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Biology Department
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Module : Mathematics 1

Series of Exercises N°1

2025-2026

Exercise n°1. Calculate the domain of the function f defined as follows :

$$(1).\ f(x)=\frac{5x+4}{x^2+3x+2}\cdot \quad (2).\ f(x)=\sqrt{x}+\sqrt[3]{x}. \quad (3).\ f(x)=\sqrt[4]{x^2-5x}.$$

$$(4).\ f(x)=\sqrt{x-1}+\sqrt{x+1}. \quad (5).\ f(x)=\frac{1}{x+1}\cdot \quad (6).\ f(x)=1+\sin x. \quad (7).\ f(x)=\sqrt{\frac{3x-1}{x+4}}.$$

$$(8).\ f(x)=\frac{x}{|2x-4|-|x-1|}\cdot$$

Note : Leave to students : (1), (2), (5), (6).

Exercise n°2.

(I) Study the parity of the following functions :

$$(1).\ f(x)=|x|-\sqrt{2x^2+4}. \qquad (2).\ f(x)=x^2+x.$$

$$(3).\ f(x)=1-\frac{1}{x^2}\cdot \qquad (4).\ f(x)=\frac{x^3-x}{4}\cdot$$

(II) Let $n\in\mathbb{N}^*,\ \alpha\in\mathbb{Z},\ \beta\in\mathbb{R}\backslash\mathbb{Z},\ x\in\mathbb{R}$. Demonstrate the following formulas :

$$(1).\ E(\alpha)+E(-\alpha)=0. \qquad (2).\ E(\beta)+E(-\beta)=-1. \qquad (3).\ E(x+n)=E(x)+n.$$

$$(4).\ nE(x)\leq E(nx). \qquad (5).\ x-1<E(x)\leq x.$$

Note : See the course : (3) and (5).

(III) Let $x,\ y\in\mathbb{R}$. Demonstrate the following inequalities :

1. $||x|-|y||\leq|x+y|\leq|x|+|y|\leq|x+y|+|x-y|$.
2. $1+|xy-1|\leq(1+|x-1|)(1+|y-1|)$.

Note : See the course : $|x+y|\leq|x|+|y|$.

Exercise n°3. Calculate the following limits without using the l'Hospital rule :

$$(1).\ \lim_{x\to 0}\frac{x^2+2|x|}{x}\cdot \qquad (2).\ \lim_{x\to 0^+}\frac{x+2}{x^2\ln x}\cdot \qquad (3).\ \lim_{x\to 1}\frac{x-1}{x^n-1}\cdot$$

$$(4).\ \lim_{x\to +\infty} x(-x+1). \qquad (5).\ \lim_{x\to \pi}\frac{\sin^2 x}{1+\cos x}\cdot$$

Exercise n°4. Determine the values of a that ensure the continuity of the function f on $[0,2]$:

$$f(x)=\begin{cases}\dfrac{6\sin\left(a(x-1)\right)}{x-1}, & x\in[0,1[\\ 5x-a, & x\in[1,2]\end{cases}$$

Exercise n°5. Let f defined by :

$$f(x)=\begin{cases} e^x-a, & x<0\\ b\ln(x+1), & x\geq 0.\end{cases}$$

where $a,\ b\in\mathbb{R}$ unknown numbers.

1. Is the function f continuous and derivable on $\mathbb{R}^*$. ?
2. Determine the values of a and b that ensure the continuity of the function f.
3. Determine the values of a and b that ensure the derivability of the function f.

Exercise n°6. Calculate the derivative of the following functions :

$$(1).\ f(x)=\tan x. \quad (2).\ f(x)=\sin(2x+6)+\cos(3x+1). \quad (3).\ f(x)=\ln(\ln x).$$

$$(4).\ f(x)=\sqrt[3]{x^3+2}, \qquad (5).\ f(x)=\sqrt{x+\sqrt{x}}.$$

Note : Leave to students : (1), (3), (5).

Exercise n°7. Let $a_0,\ a_2,\cdots,a_n\in\mathbb{R}$, such that $a_0+\dfrac{a_1}{2}+\cdots+\dfrac{a_n}{n+1}=0$. Let the function f defined by :

$$f(x)=a_0+a_1x+\cdots+a_nx^n.$$

Prove that the equation $f(x)=0$ has at least a solution on $]0,1[$.

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