Solution de contrôle TD de Physique 01

Exercice 01 : 5pts

$div(\vec{V}) = \vec{\nabla}.\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$ 0.5

$\frac{\partial V_x}{\partial x} = \frac{\partial (x)}{\partial x} = 1$

$\frac{\partial V_y}{\partial y} = \frac{\partial (2y)}{\partial y} = 2$ 1

$\frac{\partial V_z}{\partial z} = \frac{\partial (-z)}{\partial z} = -1$

$div(\vec{V}) = 1 + 2 - 1 = 2$ 0.5

2/

$x = V_0 t^2 \dashrightarrow [x] = [V_0][t]^2 \dashrightarrow L \neq \frac{L}{T}T^2 = LT$ Non homogène 1

$x = V_0 t + \frac{1}{2}at^2 \dashrightarrow [x] = [V_0 t] = \left[\frac{1}{2}at^2\right]$=L Homogène car : 1

$[x] = L$

$[V_0 t] = L$

$\left[\frac{1}{2}at^2\right] = LT^{-2}T^2 = L$

$x = V_0 t + 2at^2 \dashrightarrow [x] = [V_0 t] = [2at^2]$=L Homogène car : 1

On a $[2at^2] = LT^{-2}T^2 =$ L car la constante n'a pas de dimension.

Exercice 02 : 8pts

1/ $[\gamma] = \frac{[x]}{[t]} = \frac{L}{T} = LT^{-1}$ 0.5

$[\beta] = \frac{[y]}{[-2][t^2]} = \frac{L}{T^2} = LT^{-2}$ 0.5

2/ $\overrightarrow{OM}(t) = x\vec{i} + y\vec{j} = \gamma t\vec{i} - 2\beta t$ 0.5

3/ Equation de la trajectoire

On a : $t = \frac{x}{\gamma}$, et en remplaçant t dans l'expression de y(t), on obtient :

0.5

$y = \frac{-2\beta}{\gamma} x^2$ 0.5

La nature de la trajectoire est parabolique.

4/ Vitesse $\vec{v}(t)$

$$\vec{v}(t) = v_x \vec{\imath} + v_y \vec{\jmath}$$

Avec :

$v_x = \frac{dx(t)}{dt} = \gamma$ 0.5

$v_y = \frac{dy(t)}{dt} = -4\beta t$ 0.5

$$\vec{v}(t) = \gamma \vec{\imath} - 4\beta t \vec{\jmath}$$ 0.5

$$\|\vec{v}(t)\| = \sqrt{\gamma^2 + 16\beta^2 t^2}$$ 0.5

5/ Accélération

$$\vec{a}(t) = a_x \vec{\imath} + a_y \vec{\jmath}$$

Avec :

$a_x = \frac{dv_x(t)}{dt} = 0$ 0.5

$a_y = \frac{dv_y(t)}{dt} = -4\beta$ 0.5

$$\vec{a}(t) = -4\beta \vec{\jmath}$$ 0.5

$$\|\vec{a}(t)\| = \sqrt{16\beta^2} = 4\beta$$ 0.5

6/ Accélération tangentielle

$$a_T = \frac{d\|\vec{v}(t)\|}{dt} = \frac{16\beta^2 t}{\sqrt{\gamma^2 + 16\beta^2 t^2}}$$

Accélération normale

$$a_N = \sqrt{a^2 - a_T^2}$$ 0.5

$$a_N = \sqrt{16\beta^2 - \left(\frac{16\beta^2 t}{\sqrt{\gamma^2 + 16\beta^2 t^2}}\right)^2} = \sqrt{\frac{16\beta^2\gamma^2}{(\gamma^2 + 16\beta^2 t^2)}} = \frac{4\beta\gamma}{\sqrt{\gamma^2 + 16\beta^2 t^2}} = \frac{4\beta\gamma}{v}$$ 0.5

7/ Rayon de courbure

$R_c = \frac{v^2}{a_N} = \frac{v^2}{\frac{4\beta}{v}} = \frac{v^3}{4\beta\gamma}$ 0.5