University of Jijel - Faculty of exact sciences and computer science - Mathematics department

Series No. 03 *2023/2024*

Exercise 01: It is given that:

$$z + 2i = iz + k, k \in \mathbb{R} \ and \ \frac{w}{z} = 2 + 2i, Im(w) = 8.$$

Determine the value of k.

Exercise 02: Solving in $\mathbb{C}$ the following equations:

1- $z^2 = 3 - 4i,$
2- $2z^2 - (1 + 5i)z + 2(i - 1) = 0,$
3- $z^4 - 8z^3 + 33z^2 - 68z + 52 = 0, with \ z_1 = 2 + 3i,$
4- $z^3 = 8i,$
5- $z^6 - iz^3 - 1 - i = 0.$

Exercise 03: The following complex numbers are given:

$$z_1 = 2 - 2i, z_2 = \sqrt{3} + i \ \ and \ z_3 = a + ib \ where \ a, b \in \mathbb{R}.$$

1) If $|z_1 z_3| = 16$, find the modulus of z_3.
2) Given further that $\arg\left(\frac{z_3}{z_2}\right) = \frac{7\pi}{12}$, determine the argument of z_3.
3) Find the values of a and b, and hence show that $\frac{z_3}{z_1} = -2$.

Exercise 04:

The complex numbers z and w are such so that: $|z| = |w| = 1$.

Show clearly that: $\frac{z+w}{1+zw}$ is real.

Exercise 05:

1- Give the development of : $\cos 5x$ and $\sin 2x$.
2- Find the linear formula of the expressions: $\cos x \sin^2 x$ and $(\cos 2x)^3$.

Exercise 06: Let $z = x + iy, x, y \in \mathbb{R}$ be a complex number such that $\overline{z}$ its conjugate.

Solve the equation: $z + 2\overline{z} = |z + 2|$.

Exercise 07: The following cubic equation is given

$$z^3 + 2z^2 + az + b = 0, \qquad where\ \ a, b \in \mathbb{R}.$$

One of the roots of the above cubic equation is: $1 + i$.

a) Find the real root of the equation.
b) Find the value of a and the value of b.

Exercise 08:

1) Find the modulus and argument of the following complex numbers:

$$z = \sqrt{3} + i, w = 3i.$$

2) Determine simplified expressions for zw and $\frac{w}{z}$, giving the answers in the algebraic form.
3) Find the modulus and argument of zw and $\frac{w}{z}$.

Exercise 09: Let $n \geq 2$ be an integer number.

1) Find all complex numbers z that satisfy: $z^{2n} = 1$.
2) Find all complex numbers z which satisfy the equation: $z^n = -1$.
3) Compute the sum of the complex numbers which verify: $z^n = -1$.

Exercise 10: Let $f: \mathbb{C} \to \mathbb{C}$ be a function defined as: $f(z) = z - z^2$.

1) Find all complex numbers z that satisfy: $f(z) = z^2$.
2) Prove that if $\left|z - \frac{1}{2}\right| < \frac{1}{2}$, then $\left|f(z) - \frac{1}{4}\right| < \frac{1}{4}$.