

Series of Exercises N°4

2025-2026

Exercise n°1. Determine and represent the domain of the function f :

1. $f(x, y) = \frac{x^2}{y - 2x^2},$
2. $f(x, y) = \frac{x^2 + y^2}{x + y},$
3. $f(x, y) = \sqrt{xy},$
4. $f(x, y) = \sqrt{y - x^2},$
5. $f(x, y) = x^2 + y + \ln(x^2 + y^2 - 2).$

Exercise n°2. Calculate the following limits :

1. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2 + xy}.$
2. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 + y^3}. \quad (\text{Leave to students})$

Exercise n°3. Study the continuity of the following functions on \mathbb{R}^2 :

1. $f(x, y) = \begin{cases} y^2 \sin\left(\frac{x}{y}\right), & y \neq 0 \\ 0 & y = 0 \end{cases}$
2. $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases} \quad (\text{Leave to students})$

Exercise n°4.

(I) Calculate the gradient of the function f at the considerate point :

$$(i). f(x, y) = \frac{x^2}{y - 2x^2}, \quad \text{at } a = (1, 1), \quad (ii). f(x, y) = \sqrt{1 + x^2 y^2}, \quad \text{at } a = (1, 2).$$

(II) Calculate the first partial derivatives of the following functions :

1. $f(x, y) = e^x \tan(y)$.
2. $f(x, y) = x^3 + y^3 - 3xy$.
3. $f(x, y) = \sqrt{x} \cos(y)$.
4. $f(x, y, z) = x e^{\frac{y}{z}}$.
5. $f(x, y, z) = \frac{xz}{x^2 + y^2}$.
6. $f(x, y) = \ln \left(\frac{x + y}{y} \right)$.

Leave to students : (1), (2) and (3).

(III) Calculate the second partial derivatives of the following functions :

1. $f(x, y) = x^2(x - y)$.
2. $f(x, y) = \ln(x + \sqrt{x^2 + y^2})$.
3. $f(x, y) = x^4 + y^3 + 2y \cos(x) + 5y$.
4. $f(x, y) = \sqrt{1 + x^2 + y^2}$.

Leave to students : (1) and (4).

Dr. Kicha Abir