

Series No. 05

Exercise 01:

Give the maximum domains of definition in $\mathbb{R}$ of the following functions:

$$1)f(x) = \frac{x^3 + 1}{1 - |x|}, \quad 2)f(x) = \sqrt{\frac{x-2}{x+1}}, \quad 3)f(x) = \ln(4x+3), \quad 4)f(x) = \frac{\cos x}{1 + \sin 2x}.$$

$$5)f(x) = 2\sqrt{x(1-x)}+1, \quad 6)f(x) = 2\sqrt{(2-x)(1-x)}+3 \text{ (Left to the students)}$$

Exercise 02: Study the parity (even or odd) of the functions:

$$1)f(x) = \frac{\tan x - x}{x^3 \cos x} \quad 2)f(x) = \frac{\sin^2 2x - \cos 3x}{\tan x}, \quad 3)f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

Exercise 03:

$$f : \mathbb{R} \rightarrow [-1, 1], x \mapsto \sin x, \quad g : \mathbb{R}^+ \rightarrow \mathbb{R}^+, x \mapsto \sqrt{x}, \quad h : \mathbb{R} \rightarrow \mathbb{R}^+, x \mapsto x^2$$

Can we define the functions: $f \circ g, g \circ f, g \circ h, h \circ g$? if so, give their definition.

Exercise 04:

1. Prove that there exists an integer N such that, for all $n > N$:

$$\left| \sin \frac{1}{n} \right| \leq \frac{1}{16}.$$

2. What is the nature of the sequence

$$u_n = \left(2 \sin \frac{1}{n} + \frac{3}{4} \cos n \right)^n.$$

3. By definition, prove that :

$$1) \lim_{x \rightarrow 3} \sqrt{x+1} = 2, \quad 2) \lim_{x \rightarrow 2} (x^2 - x + 1) = 3 \quad 3) \lim_{x \rightarrow 1^+} \frac{x+2}{x-1} = +\infty, \quad 4) \lim_{x \rightarrow +\infty} \frac{1}{x^2} = 0.$$

Exercise 05: Calculate the following limits:

$$\begin{aligned} & 1) \lim_{x \rightarrow 0} \frac{x^2 + 2|x|}{x}, \quad 2) \lim_{x \rightarrow 0} \frac{\sqrt{2x^2 + 5x + 9} - 3}{x}, \quad 3) \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x + \sqrt{x}}}}{\sqrt{1 + x}}, \quad 4) \lim_{x \rightarrow +\infty} \frac{x + \cos x}{x + \sin x} \\ & 5) \lim_{x \rightarrow +\infty} \frac{e^{3x} + 2x + 7}{e^x + e^{-x}}, \quad 6) \lim_{x \rightarrow +\infty} \frac{x \ln x + 7}{x^2 + 4}, \quad 7) \lim_{x \rightarrow +\infty} \frac{4 \sin^2 x + 3 \cos 5x}{x}, \quad 8) \lim_{x \rightarrow +\infty} \frac{\ln x}{e^{\sqrt{3x}}} \\ & 9) \lim_{x \rightarrow 0} \frac{e^{3x+2} - e^2}{x}, \quad 10) \lim_{x \rightarrow +\infty} \frac{e^{\cos x} - 1}{x - \frac{\pi}{2}}, \quad 11) \lim_{x \rightarrow 0} \frac{\sqrt{1 + x^m} - \sqrt{1 - x^m}}{x^n}. \end{aligned}$$