TD Series No. 01

Heat and mass transfer

Fourier's law - Newton's law and Stefan Boltzmann's law

Exercise 01 (Heat flow)

A cylindrical electrical resistor (D=0.4cm, L=1.5cm) on a printed circuit board dissipates a power of 0.6 W. Assuming that heat is transferred uniformly across all surfaces.

Determine: (a) the amount of heat dissipated by this resistor over a 24-hour period, (b) the heat flux, (c) the fraction of heat dissipated by the top and bottom surfaces.

Solution

Assumptions: Heat is transferred uniformly across all surfaces.

Analysis :

(a) the heat dissipated by this resistor over a 24-hour period is :

$$Q_{total} = \dot{Q}_{total}\Delta t = (0.6\ \text{W})(24\ \text{h}) = \mathbf{14.4\ Wh = 51.84\ kJ} \quad (1\ \text{W.h} = 3600\ \text{W.s} = 3.6\ \text{kJ})$$

a) The heat flux at the resistor surface is :

$$S_{total} = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.4\,\text{cm})^2}{4} + \pi(0.4\,\text{cm})(1.5\,\text{cm}) = 0.251 + 1.885 = 2.136\,\text{cm}^2$$

$$\dot{q} = \frac{\dot{Q}_{total}}{S_{total}} = \frac{0.60\ \text{W}}{2.136\,\text{cm}^2} = \mathbf{0.2809\ W/cm^2}$$

(c) **Assuming** the heat transfer coefficient is uniform, heat transfer is proportional to surface area. Then, the fraction of heat dissipated by the top and bottom surfaces of the resistor becomes :

$$\frac{Q_{haut-\text{bas}}}{Q_{\text{total}}} = \frac{S_{\text{haut-bas}}}{S_{\text{total}}} = \frac{0.251}{2.136} = \mathbf{0.118} \quad \text{or} \quad (11.8\%)$$

Discussion: Heat transfer from the top and bottom of surfaces is low compared with that from the cylindrical surface. This is the case for walls in the thermal sense.

Discussion : Le transfert de chaleur par le haut et le bas des surfaces sont faibles par rapport à celle transféré par la surface cylindrique. C’est le cas des murs au sens thermique du terme.

Exercise 02 (Flux density and thermal resistance)

A tank contains 3 m^3 of hot water at $Ti = 80\ °C$. It is perfectly heat-insulated, except for a section with a surface area of S=0.3 m^2 . After Δt=5 hours, the water temperature has dropped by 0.6 °C at an ambient temperature of 20°C. Assuming that the heat capacity of the tank is $10^3\ kcal/°C$. Calculate:

1. the amount of heat lost in 5 hours,
2. heat flux through the lid,
3. the heat flux density through the lid,
4. thermal resistance of the lid.

Solution

1. **The amount of heat lost in 5 hours is :**

$$Q(t=5h) = \left((m.c)_{eau} + (m.c)_{réservoir}\right)\left(T_{eau}(t=5h) - T_{eau}(t=0)\right)$$

$$Q(t=5h) = \left(\left(3*10^3 * 1kcal/°C\right) + \left(10^3\ kcal/°C\right)\right)\left(-0.6°C\right) = -2.4\ 10^3\ kcal \quad \text{(Système MKH)}$$

$$Q(t=5h) = -2.4\ 10^3\ kcal = -2.4\ 10^3 * 4.185 kJ = -10044 kJ \quad \text{(Système International)}$$

4.186 J/g°C

Water has a specific heat capacity of 4.186 J/g°C, meaning that it requires 4.186 J of energy (1 calorie) to heat a gram by one degree.

The calorie was originally defined as the amount of heat required at a pressure of 1 standard atmosphere to raise the temperature of 1 gram of water 1° Celsius. Since 1925 this calorie has been defined in terms of the joule, the definition since 1948 being that one calorie is equal to approximately 4.2 joules. Nov 10, 2023

2. **Calculation of heat flux through the cover (the lid):**

$$\phi(t=5h) = \frac{Q(t=5h)}{\Delta t} = \frac{-2.4\ 10^3\ kcal}{5h} = -480\frac{kcal}{h} \quad \text{(Système MKH)}$$

$$\phi(t=5h) = -480\frac{4185 J}{60*60 s} = -480*1.1625\ W = -558\ W \quad \text{(SI)}$$

3. **Calculation of heat flux density through the cover**

$$\varphi(t=5h) = \frac{\phi(t=5h)}{S} = \frac{-480\frac{kcal}{h}}{0.3m^2} = -1600\frac{kcal}{m^2.h} \quad \text{(Système MKH)}$$

$$\varphi(t=5h) = -1600\frac{kcal}{m^2.h} = -1600\frac{4185}{3600}\frac{J}{m^2.s} = -1600*1.1625\frac{W}{m^2} = -1860\frac{W}{m^2} \quad \text{(SI)}$$

