PART A :

Heat transfer

Chapter 3 :

Conductive heat transfer in variable regime

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As introduction

1. What examine **conductive heat transfer in variable regimes ?**
   - Conductive heat transfer in variable regimes examines how heat flows through materials when subjected to changing conditions.
2. **What it investigate ?**
   - This field delves into the dynamic interplay between temperature, material properties, and boundary conditions.
3. **Why understanding the conductive heat transfer in variable regimes is crucial ?**
   - Understanding heat conduction in variable regimes is **crucial** for **optimizing thermal processes, enhancing material performance, and designing resilient (مرن) systems capable of adapting to fluctuating environmental conditions**.
4. **To it contributes ?**
   - This scientific studies contribute to **advancements in energy efficiency, material design**, and the **overall optimization of thermal systems**.
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- Conductive heat transfer in a **variable regime** **refers to the process where heat is transferred through a material due to a temperature gradient, but the system's temperature distribution changes with time.**

- **This is also known as** transient heat conduction**, as opposed to** steady-state heat conduction**, where temperature remains constant over time.**

II. 1. Heat equation: energy balance (L’équation de la Chaleur : bilan d’énergie)

The heat equation takes the most general form as we have seen in the previous chapter:

$$\boldsymbol{\rho . C . \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_x \ \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \ \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \ \frac{\partial T}{\partial z}\right) + \dot{q}}$$

Case of no heat generation:

In the case of **no** heat generation within the system, $\dot{q} \; = 0$ and the equation will have the form :

$$\boldsymbol{\rho . C . \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_x \; \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \; \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \; \frac{\partial T}{\partial z}\right)}$$

Case of an isotropic medium

In the case where the system medium is isotropic, we have:

$$\lambda_x = \lambda_y = \lambda_z = \lambda$$

And the equation takes the form :

$$\rho . C . \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \ \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \ \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \ \frac{\partial T}{\partial z}\right) + \dot{q}$$

Case of isotropic, homogeneous medium and absence of heat generation

$$(\dot{q} = 0)$$

The heat equation takes the form

$$\rho . C . \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{d\lambda}{dT} \left[\left(\frac{dT}{dx} \right)^2 + \left(\frac{dT}{dy} \right)^2 + \left(\frac{dT}{dz} \right)^2 \right]$$

Case of isotropic, homogeneous medium, no heat generation **and λ independent of T**

$$\left(\dot{q} = 0 \ et \ \frac{d\lambda}{dT} = 0\right)$$

$$\rho.C.\frac{\partial T}{\partial t} = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \lambda \, \nabla^2 T = \lambda \, \Delta T$$

Or again

$$\frac{\rho.C}{\lambda}\frac{\partial T}{\partial t} = \frac{1}{a}\frac{\partial T}{\partial t} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \nabla^2 T = \Delta T$$

The ration : $\frac{\rho.C}{\lambda} = \boldsymbol{a}$ is called the thermal diffusivity, its unit is :$(\frac{m^2}{s})$

and the equation takes the form :

$$\rho.C.\frac{\partial T}{\partial t} = \lambda\ \nabla^2 T = \lambda\ \Delta T$$

$$Or$$

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho.C.}\ \nabla^2 T = \frac{1}{\frac{\lambda}{\rho.C.}}\ \nabla^2 T = a\ \nabla^2 T = a\ \Delta T$$

This equation is valid in the case of an isotropic, homogeneous medium,

no heat generation and λ independent of T.

III-1. Considerations in conduction variable regime

In variable or transient regimes, we distinguish between two cases, depending on the thermal situation of the system:

- Thermally thin body (Corps thermiquement mince) ;
- Thermally thick body (Corps thermiquement épais) ;

- **Definition of thermally thin substance**

A thermally thin substance is one that is able to store a small amount of heat energy per unit volume. This means that it is able to absorb and release heat energy relatively quickly.

In other words

A **thermally thin body** refers to an object whose thermal mass and dimensions are small enough that it reaches thermal equilibrium quickly when subjected to a change in temperature or heat flux.

This condition means that temperature gradients within the body are negligible or nonexistent, and the entire body can be assumed to have a uniform temperature at any given time.

▪ Characteristics of a Thermally Thin Body

1. **Low Thermal Mass**: The product of its specific heat capacity, density, and volume is small, enabling rapid temperature changes.

$$\text{Thermal mass} = \rho \cdot c_p \cdot V$$

where ρ is the density, c_p is the specific heat capacity, and V is the volume.

