

1/ Eq diff de mouvement (Lagrange)

$$L = E_c - E_p$$

$$E_c = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2$$

$$E_p = m_1 g l_1 (1 - \cos \theta_1) + m_2 g l_2 (1 - \cos \theta_2) + \frac{1}{2} k (l_1 \theta_1 - l_1 \theta_2)^2$$

$$D = \frac{1}{2} \alpha (l_1 \dot{\theta}_1 - l_1 \dot{\theta}_2)^2$$

$$\textcircled{1} \frac{d}{dt} \left(\frac{dL}{d\dot{\theta}_1} \right) - \frac{dL}{d\theta_1} = - \frac{dD}{d\dot{\theta}_1}$$

$$\frac{dL}{d\theta_1} = -m_1 g l_1 \sin \theta_1 - k l_1 (l_1 \theta_1 - l_1 \theta_2)$$

$$\frac{dL}{d\dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1 \rightarrow \frac{d}{dt} \left(\frac{dL}{d\dot{\theta}_1} \right) = m_1 l_1^2 \ddot{\theta}_1$$

$$\frac{dD}{d\dot{\theta}_1} = \alpha l_1 (l_1 \dot{\theta}_1 - l_1 \dot{\theta}_2) = \alpha l_1^2 (\dot{\theta}_1 - \dot{\theta}_2)$$

On a donc :

$$m_1 l_1^2 \ddot{\theta}_1 + m_1 g l_1 \sin \theta_1 + k l_1^2 \theta_1 - k l_1^2 \theta_2 = -\alpha l_1^2 \dot{\theta}_1 + \alpha l_1^2 \dot{\theta}_2$$

$$m_1 l_1^2 \ddot{\theta}_1 + \alpha l_1^2 \dot{\theta}_1 + (m_1 g l_1 + k l_1^2) \theta_1 - k l_1^2 \theta_2 - \alpha l_1^2 \dot{\theta}_2 = 0$$

$$\frac{dL}{dt} = -m_2 g l_2 \dot{\theta}_2 + k l_1^2 (\dot{\theta}_1 - \dot{\theta}_2)$$

$$\frac{dL}{d\dot{\theta}_2} = m_2 l_2 \dot{\theta}_2 \rightarrow \frac{d}{dt} \left(\frac{dL}{d\dot{\theta}_2} \right) = m_2 l_2 \ddot{\theta}_2$$

$$\frac{dD}{d\dot{\theta}_2} = -\alpha l_1^2 (\dot{\theta}_1 - \dot{\theta}_2)$$

$$\frac{d}{dt} \left(\frac{dL}{d\dot{\theta}_2} \right) - \frac{dL}{d\dot{\theta}_2} = -\frac{dD}{d\dot{\theta}_2}$$

$$m_2 l_2 \ddot{\theta}_2 + (m_2 g l_2 + k l_1^2) \theta_2 + \alpha l_1^2 \dot{\theta}_2 = k l_1^2 \theta_1 + \alpha l_1^2 \dot{\theta}_1$$

→ circuit RLC équivalent.

$$m_1 \rightarrow L_1$$

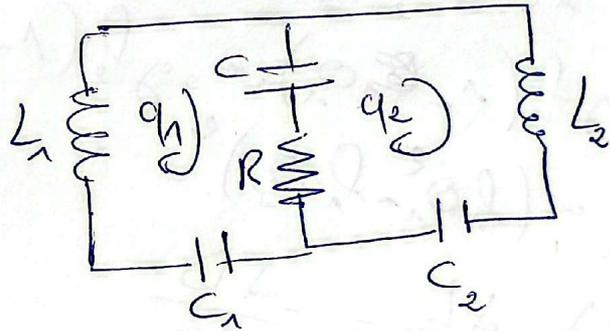
$$m_2 \rightarrow L_2$$

$$m_1 g l_1 \rightarrow C_1$$

$$m_2 g l_2 \rightarrow C_2$$

$$k \rightarrow C$$

$$\alpha \rightarrow R$$



$$\frac{m_1 l_1^2 \ddot{\theta}_1}{L_1} + \frac{\alpha l_1^2 \dot{\theta}_1}{R} + \frac{(k l_1^2 + m_1 g l_1) \theta_1}{C} = \frac{k l_1^2 \theta_2}{C} + \frac{\alpha l_1^2 \dot{\theta}_2}{R}$$

$$\frac{m_2 l_2^2 \ddot{\theta}_2}{L_2} + \frac{\alpha l_1^2 \dot{\theta}_2}{R} + \frac{(m_2 g l_2 + k l_1^2) \theta_2}{C} = \frac{k l_1^2 \theta_1}{C} + \frac{\alpha l_1^2 \dot{\theta}_1}{R}$$