

*University of Jijel - Faculty of exact sciences and computer  
science - Mathematics department*

*Series No. 03 Analysis 02*

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**Exercise 01:** Determine the type of each differential equation, and then solve one of them:

1)  $xyy' = y^2 - x(x - y)$     and    2)  $e^{y-x}y' = x^2 - x + 2.$

**Exercise 02:**

1- Compute the value of  $I = \int \sin x e^{-\cos x} dx$

2- Find the solution of the following equations:

$$\begin{cases} y' + y \sin x = 2 \sin x \\ y(0) = 0. \end{cases} \quad (E)$$

**Exercise 03:** Solve the following differential equations and determine their type:

- 1)  $(x^2 + 1)y' = y^2 + 1,$     2)  $(3y + x)dx - (3x + y)dy = 0,$   
3)  $xy' + y(x + 1) = e^x,$     4)  $y' + y \tan x = \sin 2x,$  with  $y(0) = 1,$   
5)  $xyy' = y^2 - x(x - y),$     6)  $(y + x - 3)dx + (x - y - 1)dy = 0,$   
7)  $(2x - 3)y' = x + 2y + 1,$     8)  $y' = e^x y^2 - 2y,$   
9)  $y' = xy(x^2 y^2 - 1),$     10)  $y' \sin x = y \ln x,$  avec  $y\left(\frac{\pi}{2}\right) = e,$

11)  $y' + y^2 - 2y \sin x + \sin x^2 - \cos x = 0,$  where  $y = \sin x$  is a particular solution.

**Exercise 04:** Let  $(E)$  be the following differential equation:

$$y'' - 3y' + 2y = x e^x + \cos x^2 - \sin x^2 \quad (E)$$

- 1- Find the homogeneous solution  $(y_h).$
- 2- Find the particular solution, denoted  $(y_p),$  of the equation  $(E).$
- 3- Deduce the general solution, denoted  $(y),$  of the equation  $(E).$
- 4- Find the value of  $(y)$  that satisfies:  $y(0) = 1$  and  $y'(0) = -1.$
- 5- The same questions are asked for the following differential equation:

$$\begin{cases} y'' - 2y' = x + 2 - 4e^{2x} \\ y(0) = 3, y'(0) = \frac{3}{4}. \end{cases} \quad (E')$$

**Exercise 05:** Solve the following differential equations and determine their type:

1)  $y'' + 2y' + y = 0$ ,

2)  $y'' - y' + 2y = 0$ , with  $y(0) = 1$  and  $y'(0) = -1$ ,

3)  $y'' + 3y' + 2y = 0$ ,

4)  $y'' + 4y' + 3y = 3(x^2 + 1)$ , with  $y(0) = 1$  and  $y'(0) = -1$ ,

5)  $y'' - 2y' + 5y = 2\sin x \cos x$ ,

6)  $y'' + 4y' + 3y = (x^2 + 1)e^x$ ,

7)  $y'' - 4y' + 3y = 3x + 2 + 4e^x + 5e^{-x}$ ,

8)  $y'' - 2y' = e^x \sin x$ .

### Applications of differential equations of first and second order:

#### Problem1: Exponential Growth (Application to Bacteria)

A culture of bacteria grows at a rate proportional to its size. Initially, there are 100 bacteria. After 2 hours, there are 200 bacteria.

1. Find the population  $P(t)$  (or determine the law of growth  $P(t)$  of the bacteria).
2. Find the population after 4 hours.

#### Problem2: (Mass–Spring System)

A mass attached to a spring satisfies the equation:  $x'' + 4x = 0$ ,

with initial conditions:  $x(0) = 3, x'(0) = 2$ .

1. Find the displacement  $x(t)$ .
2. Write the solution in the form:  $x(t) = R \cos(2t - \varphi)$ ,

And determine the values of  $R$  and  $\varphi$ .

where:

• **Amplitude  $R$**  : the maximum displacement from equilibrium; the motion varies between  $-R$  and  $R$ .

• **Phase angle  $\varphi$**  : determines the initial position and phase shift of the oscillation in time.