University of Jijel - Faculty of exact sciences and computer science - Mathematics department

Series No. **05** *2023/2024*

Exercise 01: Find the domain of definition of the following functions:

$$f(x)=\sqrt{\frac{x-2}{x+1}}, g(x)=\sqrt{x^2+2x+3}, h(x)=\frac{x^3+3}{1-|x|},$$

$$k(x)=\frac{\cos x}{1+\sin 2x}, l(x)=\sqrt{2\cos x-1}.$$

Exercise 02: Study the parity of the following functions:

$$f(x)=\frac{\tan x-x}{x\cos x}, g(x)=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), h(x)=\ln(x+\sqrt{x^2+1}).$$

Exercise 03: 1- Prove that the function $f(x)=x-E(x), for\ all\ x\in\mathbb{R}$ is periodic and that its period is 1.

2- Let g be a periodic function its period is T and let h be a function defined as:

$$\forall x\in\mathbb{R}: h(x)=g(ax+b), (a,b)\in\mathbb{R}^*\times\mathbb{R}.$$

- *a*- Prove that the period of h is $\frac{T}{a}$.
- *b*- **N.A:** Deduce the period of the function $h(x)=\sin(5x+2)$.

Exercise 04: Prove that all function f defined on a symmetric interval D can be written as a sum of two functions, one even denoted by f_1 and the other odd denoted by f_2.

N.A: Determine f_1 and f_2 in the case where f is defined as :

$$f:]-\infty,-\frac{1}{2}[\cup]-\frac{1}{2},\frac{1}{2}[\cup]\frac{1}{2},+\infty[\ \to\ \mathbb{R}$$
$$x \mapsto \frac{x+2}{2|x|-1}$$

Exercise 05: Answer the following questions for the piecewise defined function f(t) described on the right hand side.

(a) $f\left(-\frac{3}{2}\right)=$

(b) $f(2)=$

(c) $f\left(\frac{3}{2}\right)=$

(d) $\lim_{t\to-2} f(t)=$

$$f(t)=\begin{cases} t^2 & \text{for } t<-2 \\ \frac{t+6}{t^2-t} & \text{for } -1<t<2 \\ 3t-2 & \text{for } t\geq 2. \end{cases}$$

(e) $\lim_{t\to-1^+} f(t)=$

(f) $\lim_{t\to2} f(t)=$

(g) $\lim_{t\to0} f(t)=$

Exercise 06: For the functions:

$$f(x)=\begin{cases}0, & x=0,\\ -2, & x=1,\\ 2+x, & x\neq 0,1,\end{cases} \text{ and } g(x)=\begin{cases}0, & x=0,\\ 1+x, & x\neq 0.\end{cases}$$

Determine $(fog)(x), (fog)(0), f\left(\lim_{x\to 0} g(x)\right)$ and $\lim_{x\to 0} f\big(g(x)\big)$.

Exercise 07: Calculate the following limits:

$$1)\ \lim_{x\to+\infty}\sqrt{x+1}-\sqrt{x},\quad 2)\ \lim_{x\to+\infty}\left(\frac{2x-1}{2x+3}\right)^{x},\quad 3)\ \lim_{x\to 0} x^2\cos\frac{1}{x},\ 4)\ \lim_{x\to 0}\frac{1-\cos\sqrt{x}}{x},\ 5)\ \lim_{x\to 2}\frac{\sqrt{x^2-1}-\sqrt{3}}{x-2},$$

$$6)\ \lim_{x\to+\infty}\frac{\ln(e^{2x}+1)}{x},\ 7)\ \lim_{x\to 0}\frac{x}{a}\,E\left(\frac{b}{x}\right), a\neq 0, 8)\ \lim_{x\to 0}\frac{\cos ax-\cos bx}{x^2},\ 9)\ \lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}},$$

$$10)\ \lim_{x\to 0}\frac{1}{2x}\ln\sqrt{\frac{1+x}{1-x}},\ 11)\lim_{x\to 0}\sqrt{\frac{x\sin x}{1-\cos x}},\ 12)\lim_{x\to 0}\frac{2\tan x-\sin 2x}{2\sin x},\ 13)\ \lim_{x\to+\infty}\frac{x-\sqrt{x^2-x+1}}{2x-\sqrt{4x^2+x}}.$$

Exercise 08: Using the mathematical definition of the limit, prove that:

$$1)\ \lim_{x\to+\infty}\frac{1}{x^2}=0,\ 2)\ \lim_{x\to 1^+}\frac{x+2}{x-1}=+\infty,\ 3)\ \lim_{x\to 3}\sqrt{x+1}=2,\ 4)\ \lim_{x\to 0}x\sin\frac{1}{x}=0.$$