## 1/ Introduction:

This chapter deals with three equations commonly used in fluid mechanics: the mass's conservation (continuity equation), Bernoulli's theorem (conservation of energy), and Euler's theorem (conservation of momentum). From which the equations giving the dynamic force exerted by moving fluids (water jets) are established.

In Fluid Mechanics, the knowledge of flow behavior is important as the analysis and calculations depends on the flow conditions.

## 2/ Types of flow:

### a- Steady flow:

In steady flow fluid parameters such as velocity, density, pressure, acceleration at a point do not change with time.

$$\frac{\partial \mu}{\partial t} = 0;\ \frac{\partial \rho}{\partial t} = 0;\ \frac{\partial p}{\partial t} = 0; \frac{\partial a}{\partial t} = 0$$

Where $\mu$: velocity; P: Pressure; ρ: Density; a: acceleration; t: time

### b- Unsteady flow:

In unsteady flow fluid parameters such as velocity, density, pressure, acceleration at a point changes with time.

$$\frac{\partial \mu}{\partial t} \neq 0;\ \frac{\partial \rho}{\partial t} \neq 0;\ \frac{\partial p}{\partial t} \neq 0; \frac{\partial a}{\partial t} \neq 0$$

### c- Uniform flow:

In uniform flow if the velocity at a given instant of time is same in both magnitude and direction at all points in the flow, the flow is said to be uniform flow.

### d- Non-uniform flow:

When the velocity changes from point to point in a flow at any given instant of time, the flow is described as non-uniform flow.

### e- Compressible flow:

The flow in which density of the fluid varies during the flow is called compressible fluid flow. This is applicable in gas flow.

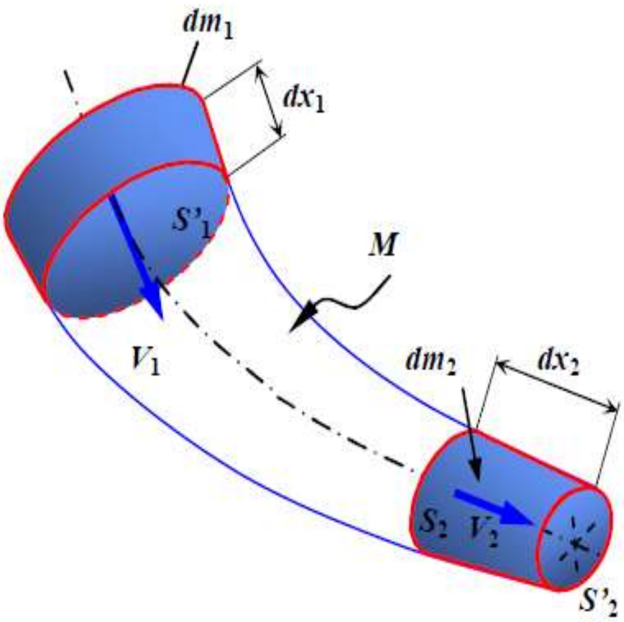
### f- Incompressible flow:

In case of in compressible fluid flow, the density of the fluid remains constant during the flow. Practically, all liquids are treated as incompressible.

## 3/Conservation of Mass (continuity equation):

The mass equation is an expression of the conservation of mass principle.

The conservation of mass relation for a closed system undergoing a change is expressed as $m_{sys}$= constant or $dm_{sys}/dt = 0$, which is a statement of the obvious that the mass of the system remains constant during a process. For a control volume (CV), mass balance is expressed in the rate form as *Conservation of mass*:




$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

Displaced mass is conserved, so: $dm_1 + m = dm_2 + m$

Or: $dm_1 = dm_2$

becomes: $\rho_1 . dV_1 = \rho_2 . dV_2$

can also be expressed as: $\rho_1 . S_1 . dx_1 = \rho_2 . S_2 . dx_2$

The mass flow rate through the entire cross-sectional area of a pipe or duct:

$$\rho_1 . S_1 . \frac{dx_1}{dt} = \rho_2 . S_2 . \frac{dx_2}{dt} \qquad [Kg . s^{-1}]$$

for incompressible flow: $\rho_1 = \rho_2 = \rho$

so we have: $S_1 . v_1 = S_2 . v_2$

The volume of the fluid flowing through a cross section per unit time is called the volume flow rate $Q = S . v$ $[m^3 . s^{-1}]$

