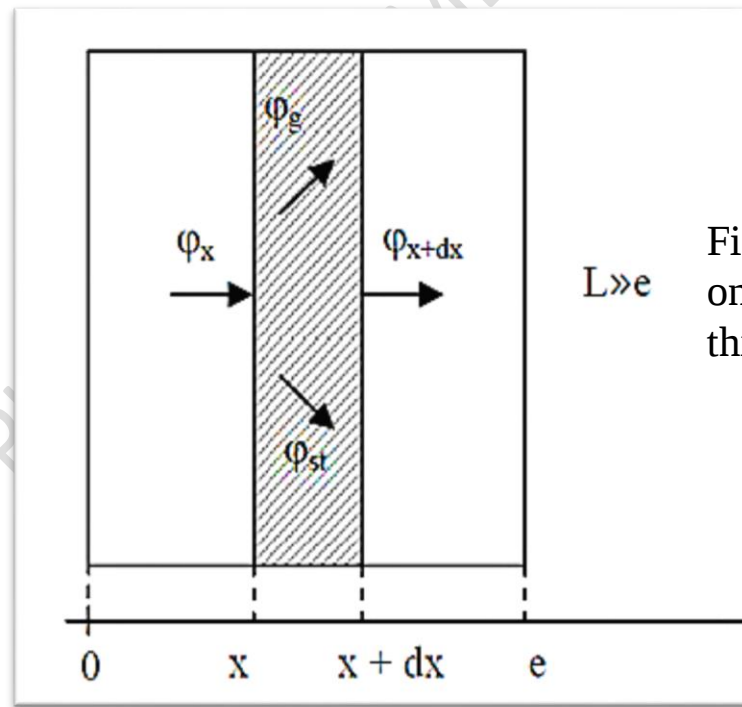


Figure II-2 Heat balance on a flat plate of thickness e and area S

$$\phi_x = -\left(\lambda S \frac{\partial T}{\partial x}\right)_x \text{ at } x \text{ et } \phi_{x+dx} = -\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} \text{ at } x + dx$$

And the heat equation takes the form

$$\rho \cdot V \cdot C \cdot \frac{\partial T}{\partial t} = -\left(\lambda S \frac{\partial T}{\partial x}\right)_x - \left[-\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx}\right] + \dot{q} \cdot V$$

$$\Rightarrow \rho \cdot S \cdot dx \cdot C \cdot \frac{\partial T}{\partial t} = \left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} - \left(\lambda S \frac{\partial T}{\partial x}\right)_x + \dot{q} \cdot S \cdot dx$$

Dividing by dx we get

$$\Rightarrow \rho \cdot S \cdot C \cdot \frac{\partial T}{\partial t} = \frac{\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} - \left(\lambda S \frac{\partial T}{\partial x}\right)_x}{dx} + \dot{q} \cdot S$$

This gives :

$$\Rightarrow \rho \cdot S \cdot C \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda S \frac{\partial T}{\partial x} \right) + \dot{q} \cdot S$$

Note that in the general case, conductivity is a function of point coordinates and time: $\lambda = \lambda(x, y, z, t)$

II. 3. Multidimensional heat conduction

In the previous section, we established the heat equation for one dimensional diffusion, expressing it in the Cartesian coordinate system and considering (Ox) as the direction of diffusion. The equation was then :

$$\rho \cdot S \cdot C \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda S \frac{\partial T}{\partial x} \right) + \dot{q} \cdot S$$

In the case of diffusion in all three directions, thermal energy will therefore propagate in the other two directions, y and z, in addition to the x direction. The heat equation then takes the most general form:

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda_z \frac{\partial T}{\partial z} \right) + \dot{q}$$

Case of no heat generation :

In the case of no heat generation within the system, $q' = 0$ and the equation will have the form :

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda_z \frac{\partial T}{\partial z} \right)$$

Case of an isotropic medium

In the case where the system medium is isotropic, we have :

$$\lambda_x = \lambda_y = \lambda_z = \lambda$$

And the equation takes the form :

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) + \dot{q}$$

Case of isotropic, homogeneous medium and absence of heat generation

$$(\dot{q} = 0)$$

The heat equation takes the form

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{d\lambda}{dT} \left[\left(\frac{dT}{dx} \right)^2 + \left(\frac{dT}{dy} \right)^2 + \left(\frac{dT}{dz} \right)^2 \right]$$

