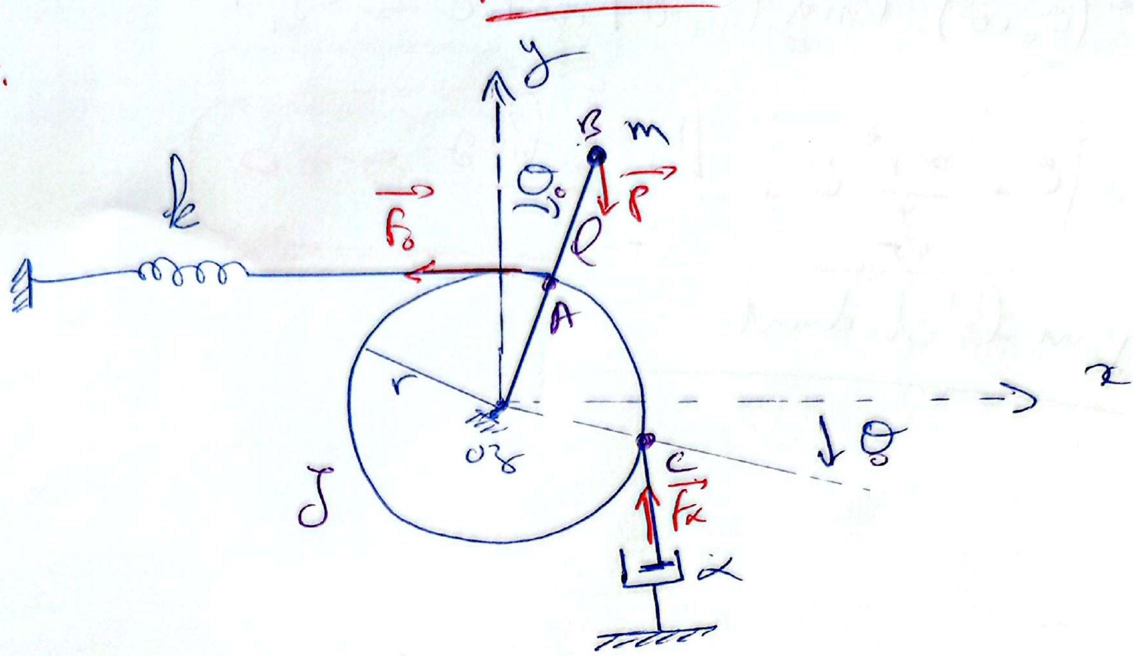
TD N3 :

Ex 1 :



1/ cdt d'équilibre :

$$\sum \vec{M}_{o_z} = \vec{0}$$

$$\vec{OA} \wedge \vec{F_0} + \vec{OB} \wedge \vec{P} = \vec{0}$$

$$\begin{pmatrix} r\sin\theta_0 \\ r\cos\theta_0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -kr\theta_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \ell\sin\theta_0 \\ \ell\cos\theta_0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ -mg \\ 0 \end{pmatrix} = \vec{0}$$

$$k r^2 \theta_0 - mg\ell\theta_0 = 0 \longrightarrow \boxed{k r^2 = mg\ell} \text{ cdt} \Leftarrow$$

2/ Eq diff (Newton)

$$\sum \vec{M}_{o_z} = -J_T \ddot{\theta}\,\vec{k}$$

/ $J_T = J + m\ell^2$
(Moment d'inertie total)
(Barre + cylindre)

$$\vec{OA} \wedge \vec{F} + \vec{OB} \wedge \vec{P} + \vec{OC} \wedge \vec{F_\alpha} = -J_T \ddot{\theta}\,\vec{k}$$

$$\begin{pmatrix} r\sin(\theta_0+\theta) \\ r\cos(\theta_0+\theta) \\ 0 \end{pmatrix} \wedge \begin{pmatrix} -kr(\theta_0+\theta) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \ell\sin(\theta_0+\theta) \\ \ell\cos(\theta_0+\theta) \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ -mg \\ 0 \end{pmatrix} +$$

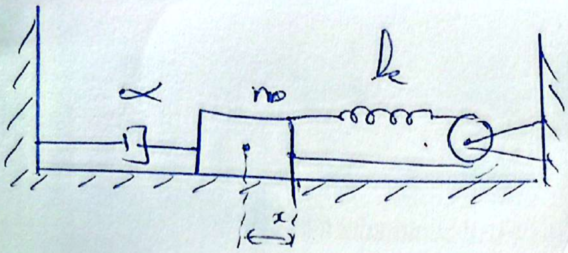
$$\begin{pmatrix} r\cos(\theta_0+\theta) \\ -r\sin(\theta_0+\theta) \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ \alpha r\dot{\theta} \\ 0 \end{pmatrix} = -J_T \ddot{\theta}$$