It is the continuity equation $Q = S . v = C^{st}$

The mass and volume flow rates are related by:

$$\rho . Q_V = Q_m$$

## 4/Conservation of Momentum (Euler's theorem):

The product of the mass and the velocity of a body is called the linear momentum or just the momentum of the body, and the momentum of a rigid body of mass m moving with a velocity $\overrightarrow{v}$ is m$\overrightarrow{v}$ . Newton's second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body. Therefore, the momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved. This is known as the conservation of momentum principle. In fluid mechanics, Newton's second law is usually referred to as the linear momentum equation.

$$\sum\vec{F} = m.\vec{a}$$

$$\vec{F} = \frac{d(m.\vec{v})}{dt} = m.\vec{a}$$

$$d(m.\vec{v}) = dm.\overrightarrow{v_1} - dm.\overrightarrow{v_2} = dm.(\overrightarrow{v_1} - \overrightarrow{v_2})$$

$$d(m.\vec{v}) = Q_m\,.dt.(\overrightarrow{v_1} - \overrightarrow{v_2})$$

$$\vec{F} = \frac{d(m.\vec{v})}{dt} = Q_m(\overrightarrow{v_1} - \overrightarrow{v_2})$$

## 5/Conservation of Energy (Bernoulli's Theorem):

Energy can be transferred to or from a closed system by heat or work, and the conservation of energy principle requires that the net energy transfer to or from a system during a process be equal to the change in the energy content of the system. Control volumes involve energy transfer via mass flow also, and the conservation of energy principle, also called the energy balance, is expressed as *Conservation of energy*:

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{CV}}{dt}$$

### a- Bernoulli's Theorem:

Bernoulli's Theorem states that "in a steady ideal flow of in compressible fluid flow, the sum of pressure energy, kinetic energy and potential energy remains constant at every section provided no energy is added or taken out by an external source"

It is based on principle of conservation of energy and is expressed by the following relation:

Pressure energy + Kinetic energy + Potential energy = constant

$$E_{tot} = PV + \frac{1}{2}mv^2 + mgz = C^{st} \quad [Joules] \quad /V \quad (\frac{m}{V} = \rho)$$

$$P + \frac{1}{2}\rho v^2 + \rho gz = C^{st} \quad [Pa]$$

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible. So, the Bernoulli equation is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

According to Bernoulli's theorem : $\frac{P}{\rho g} + \frac{v^2}{2g} + z = constant \quad [m]$

$$Pressure\ head = \frac{P}{\rho g}\ ;\ Velocity\ head = \frac{v^2}{2g}\ ;\ Potential\ head = z$$

Definition of the total head of a fluid

Energy balance on a small element of mass m, volume V, velocity $v$, pressure p and altitude z,

the Head $e_T$ of a fluid is the energy per unit mass.

$$e_T = \frac{P}{\rho} + \frac{v^2}{2} + gz = constant$$

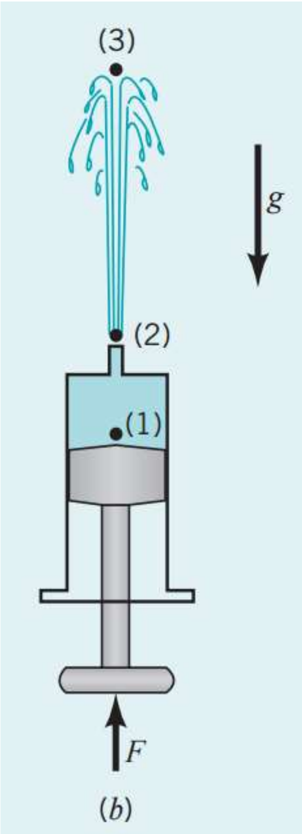
Energy per unit mass exchanged between the fluid and the external environment dw/dm, downstream, upstream.

