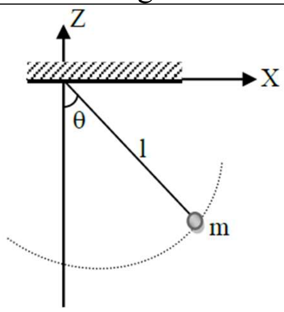
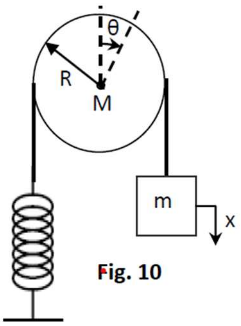
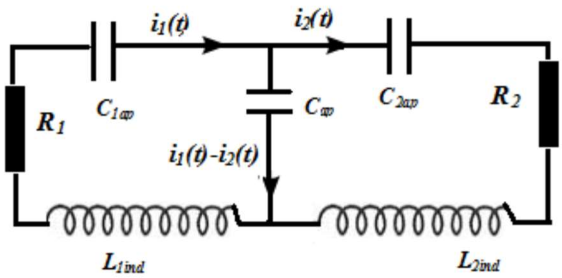
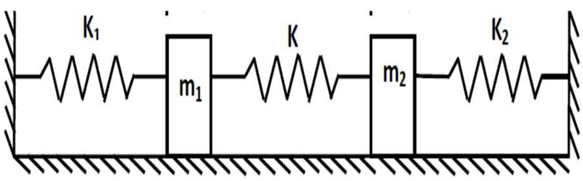
Corrigé Travaux dirigés N°2

Exercice 01

Le nombre de degrés de libertés des systèmes suivants :

	N= 3(x , y, θ) r=2 (x=l sinθ, y=l cosθ) ddl=3-2=1
	N= 2(x,θ) r=1(x=R sinθ) ddl=2-1=1
	ddl= 2
	ddl=1

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Exercice 02

Un oscillateur harmonique est décrit par l'équation : $x(t) = 0.4\sin(0.1t + 0.5)$

L'amplitude : A=0.4 m

La période : T=20π s

La fréquence : f=0.1/2π Hz

La phase à l'origine : φ=0.5 rd

La vitesse : $v(t) = 0.04\cos(0.1t + 0.5)$ m/s

L'accélération : $a(t) = -0.004\sin(0.1t + 0.5)$ m/s^2

Les conditions initiales :

$x(0) = 0.4\sin 0.5 = 0.2$ m

$v(0) = 0.04\cos(0.5) = 0.035$ m/s

$a(0) = -0.004\sin(0.5) = -0.002$ m/s^2

la position, vitesse et accélération pour t = 5s :

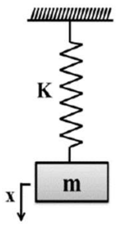
$x(5) = 0.4\sin 1 = 0.34$ m

$v(5) = 0.04\cos(1) = 0.022$ m/s

$a(5) = -0.004\sin(1) = -0.0034$ m/s^2

Exercice 03

Le Lagrangien des systèmes suivants :

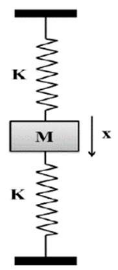


$$E_c = \frac{1}{2}m\dot{x}^2$$

$$E_p = \frac{1}{2}kx^2$$

Et

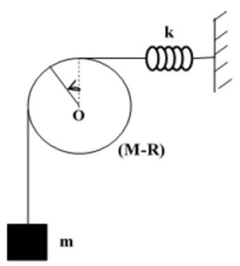
$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$



$$E_c = \frac{1}{2}M\dot{x}^2$$

$$E_P = E_{P(K)} + E_{P(K)} = \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2 = Kx^2$$

$$L = E_c - E_P$$



$$E_c = E_M + E_m$$

$$E_c = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2 \text{ , avec } x = R\theta \Rightarrow \dot{x} = R\dot{\theta}$$

$$E_c = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2 + \frac{1}{2}mR^2\dot{\theta}^2$$

$$E_c = \frac{1}{2}\left(\frac{M}{2} + m\right)R^2\dot{\theta}^2$$

$$E_P = E_{P(K)} = \frac{1}{2}kx^2 \Rightarrow E_P = \frac{1}{2}kR^2\theta^2$$

2- Fonction de Lagrange :

$$L = E_c - E_P$$

$$L = \frac{1}{2}\left(\frac{M}{2} + m\right)R^2\dot{\theta}^2 - \frac{1}{2}kR^2\theta^2$$

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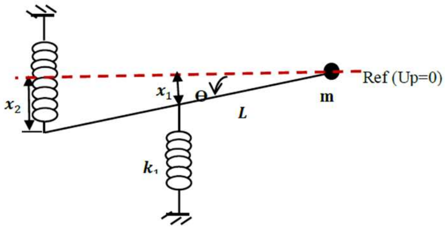


- Le nombre de degré de liberté :

$m \rightarrow rotation\ (\theta)\ \Rightarrow N = 1$

$r = 0$

Donc : $d = 1$

- L'énergie cinétique :

$T = \frac{1}{2}J\dot{\theta}^2$

où $J = ml^2$

$\Rightarrow T = \frac{1}{2}ml^2\dot{\theta}^2$

- L'énergie potentielle :

$U_{tot} = U_{k_1} + U_{k_2}$

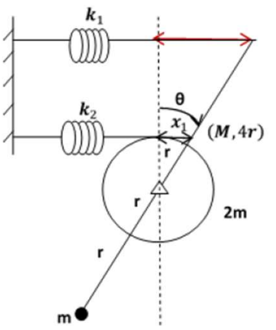
$U_{k_1} = \frac{1}{2}k_1x_1^2$ où $x_1 = \frac{l}{2}\theta$

