Series 08: Statistical Sampling

Exercise 01

Consider a population of 6 students whose scores (out of 10) in a statistics exam are: 1, 3, 5, 7, 8, 6.

1. Compute the population mean (m) and the population variance (σ^2).

2. Calculate the proportion (P) of students who scored more than 5.

Now consider selecting two random samples (E1 and E2), each of size n = 3:

E1 = {1, 3, 8}   and   E2 = {3, 5, 7}

3. Determine the number of possible samples:

   - With replacement

   - Without replacement

4. Compute the sample means ($\bar{x}1$ and $\bar{x}2$).

5. Compute the sample variances ($s'_1{}^2$ and $s'_2{}^2$).

6. Compute the sample proportions (f_1 and f_2) of students who scored more than 5.

Exercise 02 : Consider a population of students whose exam scores are: 1, 3, 5.
1. Compute the population mean (m) and variance (σ^2).

2. Compute the proportion (P) of students scoring more than 1.

A random sample of size n = 2 is drawn.

3. Determine the number of possible samples:

   - Case 1: With replacement

   - Case 2: Without replacement

Assume sampling is done with replacement.

4. Define the probability distribution of the random variable $\bar{X}$ (sample mean), and compute $E(\bar{X})$ and $V(\bar{X})$.

5. Define the probability distribution of the random variable F (sample proportion of scores > 1), and compute $E(F)$ and $V(F)$.

6. Define the probability distribution of the random variable S'^2 (sample variance), and compute $E(S'^2)$.

7. What changes if sampling is done without replacement in questions 4 and 5? Verify with calculations.

Exercise 03

In a population of 5 newborns, the birth weights (kg) are: 2.50, 2.53, 2.60, 2.62, 2.70.

1. Determine the population mean and variance.

2. Determine the proportion of newborns weighing more than 2.60 kg.

All possible samples of size 3 are drawn with replacement (non-exhaustive sampling).

3. How many samples can be formed?

4. Compute $E(\bar{X})$ and $V(\bar{X})$ for the sampling distribution of the sample means.

5. Compute $E(F)$ and $V(F)$ for the sampling distribution of the sample proportions.

6. Repeat questions 3, 4 and 5 if sampling is done without replacement.

Exercise 04

After grading a residency entrance examination, the scores have a mean of 12 and a standard deviation of 3. Random samples of 100 candidates are drawn with replacement.

1. What is the distribution of the random variable $\bar{X}$?

2. What is the probability that the sample mean is greater than 12.5?

3. What is the probability that the sample mean is between 12.5 and 12.9?

Exercise 05

We study a qualitative characteristic: G (boy) with proportion p = 0.52, and F (girl) with proportion 0.48. A sample of size 400 is drawn. The population is large enough to assume sampling with replacement.

1. Determine the probability distribution of the random variable F (sample proportion).

2. Compute the probability that the percentage of boys is between 50% and 54%.

3. Compute the probability that the percentage of girls is less than 45%.