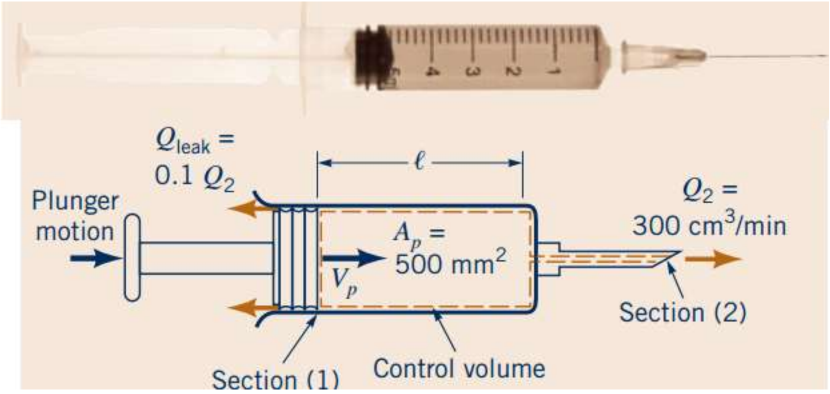
$$\frac{dw}{dm} = \frac{dw}{dt} \cdot \frac{dt}{dm} = \frac{P_e}{Q_m}$$

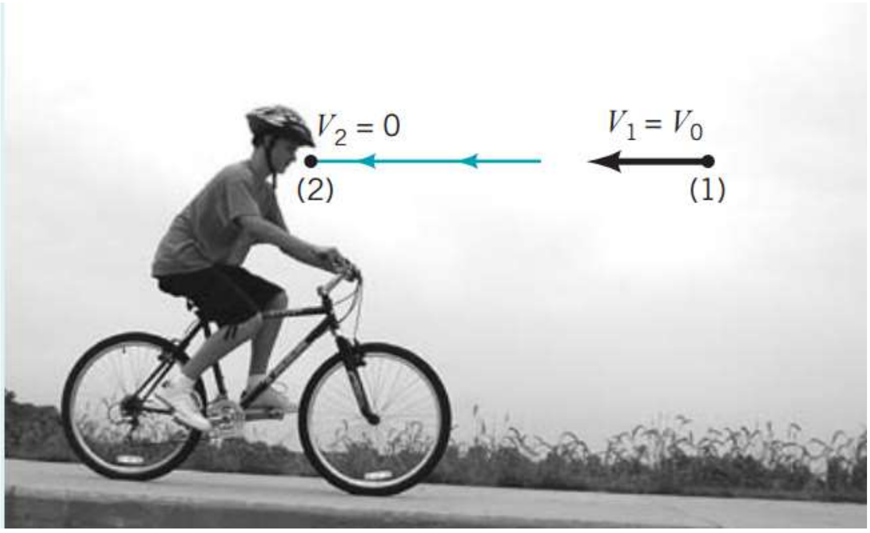
$P_e$: Pumping (positive) or turbine (negative) power, $Q_m$: the mass flow rate.

## b- Application of Bernoulli's Theorem:

Bernoulli's theorem has number of applications. The working of many flow measuring devices are based on the principle of Bernoulli's theorem. Some equations used in fluid mechanics are also derived using the concept of fluid mechanics.



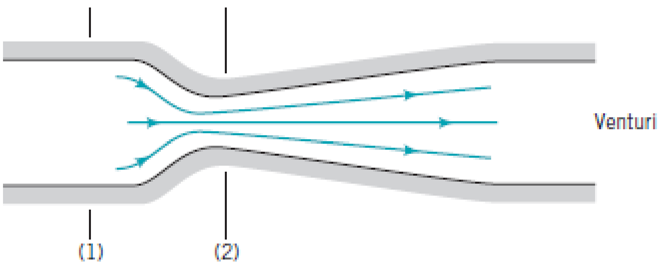




### b-1- Venturimeter:

Venturimeter is used to the measurement of flow rate. It is generally used for large diameter pipes.

If we assume the velocity profiles are uniform at sections (1) and (2), the continuity equation can be written as: $Q = S_1 . v_1 = S_2 . v_2$



We assume the flow is horizontal steady, and incompressible between points (1) and (2). The Bernoulli equation becomes:

$$P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2$$

Combination of these two equations results in the following theoretical flowrate:

$$Q = S_2.\sqrt{\frac{2.(P_1 - P_2)}{\rho.\left(1-\left(\frac{S_2}{S_1}\right)^2\right)}}$$

## b-2-Pitot Tube:

Pitot tube is used for the measurement of fluid velocity.

Outer body of Pitot tube consists of ports at point A, the inner tube is for sensing the stagnation pressure. At point B, fluid velocity becomes zero and for sensing the static pressure of fluid.

The two ends of Pitot tube are connected to U-tube manometer for measuring the pressure difference between the points A and B.