4. **lid thermal resistance by electrical thermal analogy :**

$$\Delta U = R_{électrique} * I \qquad \Delta T = R_{thermique} * \phi \quad \Rightarrow \quad R_{thermique} = \frac{\Delta T}{\phi}$$

$$R_{thermique} = \frac{\Delta T}{\phi} = \frac{T_{air} - T_{eau}(t=5h)}{\phi(t=5h)} = \frac{\left(20-(80-0.6)\right)°C}{-480\frac{kcal}{h}} = 0.12375\frac{°C.h}{kcal} \quad \text{(Système MKH)}$$

$$R_{thermique} = 0.12375\frac{°C.h}{kcal} = 0.12375*\frac{°C.3600.s}{4185.J} = 0.12375*\frac{1}{1.1625}\frac{°C}{W} = 0.1064\frac{°C}{W} \quad \text{(SI)}$$

Exercise 03 (Fourier's law)

Determine the steady-state heat transfer rate per unit area (flux density) through a 4 cm thick slab, assuming it is homogeneous with uniform temperature maintenance on both sides at 38 °C and 21 °C, respectively. The thermal conductivity of its material is 0.19 W.m-1K-1. The temperature profile is assumed to be linear.

Solution

we know Fourier's law, which relates heat flux density to temperature gradient:

$$\vec{\varphi} = -\lambda\,\overrightarrow{\mathrm{grad}}\mathrm{T} = -\lambda\,\vec{\nabla}\mathrm{T}$$

In cartesian coordinates:

$$\overrightarrow{\mathrm{grad}}\mathrm{T} = \frac{\partial \mathrm{T}}{\partial \mathrm{x}}\,\vec{\imath} + \frac{\partial \mathrm{T}}{\partial \mathrm{y}}\,\vec{\jmath} + \frac{\partial \mathrm{T}}{\partial \mathrm{z}}\,\vec{k}$$

In the present case T is a function of x only; then: $\frac{\partial \mathrm{T}}{\partial \mathrm{y}} = 0$ et $\frac{\partial \mathrm{T}}{\partial \mathrm{z}} = 0$

So the flux ϕ will only take place in the direction of x. So, it is gven by $\varphi = -\lambda\,\frac{\partial \mathrm{T}}{\partial \mathrm{x}}$

φ: is the amount of heat transferred per unit time per unit area; therefore

$$\varphi = -\lambda\,\frac{\partial \mathrm{T}}{\partial \mathrm{x}} = -\lambda\,\frac{\Delta \mathrm{T}}{\Delta \mathrm{x}} = -\lambda\,\frac{\mathrm{T}_2 - \mathrm{T}_1}{\mathrm{x}_2 - \mathrm{x}_1} = -0.19\,\frac{21 - 38}{0.04} = +80.75W/m^2$$

Exercise 04 (Newton's Law)

The heat transfer coefficient for forced convection of a fluid flowing over a cold surface is 225 W/m².°C for a particular situation. The temperature of the flowing fluid is 120°C and the surface is maintained at 10°C.

- Determine the rate of heat transfer per unit area (flux density) from the fluid to the surface.

Solution

Newton's law gives the relationship between the heat transfer coefficient, the temperature of a solid and that of a fluid, and the quantity of heat transferred. It has the form :

$$\vec{\phi} = h\, S\, (T_s - T_\infty)\, \vec{n}$$

$\vec{n}$: unit vector perpendicular to the transfer surface.

h: heat transfer coefficient.

Then, the heat flux density is :

$$\varphi = \frac{\phi}{S} = h(T_s - T_\infty) = h\left(T_{cs} - T_f\right) = 225\ (10\text{-}120) = -\ 24750\ \text{W/m}^2$$

The minus sign indicates that heat has been transferred from the fluid to the solid.

Exercise 05 (Stefan-Boltzmann law)

After sunset, an amount of radiant energy can be captured by a person standing next to a brick wall. The wall surface is heated to 44°C and the emissivity of the brick is 0.92.

- What is the heat flux per unit area of the wall at this temperature?

Solution

The Stefan-Boltzmann equation gives the relationship between the temperature of a gray body, the surface area of the body (m²), the Stephan-Boltzmann constant (5.67 10^{-8}W $m^{-2}K^{-4}$), the emissivity and the heat flux transmitted by radiation. It has the forme:

$$\phi = \varepsilon \sigma S \; {T_S}^4$$

The heat flux per unit area (flux density) is therefore : σ= 5.66 . 10^{-8} Wm^{-2} K^{-4} :cnstant of Stefan-Boltzmann

$$\varphi = \frac{\phi}{S} = \varepsilon \sigma \,.\, {T_S}^4 = (0.92)(5.6697 * 10^{-8})(44 + 273)^4 = 526.72 \, W/m^2$$

Note: Temperature T is the absolute temperature. It is in Kelvin.

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