University of Jijel
Faculty of Exact Sciences and Computer Science
Department of Mathematics

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Series N. 01

Exercise 01:

1. If a and b are two non-zero positive real numbers, show that

$$1)\ \sqrt{a}+\sqrt{b}\leq\sqrt{2(a+b)},\quad 2)\ \frac{2}{\frac{1}{a}+\frac{1}{b}}\leq\sqrt{ab}.$$

2. let a, b, c be three real numbers, show that

$$1)\ ab\leq\frac{a^2+b^2}{2},\quad 2)\ ab+ac+bc\leq a^2+b^2+c^2,\quad 3)\ 3(ab+ac+bc)\leq(a+b+c)^2.$$

Exercise 02: Determine the following sets, put these sets in the form of an interval of $\mathbb{R}$ or a union of intervals.

$$A_1=\{x\in\mathbb{R}, x^2<1\},\quad A_2=\{x\in\mathbb{R}, x^3\leq 1\},\quad A_3=\{x\in\mathbb{R}, -1<\frac{2x}{x^2+1}<1\},$$
$$A_4=\{x\in\mathbb{R}^*, \frac{1}{|x|}>1\},\quad A_5=\{x\in\mathbb{R}, -1<\frac{1}{x^2-1}<1\}.$$

Exercise 03:

1. Show that $a=\sqrt{7+4\sqrt{3}}+\sqrt{7-4\sqrt{3}}$ is an integer.
2. Let $\alpha=\sqrt{4-2\sqrt{3}}+\sqrt{4+2\sqrt{3}}$. Show that $\alpha\in\sqrt{3}\mathbb{N}$.
3. Show that $\sqrt{3}\notin\mathbb{Q}$, $\frac{\ln 3}{\ln 2}\notin\mathbb{Q}$.

Exercise 04: Solve the following equations and inequalities in $\mathbb{R}$:

$1)|x-1|+|x-2|=2,\quad 2)|x+3|=5,\quad 3)|x+3|\leq 5,$

$4)|x+2|\geq 7,\quad 5)|2x-4|\leq|x+2|,\quad 6)|x-1|<x^2-x+1,$

$7)|2|x|-3| = 2|x-1|, \quad 8)|2-x| \leq 3|x|-8, \quad 9)2 \leq |x+1| \leq 3.$

Exercise 05: Let x, y be real numbers, demonstrate the following inequalities:

1. $|x|+|y| \leq |x+y|+|x-y|$.
2. $1+|xy-1| \leq (1+|x-1|)(1+|y-1|)$.
3. $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$.
4. $||x|-|y|| \leq |x-y|$.

Exercise 06:

1. Find the integer part of the following numbers:

$$0.6, \quad \ln 3, \quad -2.6, \quad \pi, \quad 8.1, \quad -\frac{5}{3}, \quad \sqrt{5}.$$

2. Prove that:

   a. $\forall x \in \mathbb{R} : [x+1] = [x]+1$.

   b. $\forall x \in \mathbb{R}, \forall n \in \mathbb{N}^* : [\frac{[nx]}{x}] = [x]$.

   c. $\forall x, y \in \mathbb{R} : [x]+[y] \leq [x+y] \leq [x]+[y]+1$.