Case of isotropic, homogeneous medium, no heat generation and λ independent of T

$$\left(\dot{q} = 0 \text{ et } \frac{d\lambda}{dt} = 0 \right)$$

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \lambda \nabla^2 T = \lambda \Delta T$$

Or again

$$\frac{\rho \cdot C}{\lambda} \frac{\partial T}{\partial t} = \frac{1}{a} \frac{\partial T}{\partial t} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \nabla^2 T = \Delta T$$

The ratio : $\frac{\lambda}{\rho \cdot C} = \alpha$ is called the thermal diffusivity, its unit is : $(\frac{m^2}{s})$, which quantifies the speed at which heat diffuses inside the medium.

and the equation takes the form :

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} = \lambda \nabla^2 T = \Delta T$$

This local equation is valid in the case of an isotropic, homogeneous medium, no heat generation and λ independent of T.

NL

II. 1. Multidimensional heat conduction in cylindrical and spherical coordinates systems

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• in cylindrical coordinates :

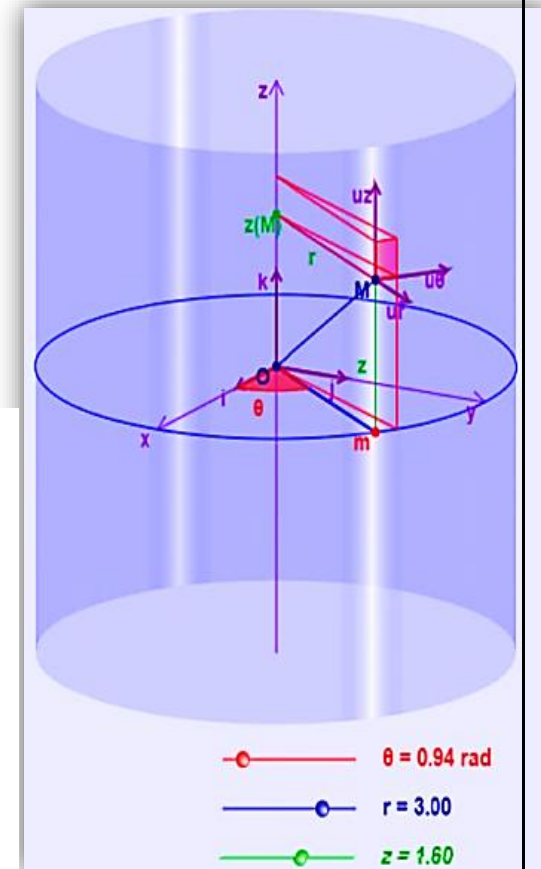
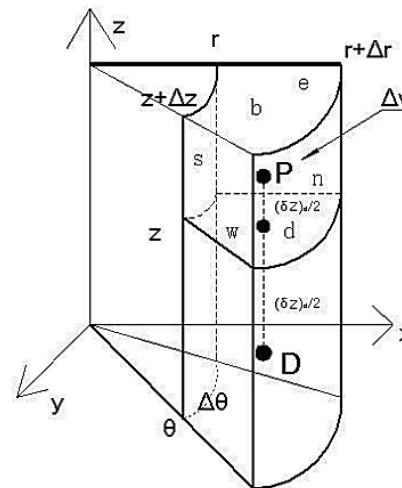
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t}$$

- In the case where T is a function only of r and t; then :

