1/ Introduction:

In this chapter we will consider the fluid is at rest. There will be no shearing stresses in the fluid, and the only forces that develop on the surfaces of the particles will be due to the pressure. Thus, our principal concern is to investigate pressure and its variation throughout a fluid and the effect of pressure on submerged surfaces.

2/ Pressure:

Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress. Since pressure is defined as force per unit area, it has the unit of Newtons per square meter (N/m^2), which is called a Pascal (Pa). That is, the actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum (i.e., absolute zero pressure).

1atm = 760 mm of mercury column

1atm = 10.3 m of water column

1atm = 101.325 kN/m^2 = 101.325 kPa

1 bar = 10^5 Pa = 0.1 MPa = 100 kPa.

3/ Pascal's law:

Consider the free-body diagram, illustrated in Figure 01, which was obtained by removing a small triangular wedge of fluid from some arbitrary location within a fluid mass. Since we are considering the situation in which there are no shearing stresses, the only external forces acting on the wedge are due to the pressure and the weight. For simplicity the forces in the *x* direction are not shown, and the *z* axis is taken as the vertical axis so the weight acts in the negative *z* direction.

The assumption of zero shearing stresses will still be valid.

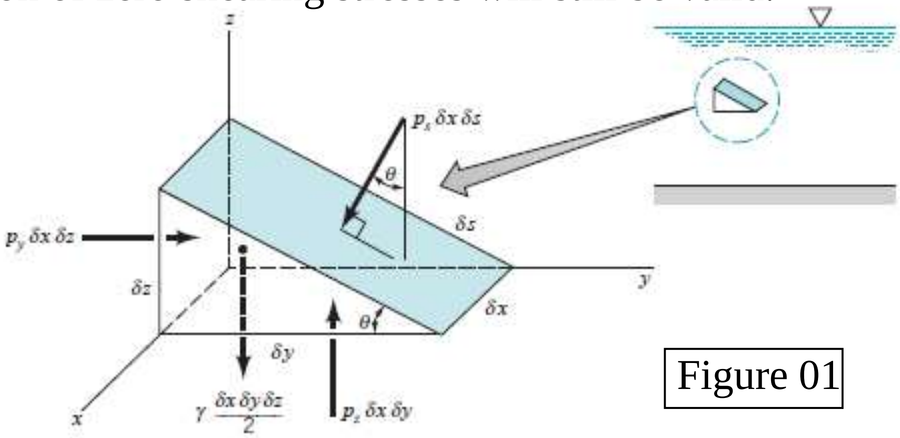


Figure 01

The equations of motion (Newton's second law F= ma) in the *y* and *z* directions are, respectively,

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s\sin\theta = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_y$$

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s\cos\theta - \gamma\frac{\delta x\,\delta y\,\delta z}{2} = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_z$$

We take the limit as $\delta x, \delta y\ and\ \delta z$ approach zero (while maintaining the angle θ), and it follows that:

$$P_y = P_s \quad \text{and} \quad P_z = P_s$$

Or $P_y = P_z = P_s$, the angle was arbitrarily chosen so we can conclude that the pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present.

This important result is known as Pascal's law.

4/ Hydrostatic Law:

Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases with depth in the fluid.

Height of fluid element = Δh

Pressure at top of fluid element = p

Force on the top of fluid element = p ΔA

Weight of fluid element = ΔA. Δh. ρ g

Upward force acting at bottom of the fluid element = $-(p + \frac{\partial p}{\partial h}\Delta h)\Delta A$

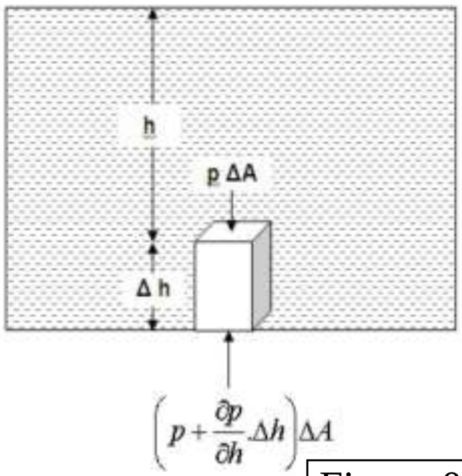


Figure 02

Under equilibrium conditions:

(downward force = upward force),

$$p.\Delta A + \Delta A.\Delta h.\rho g = (p + \frac{\partial p}{\partial h}\Delta h)\Delta A$$

$$\Delta h.\rho g = \frac{\partial p}{\partial h}\Delta h$$

$$\rho g = \frac{\partial p}{\partial h} = \gamma$$

Where, $\gamma = \frac{mg}{V} = \rho g$ (Specific Weight). Integrating,

$$\int \frac{\partial p}{\partial h} = \rho g \int \partial p = \int \rho g \partial h \qquad P = \rho g h$$

5/ Measurement of Pressure:

Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere, and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the gage pressure. Pressures below atmospheric pressure are called vacuum pressures and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure.

a- Gauge Pressure:

It is convenient to measure pressure in terms of taking atmospheric pressure as reference datum. Pressure measured above atmospheric pressure is known as gauge pressure. The atmospheric pressure on the scale is marked as zero.

b- Absolute Pressure:

Since, atmospheric pressure changes with atmospheric condition, a perfect vacuum is taken as an absolute standard of pressure. Pressure measured above perfect vacuum are called absolute pressure. The figure 2.3 explains the concept of gauge and absolute pressure.

