University of Jijel
Faculty of Exact Sciences and Computer Science
Department of Mathematics

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Series N. 03

Exercise 01: Express the following complex numbers in algebraic form:

$$1)\ \frac{9+6i}{3-4i},\quad 2)\ \frac{2+5i}{1-i}+\frac{2-5i}{1+i},\quad 3)\ \frac{(1+i)^9}{(1-i)^7}.$$

Exercise 02:

1. Express the following complex numbers in trigonomertic form:

$$1)\ 1+i,\quad 2)\ 1+\sqrt{3}i.$$

2. Calculate the real and imaginary parts of the complex number:

$$z=\left(\frac{1+\sqrt{3}i}{1+i}\right)^{125}.$$

Exercise 03:

1. Find the integers $n\in\mathbb{N}$ such that $(1+\sqrt{3}i)^n$ is a positive real number.
2. Let z be a complex number, prove that:

   a. $1+|z|^2+2Rez\geq 0$.

   b. if $z\neq 1, |z|=1 \Leftrightarrow \frac{1+z}{1-z}\in i\mathbb{R}$.
3. Let z_1, z_2 be two complex numbers, prove that:

   $|z_1-z_2|^2\leq(1+|z_1|^2)(1+|z_2|^2)$ and $|z_1+z_2|^2+|z_1-z_2|^2=2(|z_1|^2+|z_2|^2)$.

Exercise 04:

1. Solving in $\mathbb{C}$ the following equations:

1. $(3+2i)(z-1) = i$,

2. $(4-2i)z^2 = (1+5i)z$,

3. $(1+i)z^2 - (5+i)z + 6 + 4i = 0$,

4. $2z + i = \overline{z} + 1$,

5. $z^2 + z + 1 = 0$.

2. Let the equation:

$$(z^2 + 4z + 1)^2 + (3z + 5)^2 = 0 \tag{1}$$

a. Prove that (1) is equivalent to two equations of degree 2.

b. Solve the two equations, and deduce the solutions of (1).

<u>Exercise 05</u>: Let the equation:

$$z^3 - (16-i)z^2 + (89-16i)z + 89i = 0 \tag{2}$$

1. Show that (2) is equivalent to two equations of degree 1 and 2.
2. Solve the two equations, and deduce the solutions of (2).
3. Draw the triangle formed by the three points whose affixes are the solutions of (2). Then show that it is isosceles.