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial z} = 0$$

- And the equation becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t}$$



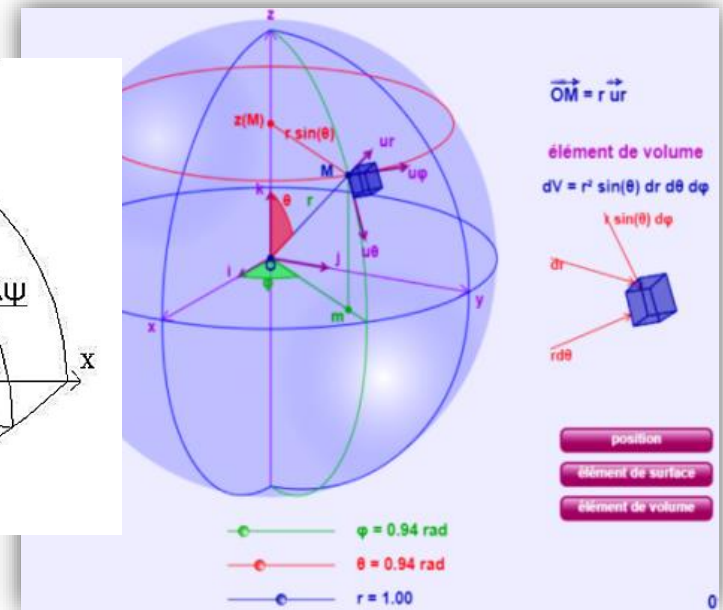
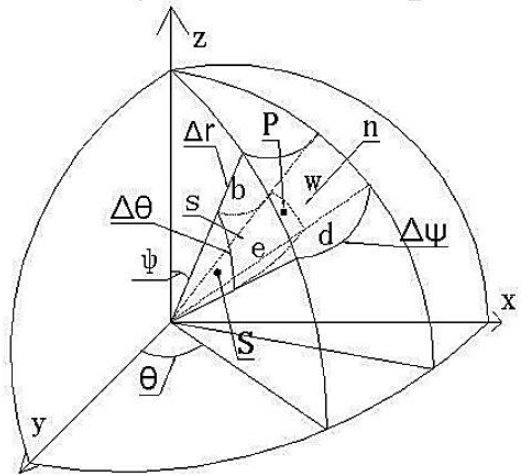
- in spherical coordinates:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} \left(\frac{\partial^2 T}{\partial \phi^2} \right) + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t}$$

Dans le cas où T n'est une fonction que de r et de t ; alors :

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \phi} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t}$$



II. 2. Heat Conduction : Steady state (permanent) vs. transient (variable) regime

In the field of heat transfer, problems are often classified as stationary (permanent) or transient (variable).

- The stationary regime indicates that the elements of the thermal study of a system (temperature, heat flow, ...) do not undergo any change with time and thus time independence.
- The unsteady (transient or variable) regime indicates the variation with time and thus the dependence on time..

In the heat equation, the term: $\frac{\partial T}{\partial t}$ is zero

II. 3. Multidimensional heat conduction in Steady-state regime

(Conduction de la chaleur multidimensionnelle en régime permanent)

Under steady-state regime : $\frac{\partial T}{\partial t} = 0$

So, the heat equation then takes the form:

$$\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda_z \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

In the case of no heat generation (in steady-state regime):

In the case of no heat generation within the system, $\dot{q} = 0$ and the equation will have the form :

$$\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda_z \frac{\partial T}{\partial z} \right) = 0$$

In the the case of isotropic medium (in steady-state regime):

$$\lambda_x = \lambda_y = \lambda_z = \lambda$$

And the equation takes the form :

$$\frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

Case of isotropic, homogeneous medium and absence of heat generation (in steady-state regime):

$$(\dot{q} = 0)$$

The heat equation takes the form

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

$$+ \frac{d\lambda}{dT} \left[\left(\frac{dT}{dx} \right)^2 + \left(\frac{dT}{dy} \right)^2 + \left(\frac{dT}{dz} \right)^2 \right] = 0$$

Case of isotropic, homogeneous medium, no heat generation and λ independent of T (in steady-state regime):

$$\left(\dot{q} = 0 \text{ et } \frac{d\lambda}{dT} = 0 \text{ et } \frac{\partial T}{\partial t} = 0 \right)$$

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \lambda \nabla^2 T = \lambda \Delta T = 0$$

Or again

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \nabla^2 T = \Delta T = 0$$

II. 4. Temperature from the heat equation

(La température à partir de l'équation de la chaleur)

The local heat equation can be used to **determine** the temperature $T(x,y,z,t)$ by **integration**.

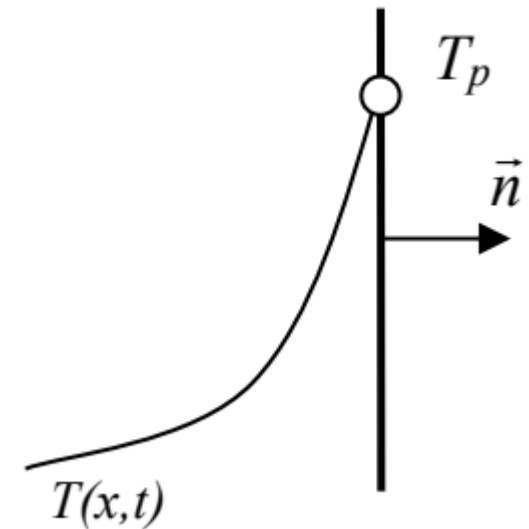
To this end, a word of clarification is in order:

1. An initial condition $T(x,y,z,t=0)$ which defines the initial thermal state of the system.
2. Two boundary conditions. These conditions can be of two types:
 - Dirichlet-type conditions
 - Neumann-type conditions

