University of Jijel - Faculty of exact sciences and computer science - Mathematics department

Series No. 01 Analysis 02 *2025/2026*

Exercise 01: Integrate by parts the following :

$$I_1 = \int \sin(3x)\,(x^3+1)dx\,, \qquad I_2 = \int e^{-4x}\cos(2x)\,dx\,, I_3 = \int \frac{\ln x}{x^5}dx,$$

$$I_4 = \int 2x(\arctan x)dx,\ I_5 = \int \left(\frac{\ln x}{x}\right)^2 dx\,,\ \ I_6 = \int x\sqrt{x+3}dx,$$

$$I_7 = \int x^5 e^{x^3} dx\,,\ \ I_8 = \int x^7\sqrt{5+3x^4}dx,\ \ I_9 = \int e^{6x}\sin(e^{3x})\,dx.$$

Exercise 02 : Calculate the following integrals by using the change of variable.

$$I_1 = \int \frac{x^2dx}{\sqrt{1-x^2}}\ (x=\sin \mathrm{t}),\ I_2 = \int \frac{xdx}{\sqrt{x^2+1}}\quad (t = x^2+1),$$

$$I_3 = \int \frac{dx}{\operatorname{ch} x}\ (t = e^x), I_4 = \int x\sqrt{x+3}dx\ \ (t = x+3),$$

$$I_5 = \int x\mathrm{e}^{x^2+5}dx\ \ (t = x^2+5), I_6 = \int \frac{dx}{x\ln x}\ (\mathrm{t} = \ln x),$$

$$I_7 = \int \sqrt{\mathrm{a}-x^2}dx\ (x = \sqrt{\mathrm{a}}\sin t)\,, I_8 = \int \sqrt{x^2-\mathrm{b}}\,dx\ \left(x = \sqrt{\mathrm{a}}\operatorname{ch} t\right),$$

$$I_9 = \int \frac{x^5dx}{1+x^{12}}\ (\mathrm{t} = x^6), I_{10} = \int \frac{dx}{(\operatorname{ch} x - \operatorname{sh} x)^{\mathrm{n}}}, I_{11} = \int \frac{dx}{x\sqrt{x+2}},$$

$$I_{12} = \int \frac{dx}{\sin x+1}\left(\mathrm{t} = \tan\frac{x}{2}\right),\ \ I_{13} = \int \frac{\sqrt[4]{x}dx}{\sqrt[6]{x}+\sqrt[3]{x}}\ (\mathrm{t} = \sqrt[12]{x}).$$

Exercice 03 : Verify that:

$$\int x\sqrt{x}dx \neq \int xdx.\ \int \sqrt{x}dx \quad \text{and}\ \ 2)\ \int x(x+1)dx \neq x\int (x+1)dx\,.$$

Exercise 04 : Compute the following integrals :

$$1)\int \sin(3x)\cos(5x)\,dx,\ 2)\int \cos^5 x\sin^3 x\,dx, 3)\int \cos^2 x\sin^4 x\,dx,$$

$$4)\int \frac{\sin^3 x}{\cos^4 x}dx,\ 5)\int \frac{\sqrt{x}dx}{1+\sqrt[3]{x}}, 6)\int \cos^3 x\sin^4 x\,dx,$$

$$7)\int \cos^2 x\sin^3 x\,dx\,, 8)\int \frac{dx}{x(1-x^2)},\ 9)\int \frac{dx}{1+x^3}.$$

Exercice 05 : Let F, G and H be the following functions:

$F(a) = \int_0^a \frac{\sin x}{1+\cos x}dx\,, G(a) = \int_0^a \frac{\cos x}{1+\cos x}dx$ and $H(a) = \int_0^a \frac{1}{1+\cos x}dx.$

Where $a \in]-\pi, \pi[$.

1- Compute the values of $F(a)$ and $G(a)$. (You can integrate by using variable change).
2- Find an elementary relationship between $G(a)$ and $H(a)$.
3- Deduce the value of $H(a)$.

Exercice 06 : Let I_n be the following integral:

$$I_n = \int_1^e (\ln x)^n dx$$

1- Compute the values of I_0 and I_1.
2- For all $n \geq 0$ find a relation between I_n and I_{n+1}.
3- Deduce the value of the integral I_4.
4- Establish that for all $n \geq 0$, we have : $0 < I_n < \frac{e}{n+1}$.