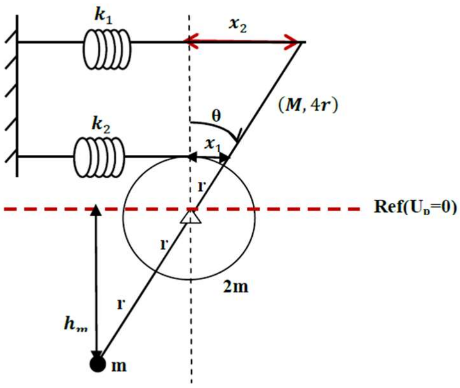
$\Rightarrow U_{k_1} = \frac{1}{2}\left(\frac{k_1}{4}\right)l^2\theta^2$

$U_{k_2} = \frac{1}{2}k_2x_2^2$ où $x_2 = l\theta$

$\Rightarrow U_{k_2} = \frac{1}{2}k_2l^2\theta^2$

$U_{tot} = \frac{1}{2}\left[\frac{1}{4}k_1 + k_2\right]l^2$





- Le nombre de degré de liberté :

$2m \rightarrow rotation(\theta)$

$m \rightarrow rotation(\theta)\ \Rightarrow N = 1\ ,\ r = 0$

$M \rightarrow rotation(\theta)$

Donc : $d = 1$

- L'énergie cinétique :

$T_{tot} = T_m + T_M + T_{2m}$

- $T_m = \frac{1}{2}J_m\dot{\theta}^2$ où $J_m = m(2r)^2 = 4mr^2$

Donc : $T_m = \frac{1}{2}m(2r)^2\dot{\theta}^2 = \frac{1}{2}(4m)r^2\dot{\theta}^2$

- $T_{2m} = \frac{1}{2}J_{2m}\dot{\theta}^2$ où $J_{2m} = \frac{1}{2}(2m)r^2 = mr^2$

Donc : $T_{2m} = \frac{1}{2}(m)r^2\dot{\theta}^2$

- $T_M = \frac{1}{2}J_M\dot{\theta}^2$ où $J_M = \frac{1}{12}ML^2 = \frac{1}{12}M(4r)^2 = \frac{4}{3}Mr^2$

Donc : $T_M = \frac{1}{2}\left(\frac{4}{3}M\right)r^2\dot{\theta}^2$

$$T_{tot} = \frac{1}{2}\left(4m + m + \frac{4}{3}M\right)r^2\dot{\theta}^2 = \frac{1}{2}\left(5m + \frac{4}{3}M\right)r^2\dot{\theta}^2$$

- L'énergie potentielle :

$$U_{tot} = U_m + \overbrace{U_{2m}}^{0} + \overbrace{U_M}^{0} + U_{k_1} + U_{k_2}$$

- $U_m = -mgh_m$ avec $h_m = 2rcos\theta$

Donc : $U_m = -2mgrcos\theta$

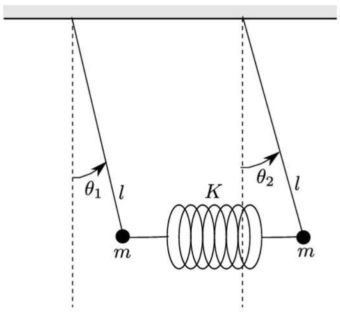
- $U_{k_1} = \frac{1}{2}k_1x_1^2$ avec $x_1 = r\theta$

Donc : $U_{K_1} = \frac{1}{2}k_1r^2\theta^2$

- $U_{K_2} = \frac{1}{2}k_2x_2^2$ avec $x_2 = 2r\theta$

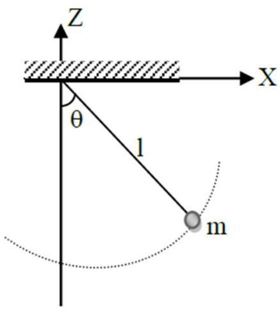
Donc : $U_{k_2} = \frac{1}{2}(4k_2)r^2\theta^2$

$$U_{tot} = \frac{1}{2}(k_1 + 4k_2)r^2\theta^2 - 2mgrcos\theta$$



$$T = \frac{1}{2}ml^2\,\dot\theta_1^2 + \frac{1}{2}ml^2\,\dot\theta_2^2$$

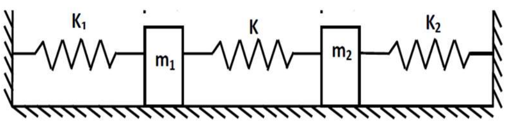
$$U = \frac{1}{2}\left[Kl^2 + mgl\right]\theta_1^2 + \frac{1}{2}\left[Kl^2 + mgl\right]\theta_2^2 - Kl^2\theta_1\theta_2$$



$$L = T - U$$

$$T = \frac{1}{2}I_{/D}\dot\theta^2 = \frac{1}{2}ml^2\dot\theta^2$$

$$U_{mas} = +mgh = +mg(l - l\cos\theta) = +mgl(1-\cos\theta)$$



$$T = \frac{1}{2}m_1\dot x_1^2 + \frac{1}{2}m_2\dot x_2^2$$

$$U = \frac{1}{2}kx_1^2 + \frac{1}{2}k_0(x_1 - x_2)^2 + \frac{1}{2}kx_2^2$$