- **Dirichlet-type conditions**

In this case, the heat flow across the boundary is unknown (resulting from exchanges). It can be calculated using Fourier's law applied to the boundary.

$$\phi_{\Sigma} = -\vec{\phi} \cdot \vec{n} = \lambda \vec{\nabla} T \cdot \vec{n} = \lambda \left. \frac{\partial T}{\partial n} \right|_{\Sigma}$$



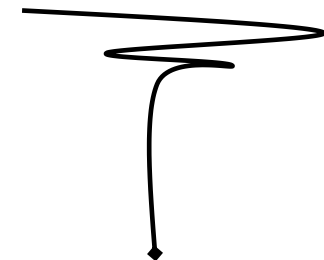
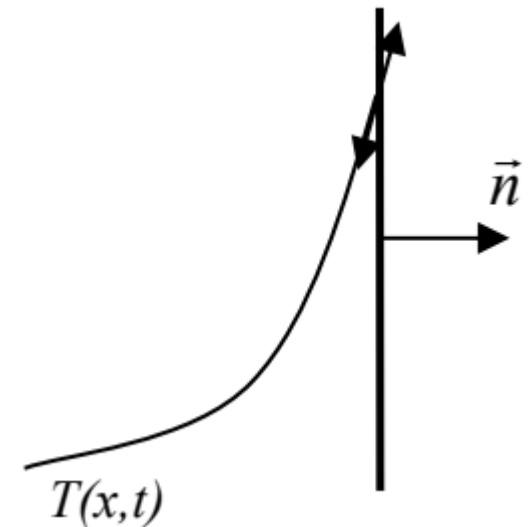
- **Neumann-type conditions**

In this case, the temperature of the boundary is unknown (resulting from exchanges). In general :

$$\varphi_{\Sigma} = -\vec{\phi} \cdot \vec{n} = \lambda \vec{\nabla} T \cdot \vec{n} = \lambda \left. \frac{\partial T}{\partial n} \right|_{\Sigma}$$

we impose a zero slope on the temperature
 $\Rightarrow$ temperature at the boundary.

$\Rightarrow$ on impose un gradient de température $\Rightarrow$. on impose la pente du profil de température à la frontière.





Exercise 01 (Heat flow)

A cylindrical electrical resistor ($D=0.4\text{cm}$, $L=1.5\text{cm}$) on a printed circuit board dissipates a power of 0.6 W . Assuming that heat is transferred uniformly across all surfaces.

Determine: (a) the amount of heat dissipated by this resistor over a 24-hour period, (b) the heat flux, (c) the fraction of heat dissipated by the top and bottom surfaces.

Exercise 02 (Flux density and thermal resistance)

A tank contains 3 m^3 of hot water at $T_i=80\text{ }^\circ\text{C}$. It is perfectly heat-insulated, except for a section with a surface area of $S=0.3\text{ m}^2$. After $\Delta t=5$ hours, the water temperature has dropped by $0.6\text{ }^\circ\text{C}$ at an ambient temperature of $20\text{ }^\circ\text{C}$. Assuming that the heat capacity of the tank is $103\text{ kcal}/^\circ\text{C}$. Calculate:

1. the amount of heat lost in 5 hours,
2. heat flux through the lid,
3. the heat flux density through the lid,
4. thermal resistance of the lid.

Exercise 03 (Fourier's law)

Determine the steady-state heat transfer rate per unit area (flux density) through a 4 cm thick slab, assuming it is homogeneous with uniform temperature maintenance on both sides at $38\text{ }^\circ\text{C}$ and $21\text{ }^\circ\text{C}$, respectively. The thermal conductivity of its material is $0.19\text{ W.m}^{-1}\text{K}^{-1}$. The temperature profile is assumed to be linear.

Exercise 04 (Newton's Law)

The heat transfer coefficient for forced convection of a fluid flowing over a cold surface is $225\text{ W/m}^2\cdot^\circ\text{C}$ for a particular situation. The temperature of the flowing fluid is $120\text{ }^\circ\text{C}$ and the surface is maintained at $10\text{ }^\circ\text{C}$.

- Determine the rate of heat transfer per unit area (flux density) from the fluid to the surface.

Exercise 05 (Stefan-Boltzmann law)

After sunset, an amount of radiant energy can be captured by a person standing next to a brick wall. The wall surface is heated to $44\text{ }^\circ\text{C}$ and the emissivity of the brick is 0.92 .

