

University of Jijel
Department of Physics
Functions of a complex variable

TD session three

Exercise 1: Let γ be the curve given by $z : [0, 1] \rightarrow \mathbb{C}$, $z(t) = t + it^2$, $t \in [0, 1]$ and $f(z) = z - 1$. Compute the integral $\int_{\gamma} f(z) dz$.

Exercise 2: Compute the integral $\int_{\gamma} f(z) dz$ where $f(z) = z + \frac{1}{z}$, $\forall z \in \mathbb{C}^*$ and γ is the upper semi-circle at the origin of radius $r = 1$.

Exercise 3: Let γ be the curve given by $z : [0, 2\pi] \rightarrow \mathbb{C}$, $z(t) = 2e^{it}$, $t \in [0, 2\pi]$.

Show that

$$\left| \int_{\gamma} \frac{e^z}{z^2 + 1} dz \right| \leq \frac{4\pi e^2}{3}.$$

Exercise 4: Compute the integral $\int_{\gamma} e^z dz$ where γ is the part of the unit circle joining 1 to i in the counterclockwise direction.

Exercise 5: Compute the integral $\int_{\gamma} \frac{e^z}{z^2 - 16} dz$ where γ is the unit circle traversed once in the counterclockwise direction.

Exercise 6: Compute the integral $\int_{\gamma} \frac{e^{2z} + \sin z}{z - \pi} dz$ where γ is the circle $\{z \in \mathbb{C}, |z - 2| = 2\}$ traversed once in the counterclockwise direction.

Exercise 7: Compute the integral $\int_{\gamma} \frac{\sin(3z)}{z^4} dz$ where γ is the unit circle traversed once in the counterclockwise direction.

Exercise 8: Determine the zeros and the order of each of them of the following functions

$$z \mapsto f(z) = e^{2z} - e^z, \quad z \mapsto g(z) = z^2 \sinh z, \quad z \mapsto h(z) = z^4 \cos^2 z, \quad z \mapsto H(z) = z^3 \cos z^2.$$

Exercise 9: Determine and classify the singularities of each of the following functions

$$z \mapsto f(z) = \frac{z^2 + 2}{z^3(z + i)}, \quad z \mapsto g(z) = z^2 e^{\frac{1}{z}}, \quad z \mapsto h(z) = \frac{z + 1}{\sin z},$$