Absolute pressure = atmospheric pressure + gauge pressure

$$P_{abs} = P_{atm} + P_{gauge}$$

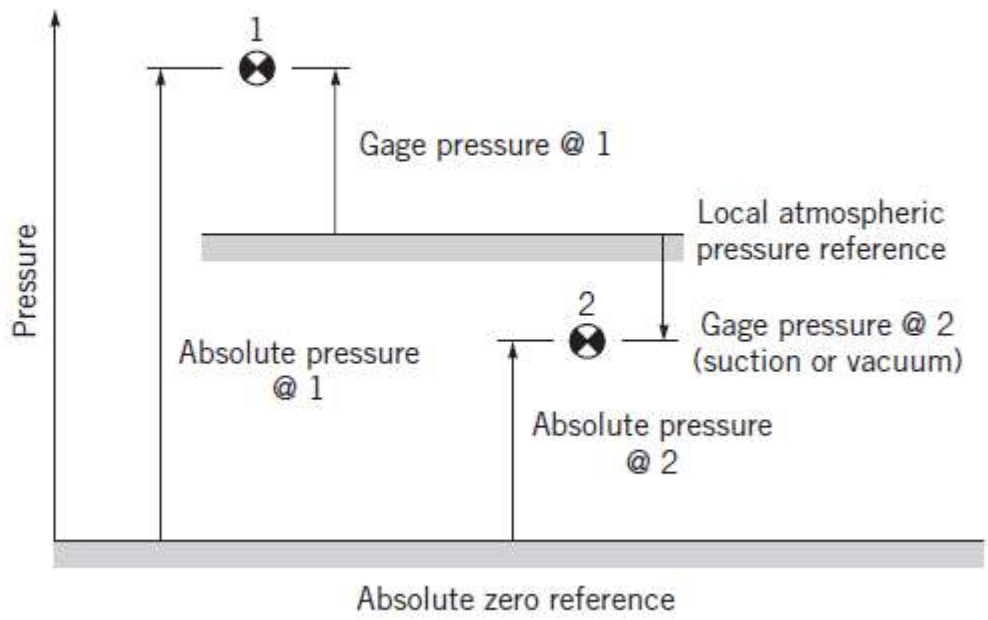
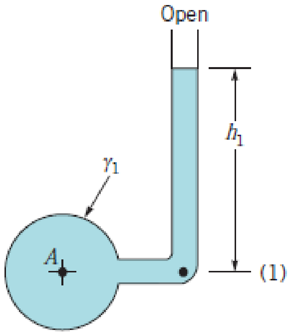


Figure 03: Absolute and gage pressures

6/ Manometers:

a- Piezometer Tube:

Figure 03: (Piezometer Tube)



The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Figure 03.

The pressure P_A can be determined by a measurement of h_1 through the relationship: $P_1 = \gamma_1 . h_1$

Where γ_1 is the specific weight of the liquid in the container. Note that since the tube is open at the top, the pressure p_0 can be set equal to P_{atm}.

b- U-Tube Manometer:

$P_A = P_1$ and $P_2 = P_3$ (the same elevation).

$$P_2 = P_1 + \gamma_1 h_1$$

$$P_3 = P_{atm} + \gamma_2 h_2$$

$$P_1 + \gamma_1 h_1 = P_{atm} + \gamma_2 h_2$$

$$P_A = P_{atm} + \gamma_2 h_2 - \gamma_1 h_1$$

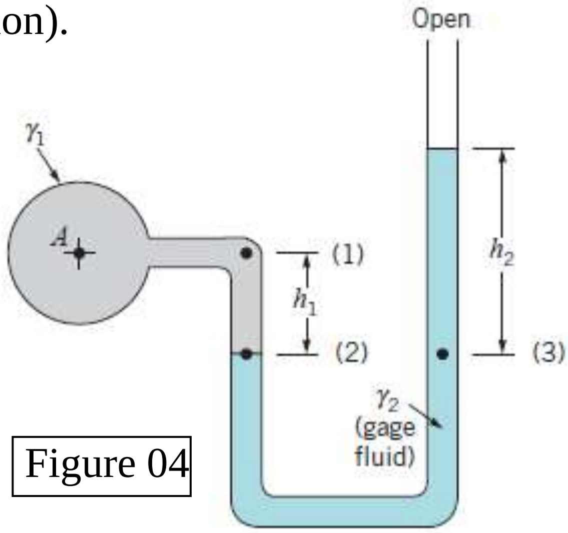


Figure 04

Archimedes' Principle:

When a stationary body is completely submerged in a fluid, the resultant fluid force acting on the body is called the buoyant force. A net upward vertical force results because pressure increases with depth and the pressure forces acting from below are larger than the pressure forces acting from above.

Archimedes Principle states that the buoyant force acting on the body immersed in fluid is equal to the weight of fluid displaced by the body.

This principle explains the loss of weight in a body immersed in fluid, which is equal to the weight of fluid displaced by it. The volume of fluid displaced by the floating body is just enough to balance its weight.

$$F_B = \gamma \forall$$

Where: F_B is the buoyant force, γ is the specific weight of the fluid and $\forall$ is the volume of the body.