University of Jijel - Faculty of exact sciences and computer science - Mathematics department

Series No. 02 Analysis 02 2025/2026

Exercise 01: Let d and d' be two subdivisions defined as:

$$d = \{-1/2, 0, 3/4, 5/2, 3, 9/2, 11/2\} \text{ and } d' = \{-3/2, -1, -1/2, 0, 1/2, 1, 3/2\}$$

for the intervals $[-1/2, 11/2]$ and $[-3/2, 3/2]$, respectively.

1- Find the subintervals of the subdivisions d and d'.
2- Find the step of each subdivision.
3- What can you deduce?

Exercise 02 : Compute $\frac{d}{dx}$ for

$$1) \int_1^x e^{t^2}\, dt, \quad 2) \int_{x^2}^{x^3} e^{t^2}\, dt, \quad 3) \int_{g(x)}^{l(x)} f(t)\, dt.$$

Hint. Use the fundamental theorem of calculus (It's impossible to get explicit formula for $F(t) = \int e^{t^2}\, dt$.

Exercise 03 : Evaluate $\int_0^1 x^2\, dx$ using :

1- The right Riemann sum with n equally spaced subintervals.
2- The left Riemann sum with n equally spaced subintervals.
3- What can you deduce?

Exercise 04: Let h be a continuous function defined on the interval $[1,2]$.

- Prove that : $\exists c \in]1,2[: \int_1^2 \frac{h(x)}{x} dx = h(c) \ln 2$.

Exercise 05: Let f be a function defined as: $f(x) = x - 1$.

1- Prove that f is Riemann integral over the interval $[-1,1]$.
2- Deduce the value of $\int_{-1}^1 f(x)dx$.

Exercise 06: Let $f \in \mathcal{R}[a,b]$ and suppose that g is a function defined as:

$$g(x) = f\left(\frac{x}{c}\right), \forall x \in [ac, bc],$$

where c is a strictly positive constant, prove that:

1- $g \in \mathcal{R}[ac, bc]$.
2- $\int_{ac}^{bc} g(x)dx = c \int_{a}^{b} f(x)dx$.

Exercise 07 : Let $f, g \in \mathcal{R}[a, b]$ and α, β be two real constants different to zero.

1- Prove that:
   a. $\alpha f \in \mathcal{R}[a, b]$.
   b. $f + g \in \mathcal{R}[a, b]$.
2- Deduce that $\alpha f + \beta g \in \mathcal{R}[a, b]$.

Applications of definite integrals:

Exercise 08: An object moves along x -axis towards right with speed $v(t) = t^2 \, m/s$.

Calculate the distance traveled from $t = 0$ to $t = 3 \, s$.

Hint. Let $S(t)$ be the position at t. Then, $S'(t) = v(t)$.

Exercise 09: Let $L(t)$ be the level of carbon monoxide (CO). Given that $L'(t) = 0.1t + 0.1$ parts per million (ppm).
How much will the pollution change from $t = 0$ to $t = 3$?