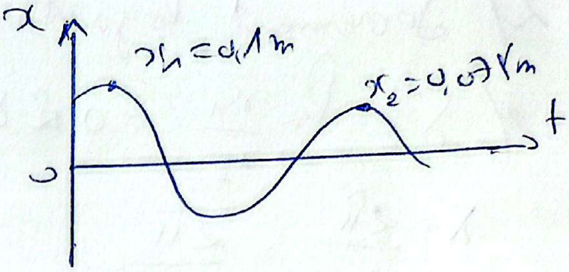
$$k r^2 (\theta_0 + \theta) - \ell m g (\theta_0 + \theta) + \alpha r^2 \dot{\theta} = - J_T r \ddot{\theta}$$

$$\rightarrow \boxed{\ddot{\theta} + \frac{\alpha r^2}{J_T} \dot{\theta} + \frac{k r^2 - \ell m g}{J_T} \theta = 0}$$

3/ pour les étudiants.

Exo2 :





1/ Eq diff du mvt (Lagrange)

$$E_c = \frac{1}{2} m V^2 = \frac{1}{2} m \dot{x}^2$$

$$E_p = \frac{1}{2} k (x+x)^2 = 2 k x^2$$

$$D = \frac{1}{2} \alpha \dot{x}^2$$

$$L = E_c - E_p = \frac{1}{2} m \dot{x}^2 - 2 k x^2$$

Éq de Lagrange

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) - \frac{dL}{dx} = -\frac{dD}{d\dot{x}}$$

$$\begin{cases} \frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = m\ddot{x} \\ \frac{dL}{dx} = -4 k x \\ \frac{dD}{d\dot{x}} = \alpha \dot{x} \end{cases}$$

$$m\ddot{x} + 4 k x = -\alpha \dot{x}$$

↳ donc on a :

$$\ddot{x} + \frac{\alpha}{m} \dot{x} + \frac{4k}{m} x = 0$$

(système libre amortie $\ddot{x} + 2\lambda \dot{x} + w_0^2 x = 0$

$$\longrightarrow 2\lambda = \frac{\alpha}{m} \longrightarrow \lambda = \frac{\alpha}{2m}$$

$$\longrightarrow \frac{4k}{m} = w_0^2 \longrightarrow w_0^2 = \frac{4 \times 1}{1} = 4$$

A

2/ Décrément logarithmique :

a/ $\delta = \ln \frac{x_1}{x_2} = 0{,}287$

$\lambda = \frac{2\pi}{w_0} = \frac{2\pi}{\sqrt{w_0^2 - \lambda^2}}$

on $\delta = \lambda T^a = \lambda \frac{2\pi}{\sqrt{w_0^2 - \lambda^2}} \Rightarrow \delta^2 = \frac{\lambda^2 4\pi^2}{w_0^2 - \lambda^2}$

$$\delta^2 w_0^2 - \delta^2 \lambda^2 = 4\pi^2 \lambda^2$$

$$\lambda^2 (4\pi^2 + \delta^2) = \delta^2 w_0^2$$

$$\lambda = \frac{\delta w_0}{\sqrt{4\pi^2 + \delta^2}} \quad B$$

de A et B on aura :

$$\lambda = \frac{\alpha}{2m} = \frac{\delta w_0}{\sqrt{4\pi^2 + \delta^2}}$$

$$\rightarrow \alpha = \frac{2 w_0 m \delta}{\sqrt{4\pi^2 + \delta^2}}$$

coefficient de frottement

$w_0 = \sqrt{\frac{4K}{m}} = 2 \text{ rad/s} \longrightarrow \boxed{\alpha = 0{,}1826 \text{ N/m}}$

b/ $\delta = \lambda T^a = \frac{\alpha}{2m} T^a \longrightarrow T^a = \frac{2m\delta}{\alpha} = 3{,}143 \text{ s}$

$\boxed{T^a = 3{,}143 \text{ s}}$ pseudo-période

c/ constante de temps

$\tau = \frac{1}{\lambda} = \frac{2m}{\alpha} = 10{,}91$ $\quad \boxed{\tau = 10{,}91 \text{ s}}$