Example : Thin Aluminum Sheet

Properties of Aluminum:

Density (ρ): $2700\ \mathrm{kg/m^3}$; **Specific Heat Capacity (c_p)**: $900\ \mathrm{J/kg.K}$; **Thickness (d)**: $1\ \mathrm{mm} = 0.001\ \mathrm{m}$; **Area ($A$)**: $1\ \mathrm{m^2}$

Volume of the Sheet: $\boldsymbol{V = A \cdot d = 1\, m^2 \cdot 0.001\, m = 0.001\, m^3}$

Thermal Mass $\boldsymbol{= \rho \cdot c_p \cdot V = 2430\, J/K}$

This low thermal mass allows the sheet to heat up or cool down quickly when exposed to a heat flux, making it thermally responsive and suitable for situations where uniform temperature or rapid thermal response is needed.

2. **Thin Thickness Relative to Heat Diffusion**: The characteristic thickness of the body is small enough compared to the heat penetration depth for the time scale of interest.

3. **Uniform Temperature Distribution**: Due to the small size or high thermal conductivity, the body doesn't experience significant internal temperature gradients.

4. **Heat Transfer Dominated by Surface Effects**: Heat exchange occurs primarily at the surface (e.g., through convection or radiation), with negligible resistance within the body.

- **Définition of thermally thick substance :**

A thermally thick substance is one that is able to store a **large** amount of heat energy per unit volume. This means that it is able to absorb and release heat energy relatively slowly.

Examples : metals, water and concretes.

Note :

In general, thermally thick substances have a higher thermal mass (which is a property that describes how well a substance can retain heat energy.) than thermally thin substances.

This means that they are better able to regulate temperature and maintain a relatively constant temperature over time. This is why it takes longer to heat up or cool down a room with a lot of metal, water and concrete in it, compared to a room with a lot of air, glass and plastic.

How can we distinguish thermally thin from thick ?

- Thermally thin or thick is defined by the heat transfer resistance ratio (i.e) Biot's number.
  - Materials having a Biot number smaller than 0.1 labels a substance as "thermally thin," and temperature can be assumed to be constant throughout the material's volume.
  - Materials having a Biot number greater than 0.1 (a "thermally thick" substance) indicates that one cannot make this assumption, and more complicated heat transfer equations for "transient heat conduction" will be required to describe the time-varying and non-spatially-uniform temperature field within the material body

i. **Internal thermal resistance (of conduction)**

- **Definition**

Internal thermal resistance in conduction refers to the resistance to heat flow within a material as it conducts thermal energy. When heat travels through a material, it must overcome the material's internal properties, such as its thermal conductivity, which is influenced by the structure and composition of the material.

- **Expression**

The internal thermal conduction resistance of a solid is given by:

$$R_i = \frac{l}{\lambda . S}$$

l : characteristic length (longueur caractéristique) ;

λ : Thermal conductivity;

S : transfer surface (area);

Note : The characteristic length in heat transfer is a representative length scale used to describe the dimensions of a system or component undergoing heat transfer. It is a parameter that helps determine the rate of heat transfer and is crucial in defining dimensionless numbers such as the Reynolds number, Prandtl number, and Nusselt number.

For different geometries, the characteristic length varies. Here are some examples of caracteristic length:

1. Flat Plate: For heat transfer over a flat plate, the characteristic length is typically the length of the plate perpendicular to the direction of flow.
2. Cylinder: For a cylinder, the characteristic length is often the diameter of the cylinder.
3. Sphere: For a sphere, the characteristic length is the diameter of the sphere.
4. Rectangular Fin: In the case of a rectangular fin, the characteristic length is the thickness of the fin.

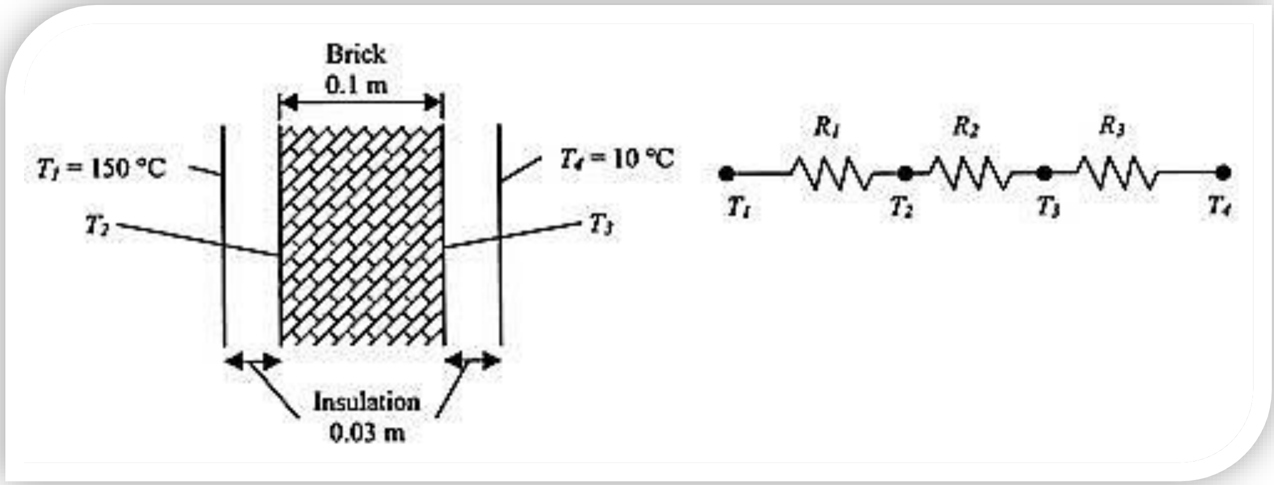
- ### **Internal resistance of systems involving multiple layers or components**

