University of Jijel - Faculty of exact sciences and computer science - Mathematics department

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Exercise 01: Calculate the following integrals by using the integration by part:

$$I_1 = \int \sin(\alpha x)\, e^{3x} dx\,, \qquad I_2 = \int (\mathrm{x}^2 + 1)\cos(2x)\, dx,$$

$$I_3 = \int \ln(\mathrm{x} + \sqrt{1 + \mathrm{x}^2})\, dx\,, \quad I_4 = \int (\arcsin \mathrm{x})^2 dx.$$

Exercise 02 : Calculate the following integrals by using the change of variable.

$$I_1 = \int \cos(\sqrt{x})\, dx\,, I_2 = \int \sqrt{4 - \mathrm{x}^2} dx\,, I_3 = \int \sqrt{\mathrm{x}^2 - 1} dx, I_4 = \int x\sqrt{\mathrm{x} - 1} dx$$

$$I_5 = \int \frac{\mathrm{e}^{\mathrm{x}}}{(1 + \mathrm{e}^{\mathrm{x}})\sqrt{\mathrm{e}^{\mathrm{x}} - 1}} dx\,, I_6 = \int \frac{2\mathrm{x}}{\sqrt{2\mathrm{x} + 1}} dx, I_7 = \int \frac{dx}{\sqrt{\mathrm{x}^2 + 1}},$$

$$I_8 = \int \frac{x^7 dx}{1 + x^{16}}, I_9 = \int \frac{\sin x\, dx}{\sin^2 \mathrm{x} + 1}, \ I_{10} = \int \frac{dx}{\mathrm{ch\, x}}, \ I_{11} = \int \frac{dx}{\mathrm{x}\sqrt{\mathrm{x} + 1}},$$

$$I_{12} = \int \frac{dx}{\sqrt{\mathrm{x}^2 - 1}}, \ I_{13} = \int \frac{\mathrm{e}^{\mathrm{x}} dx}{1 + \mathrm{e}^{\mathrm{x}}}, \ I_{14} = \int \frac{dx}{(\mathrm{ch\, x} + \mathrm{sh\, x})^{\mathrm{n}}}, I_{15} = \int \frac{dx}{\cos^4 \mathrm{x}},$$

$$I_{16} = \int \frac{chx - 1}{\mathrm{ch\, x} + 1} \mathrm{dx}, \ I_{17} = \int \frac{dx}{\sin \mathrm{x} + 2}, \ I_{18} = \int \frac{dx}{\sin \mathrm{x} - \cos \mathrm{x} - 1},$$

$$I_{19} = \int \sin(2x)\cos(3x)\, dx, I_{20} = \int \cos^3 x \sin^3 \mathrm{x}\, dx, I_{21} = \int \cos^4 x \sin^2 \mathrm{x}\, dx,$$

$$I_{22} = \int \frac{\cos^3 x}{\sin^4 \mathrm{x}} dx, I_{23} = \int \frac{dx}{\sqrt{1 - \mathrm{x}^2}}, \ I_{24} = \int \frac{\sqrt{x} dx}{1 + \sqrt[3]{\mathrm{x}}}.$$

Exercice 03 : We consider the integral : $\forall n \in \mathbb{N}$: $I_n = \int \frac{dx}{(\mathrm{x}^2 + 1)^{\mathrm{n}}}$,

1- Compute I_0, I_1 and I_2.
2- Find a recurrence relation between I_n and I_{n+1}.
3- Deduce the value of the integral I_3.