- - What is the heat flux per unit area of the wall at this temperature?





Exercice 01 (Flux de chaleur)

Une résistance électrique de forme cylindrique ($D=0,4\text{cm}$, $L=1,5\text{cm}$) sur un circuit imprimé dissipe une puissance de $0,6\text{ W}$. En supposant que la chaleur est transférée de manière uniforme à travers toutes les surfaces.

Déterminer : (a) la quantité de chaleur dissipée par cette résistance au cours d'une période de 24 heures, (b) le flux de chaleur, (c) la fraction de la chaleur dissipée par les surfaces du haut et du bas.

Exercice 02 (Densité de flux et résistance thermiques)

Un réservoir contient 3m^3 d'eau chaude à $T_i=80^\circ\text{C}$. Il est parfaitement calorifugé sauf sur une partie dont la surface est $S=0,3\text{m}^2$. On constate qu'au bout de $\Delta t=5$ heures, la température de l'eau a baissé de $0,6^\circ\text{C}$ quand la température ambiante est de 20°C . En supposant que la capacité calorifique du réservoir est de $103\text{ kcal}/^\circ\text{C}$. Calculer:

- 1) la quantité de chaleur perdue en 5 heures,
- 2) le flux de chaleur à travers le couvercle,
- 3) la densité de flux thermique à travers le couvercle,
- 4) la résistance thermique du couvercle.

Exercice 03 (Loi de Fourier)

- Déterminer le taux de transfert thermique en régime permanent par unité de surface (densité de flux) à travers une dalle de 4 cm d'épaisseur en supposant qu'elle est homogène avec un maintien de température uniforme sur ses deux faces à 38°C et à 21°C . La conductivité thermique de son matériau est $0,19\text{ W}\cdot\text{m}^{-1}\text{K}^{-1}$. On accepte que le profil de la température soit linéaire.

Exercice 04 (Loi de Newton)

Le coefficient de transfert de chaleur d'une convection forcée d'un fluide coulant sur une surface froide est $225\text{ W}/\text{m}^2\cdot^\circ\text{C}$ pour une situation particulière. La température du fluide coulant est de 120°C et la surface est maintenue à 10°C .

- Déterminer le taux de transfert thermique par unité de surface (Densité de flux) du fluide à la surface.

Exercice 05 (Loi de Stefan-Boltzmann)

Après le coucher de soleil, une quantité d'énergie rayonnante peut être attrapée par une personne debout près d'un mur en briques. La surface du mur est portée à 44°C et l'émissivité de la brique est $0,92$.

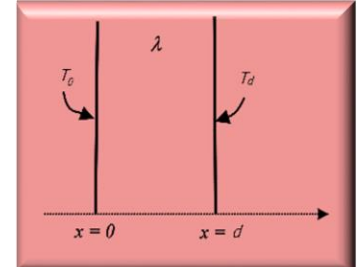
- Quel est le flux thermique par unité de surface du mur à cette température.

Proposés par N. MAHAMADIOUA



Exercise 01 (unidirectional steady-state heat conduction)

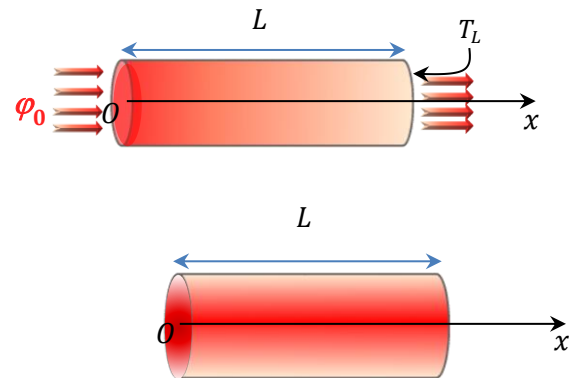
We consider a wall with a negligible thickness d compared to its dimensions. It is made of a homogeneous and undeformable material and has a constant conductivity λ . Knowing that the heat flux will propagate only in the direction of x and that the regime is permanent:



1. Show that, in the present case, the general heat equation reduces to Laplace's equation.
 2. Find the expression for the temperature distribution inside the wall.
 3. Determine the heat flux density through the wall. Deduce the heat flux through the entire wall.
 4. Express the temperature distribution as a function of flux ϕ and d , λ and the surface area S .
- N.A.** $d = 10 \text{ cm}$, $S = 12 \text{ m}$, T_0 case of brick (terracotta) $\lambda = 1,35 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$, Reinforced concrete and mortar $\lambda = 1,5 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$, marble $\lambda = 2.9 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$ and stainless steel $\lambda = 16.3 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$.

