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**Solution de la série de TD N° 03**
**Heat and mass transfer**

**Convective Heat Transfer**

# Before starting

A large number of convective heat transfer problems come down to heat flow calculations. The major problem in this calculation is determining the convective conduction coefficient h . To achieve this, a long road must be travelled. Starting with the numerous parameters describing the heat transfer phenomena of the system under study, through a dimensional analysis, after the Vaschy-Buckingham theorem, then combinations of parameters giving the dimensioned groups, to experimental measurements to determine the correlation laws between groups, leading to the determination of the coefficient h, with knowledge of the fluid characteristics.

Un grand nombre de problème du transfert thermique par convection revient au calcul du flux thermique. Le problème majeur pour ce calcul  est celui de la détermination du coefficient de conduction convective $h$. Pour y arriver, un long chemin doit être parcouru. Partons des nombreux paramètres descriptifs du phénomènes de transfert du système étudié, passant par une analyse dimensionnelle, après le théorème de Vaschy-Buckingham, puis des combinaisons des paramètres donnant les groupes adimensionnés allant aux mesures expérimentales pour déterminer les lois de corrélation entre groupe se qui conduit à la détermination du coefficient h, par connaissance des caractéristiques du fluide.

**We briefly describe the methodology used to solve convection problems.**

- **Methodology for forced convection: (caused by artificial circulation (pump, turbine, fan) of a fluid).**

1. **Calculation of the dimensionless Reynolds number, Re, and Prandtl number, Pr;;**
2. **Looking at the value of Re and the configuration, we move on to the choice of correlation;**
3. **Using the chosen correlation, we calculate the Nusselt number, Nu ; ;**
4. **The result enables us to calculate the coefficient $h = \lambda.Nu/D$ t (ou $h = \lambda.Nu/L$ et de $\Phi = h.S.(T_P - T_\infty)$ ;**

- **Methodology for natural convection:**

1. **Calculation of Grashof, Gr, and Prandtl, Pr, dimensionless numbers;**
2. **Looking at the value of Gr and the configuration, we move on to the choice of correlation;**
3. **Using the chosen correlation, we calculate the Nusselt number, Nu;**

4. The result is used to calculate the coefficient $h = \lambda . Nu/D$ t (ou $h = \lambda . Nu/L$ et de $\Phi = h.S.(T_P - T_\infty)$ ;

# Exemples de corrélations pour la détermination du nombre de Nusselt: Nu:

## 1. Cas de convection forcée sans changement d'état:

### i. Ecoulement parallèle à une surface plane isotherme

- **Ecoulement luminaire:**
  - conditions: $Re < 5.10^5$ $et$ $Pr > 0.7$
    - $Nu = 0.332\ Re^{1/2} Pr^{1/3}$ $(Local)$
    - $\overline{Nu} = 0.664\ Re^{1/2}\ Pr^{1/3}$ (Moyen)
- **Ecoulement turbulent :**
  - conditions: $Re > 5.10^5$ $et$ $0.6 < Pr < 60$
    - $Nu = 0.0296\ Re^{4/5} Pr^{1/3}$

### ii. Ecoulement autour d'un tube: (corrélation de Hilpert)

**Condition :** $Pr > 0.7$

| $Re_D$ | $C$ | $m$ |
|---|---|---|
| 0.4 – 4 | 0.989 | 0.33 |
| 4 – 40 | 0.911 | 0.385 |
| 40 – 4 10³ | 0.683 | 0.466 |
| 4 10³ - 4 10⁴ | 0.193 | 0.618 |
| 4 10⁴ - 4 10⁵ | 0.027 | 0.805 |

- $Nu = C\ Re^m . Pr^{0.33}$

### iii. Ecoulement dans une conduite : (corrélation de Colbun: Régime turbulent)

- $Nu = 0.023\ \ Re^{0.8} Pr^{1/3}$

## 2. Cas de convection naturelle sans changement d'état:

$$Nu = C.Re^n$$

**N=1/4 (régime luminaire) , n=1/3 (régime turbulent)**

| Géométrie | Dimension caractéristique | Régime laminaire | Régime turbulent |
|---|---|---|---|
| Plaque verticale | Hauteur | 0,59 | 0,13 |
| Cylindre horizontal | Diamètre extérieur | 0,53 | 0,10 |
| Plaque horizontale chauffant vers le haut | Largeur | 0,54 | 0,14 |
| Plaque horizontale chauffant vers le bas | Largeur | 0,27 | 0,07 |

# Solution

## Exercice 01

Consider a vertical wall in a natural air flow of velocity v. Find, approximately, the convective conduction coefficient h in the cases : a) $v_1 = 6m/s$ ; b) $v_2 = 12m/s$ and c) $v_3 = 18\,m/s$.