If the systems have several layers or parts, each layer (or substance) will add to the system's overall thermal resistance, and the sum of the individual resistances from each material or layer is used to determine the overall thermal resistance in such systems. The process depends on whether the layers are arranged in parallel or series:

1. **Series Configuration (e.g., layered materials in one path)**: If heat flows through each material one after the other (like heat moving from one material to another in a stack), the total thermal resistance is simply the sum of the individual resistances. Mathematically:

$$R_{\text{total}} = R_1 + R_2 + R_3 + \cdots + R_n$$

where $R_1, R_2, \dots, R_n$ are the thermal resistances of each layer or material.



2. **Parallel Configuration (e.g., heat spreading out in different paths)**:

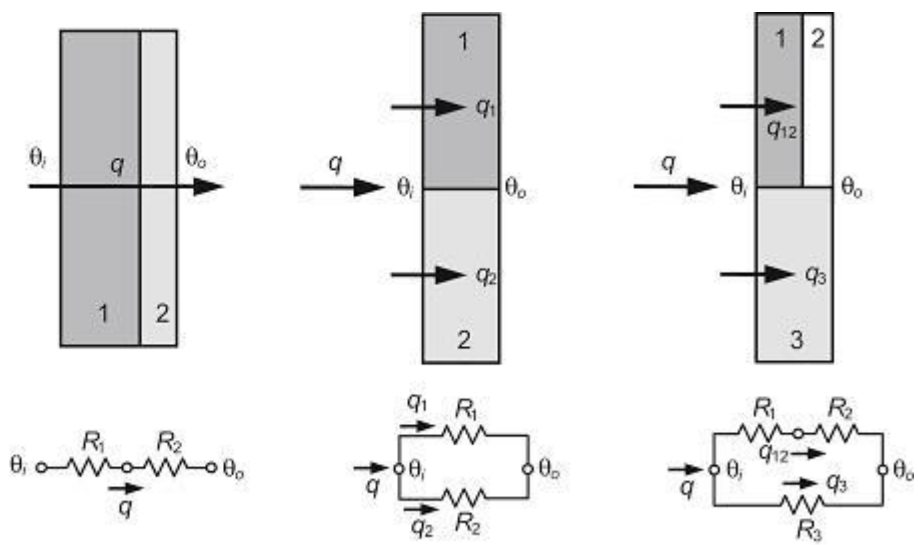
If heat can flow through multiple paths, the total thermal resistance is calculated using the reciprocal of the sum of reciprocals:

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

In this case, the heat can flow through different layers simultaneously, and the total resistance is lower than the sum of individual resistances.

Mixte configuration

In the mixte configuration we have :



(a) (b) (c)

ii. External resistance (of convection)

The external resistance (of convection) is given by :

$$R_{ext} = \frac{1}{h.S}$$

h : convection transfer coefficient;

iii. Resistance of a thermally thin body

The internal resistance of a thermally thin body is low (negligible). This condition leads us to consider the homogeneity of the temperature at any point of the body at a given instant t: T is then a function of t only. It is uniform at every point on the body.

iv. Resistance of a thermally thick body (medium)

A body (medium) is said to be thermally thick if its internal resistance is considerable (not negligible). This condition leads to the consideration of the non-homogeneity of the temperature in the body at a given instant. t : T is then a function of x, y, z et t.

We write :

$$T = T(x, y, z, t).$$

v. Biot number as a classification criterion

The Biot number is defined as the ratio between internal (conduction) and external (convection) resistance. It is denoted by B_i and given by:

$$B_i = \frac{\frac{l}{\lambda.S}}{\frac{1}{h.S}} = \frac{h.l}{\lambda}$$

- The medium (the body) is said to be thin if the number $\boldsymbol{B_i \leq 0.1}$.

  The physical meaning is that the external resistance blocks the heat flow. This means that the temperature is uniform along the direction of L.