Exercise 02 (Conduction in a rod)

Consider a cylindrical bar, perfectly insulated at its lateral surface, of length L and cross-section S , made of a homogeneous material of constant thermal conductivity λ . It is assumed that the system has reached steady state (figure opposite).

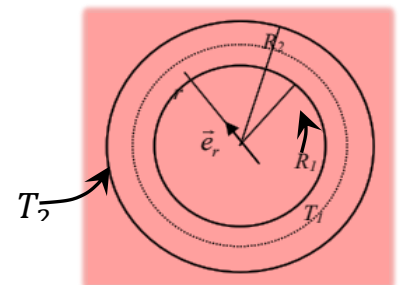


1. Using the heat equation, find the temperature expression.
2. Deduce the temperature T at $x = 0$.

Exercise 03 (Revolution-symmetric problem)

Consider a cylinder of radius r and length $\gg r$, made of a homogeneous, dimensionally stable material of constant thermal conductivity λ (see figure opposite).

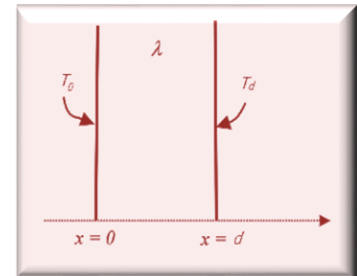
1. Find the expression for the temperature $T(r)$ ($T = T(r)$: means that T is radial. in the case of the hollow cylinder with inner radius R_1 and outer radius R_2).
2. Determine the density of heat flux through the cylinder at r .
3. Find the heat flux expression.





Exercice 01 (Transfert de chaleur unidirectionnel en régime permanent)

On considère un mur d'épaisseur d négligeable devant ses dimensions. Il est constitué d'un matériau homogène et indéformable et il a une conductivité constante λ . Sachant que le flux de chaleur se propagera seulement dans la direction de x et que le régime est permanent:

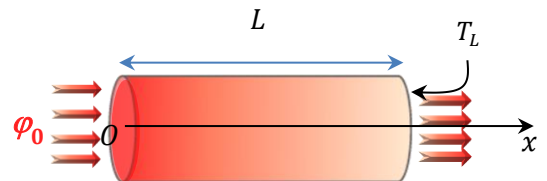


1. Montrer que, dans le présent cas, l'équation générale de la chaleur se réduit à l'équation de **Laplace**.
2. Trouvez l'expression de la distribution de la température à l'intérieur du mur.
3. Déterminer la densité de flux de chaleur traversant le mur. En déduire le flux de chaleur traversant tout le mur.
4. Exprimer la distribution de la température en fonction du flux ϕ et de λ et la surface S .

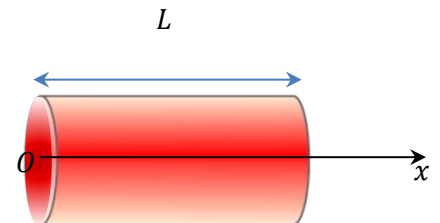
A.N. $d = 10 \text{ cm}$, $S = 12 \text{ m}$, T_0 cas de la brique (terre cuite) $\lambda = 1,35 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$, Béton armé et mortier $\lambda = 1,5 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$, le marbre $\lambda = 2.9 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$ et l'acier inoxydable $\lambda = 16.3 \text{ w}/(\text{m}^{-1}\text{K}^{-1})$.

Exercice 02 (Conduction dans un barreau)

Soit un barreau cylindrique, parfaitement isolé à sa surface latérale, de longueur L et de section S , fabriqué d'un matériau homogène de conductivité thermique λ constante. On suppose que le système a atteint le régime stationnaire (figure ci-contre).



1. En utilisant l'équation de la chaleur, trouver l'expression de la température.
2. En déduire la température T à $x = 0$.



Exercice 03 (Problème à symétrie de révolution)

Soit un cylindre de rayon r et de longueur $L \gg r$, fabriqué d'un matériau homogène et indéformable de conductivité thermique constante λ (voir figure ci-contre).

1. Trouver l'expression de la température $T(r)$ ($T = T(r)$): veut dire que T est radiale. dans le cas du cylindre creux de rayon intérieur R_1 et de rayon extérieur R_2 .
2. Déterminer la densité du flux de chaleur qui traverse le cylindre en r .
3. Trouver l'expression du flux thermique.