## Solution de l'exercice 01

The coefficient h is given, as a function of speed, by the approximate expression where

$5\,m/s < v < 18\,m/s$

$$\boldsymbol{h = 10.45 - v + 10\sqrt{v}}$$

1. $h_1 = 10.45 - 6 + 10\sqrt{6} = 28.94$
2. $h_2 = 10.45 - 12 + 10\sqrt{12} = 33.09$
3. $h_3 = 10.45 - 18 + 10\sqrt{18}$ =34.87

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## Exercice 02 (Convective heat transfer coefficient)

A pipe in a radiator with a circular cross-section, diameter D=2.4 mm, carrying hot water. The temperature difference between water and wall $T_p - T_\infty = 35\,°C$. The flowing water is steady-state and has characteristics such that the Reynolds number Re equals 53.18. Such a situation leads to the application of COLBURN's correlation, which gives the Nuselt number:

$$Nu = 0.023\,Pr^{1/3}Re^{0.8}$$

Knowing that at 52°C the thermal conductivity of the fluid $\lambda = 0.639\,\frac{W}{m.°C}$, dynamic viscosity $\mu = 0.53\,10^{-3}\mathrm{Pa.s}$, and the heat capacity at constant pressure is $\mathrm{C} = 4184\,\frac{\mathrm{J}}{\mathrm{kg.°C}}$.

1. Calculate the convective conduction coefficient h ;
2. Calculate the heat flux density ;

3. Calculate the heat flux $\phi$

**Solution de l'exercice 02:**

First, let's calculate Prandtl's dimensioned number :

$$Pr = \frac{\nu}{\alpha} = \frac{\mu\, c_p}{\lambda} = \frac{0.55\ 10^{-3}.4184}{0.639} = 3.60$$

Knowing that Re=57.124 and that the configuration follows the COLBURN correlation :

$$Nu = 0.023\, Pr^{1/3} Re^{0.8}$$

So $Nu = 224$

But : $$Nu = \frac{h.D}{\lambda}$$

so $$h = \frac{\lambda}{D}\, Nu = \frac{0.639}{0.02}.224 = 7156\, W.m^{-2}K^{-1}$$

Once h is determined, the heat flow is :

$$\phi = h.S.(T_P - T_\infty) = 15.7\ kW/m$$

**Exercice 3**

Consider the wall of a room measuring 3 m x 5 m, heated by the sun to a temperature of 38°C at its outer surface. The surrounding air temperature is 22°C.

- \- Calculate the heat power (heat flow) exchanged by convection between the wall and the outside, knowing that, for air ; ρ=1.149 kgm3, μ=18.4 10-6 kg/(m.s), λ=0.0258 W/(m.K) and Cp=1006 J/(kg.K).

## Solution de l'exercice 03

The present case is that of natural convection with no change of state between air and a vertical wall. We then apply the corresponding experimental correlation given above. This is that of a vertical plate in the case of natural convection.

This is where the Nusselt number is given by:

$$N_u = C.(R_a)^n = C.(Gr.Pr)^n$$

The quantities are calculated at the mean temperature, $T_{moy}$ :

$$T_{moy} = \frac{38\,°C + 22\,°C}{2} = 30\,°C$$

so : $\alpha = \frac{1}{T_{moy}} = \frac{1}{30+273.15} = 0.003\,K^{-1}$

Calculating the number of Grashof :

$$G_r = \frac{\alpha.g.\Delta T.\rho^2 L^3}{\mu^2} = \frac{0.003.10.(38-22)\,1.149\,^2 5^3}{(18.4\ \ 10^{-6})^2} = 2.339 \text{ x } 10^{11}$$

We calculate the Prandtl number: $Pr = \frac{\mu.C}{\lambda} = 0.717$

And the number of Raynoldz: $Ra = 1.677\, x\, 10^{11} \gg 10^9$ , then convection is turbulent

And therefore : $C = 0.13$ ; $n = 1/3$

Now let's calculate the Nusselt number :

$$Nu = C.(Ra)^n = 0.13\,.\,(\,1.677 \text{ x } 10^{11})^{\frac{1}{3}} = 926.8$$

Thus : $h = \frac{Nu.\lambda}{D} = \frac{847.84 \text{ x } 0.0258}{5} = 4.37\ \frac{W.K}{m^2}$

And thermal power (heat flow):

$$\Phi = h.(\,Tm - Ts).S = 3.64\,(38-22)(5*3) = 873.6\,W$$