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Module : Mathematics 1

Series of Exercises N°3

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Exercise n°1. Study the nature of the series of the general term u_n :

1. $u_n = \dfrac{1}{(n+1)(n+2)}, \qquad n \geq 0.$
2. $u_n = \ln(1 + \dfrac{1}{n}), \qquad n \geq 1.$
3. $u_n = \sqrt{n+1} - \sqrt{n}, \qquad n \geq 1.$
4. $u_n = \dfrac{n^2-1}{n-1}, \qquad n \geq 2.$

Exercise n°2. Let $(u_n)_{n\in\mathbb{N}}$ be defined by :

$$u_n = \frac{n + (-1)^n}{n^2+1}, \qquad \forall\, n \in \mathbb{N}.$$

1. Prove that :
$$\frac{n-1}{n^2+1} \leq u_n \leq \frac{n+1}{n^2+1}.$$
2. Deduce the limit $\lim\limits_{n\to+\infty} u_n$ and the nature of the sequence $(u_n)_{n\in\mathbb{N}}$.

Exercise n°3. Let $(v_n)_{n\in\mathbb{N}}$ be defined by :

$$v_n = \frac{(-1)^n + n}{(-1)^n + 2}, \qquad \forall\, n \in \mathbb{N}.$$

1. Prove that :
$$v_n \geq \frac{n-1}{3}, \qquad \forall\, n \in \mathbb{N}.$$
2. Deduce the limit $\lim\limits_{n\to+\infty} v_n$ and the nature of the sequence $(v_n)_{n\in\mathbb{N}}$.

Exercise n°4. Study the nature of the following series :

1. $\displaystyle\sum_{n\geq 1} \frac{n^2+1}{n^2}$.
2. $\displaystyle\sum_{n\geq 1} \frac{2}{\sqrt{n}}$.
3. $\displaystyle\sum_{n\geq 1} \frac{1}{n\cos^2(n)}$.
4. $\displaystyle\sum_{n\geq 2} \left(\frac{1}{\ln(n)}\right)^n$.
5. $\displaystyle\sum_{n\geq 1} \frac{2^n+3^n}{n^2+\ln(n)+5^n}$.
6. $\displaystyle\sum_{n\geq 0} \frac{n}{2^n}$.
7. $\displaystyle\sum_{n\geq 1} \frac{\ln(n)}{n^{\frac{3}{2}}}$.
8. $\displaystyle\sum_{n\geq 0} \left(\frac{1}{2}\right)^{\sqrt{n}}$.
9. $\displaystyle\sum_{n\geq 1} \frac{(n!)^\alpha}{(2n)!}, \qquad \alpha\in\mathbb{R}$.

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