1/Introduction:

The transport of a fluid (liquid or gas) in a closed conduit, commonly called a pipe if it is of round cross section or a duct if it is not round, is extremely important in our daily operations.

2/Laminar or Turbulent Flow:

The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow. The type of flow is determined by Reynold's Number.

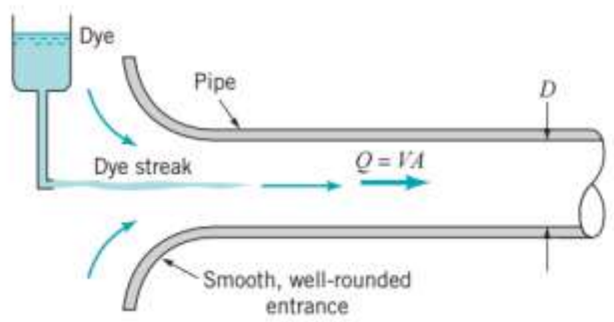
The Reynolds number ranges for which laminar, transitional, or turbulent pipe flows are obtained cannot be precisely given. The actual transition from laminar to turbulent flow may take place at various Reynolds numbers, depending on how much the flow is disturbed by vibrations of the pipe, roughness of the entrance region, and the like.

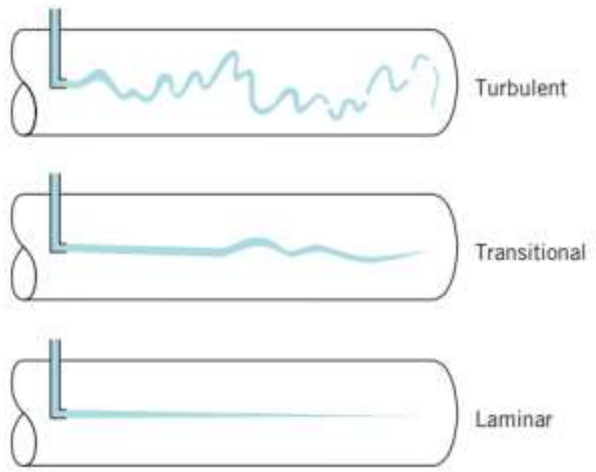
a- Laminar flow:

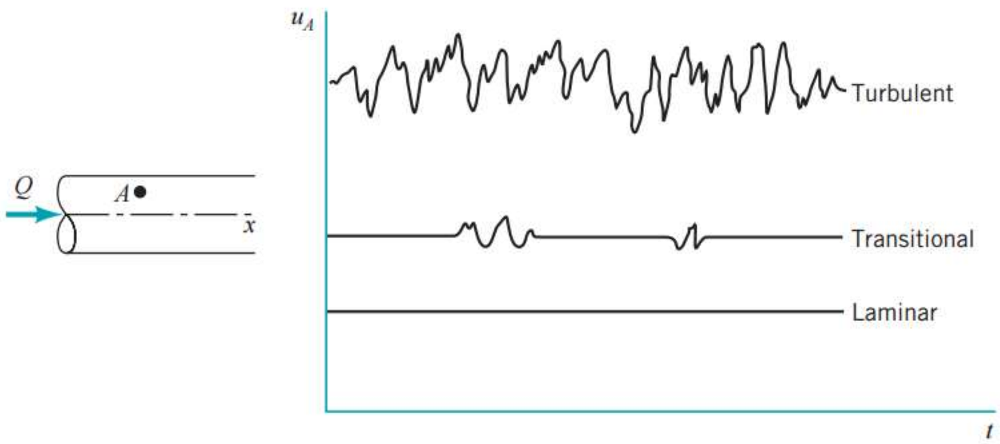
In this type of fluid flow, particles move along well defied paths or steam lines. The fluid layers moves smoothly over the adjacent layer. The fluid particles move in a definite path and their paths do not cross each other.

b- Turbulent Flow:

In turbulent fluid flow, fluid particles move in a random and zigzag way. Turbulence is characterized by the formation of eddies.







3/ Reynold's Number:

The transition from laminar to turbulent flow depends on the geometry, surface roughness, flow velocity, surface temperature, and type of fluid, among other things. Reynolds number is defined as the ratio of inertia force of the flowing fluid to the viscosity force of the fluid. In case of pipe flow, it is determined by using the following equation: $Re = \frac{\rho\, v\, D}{\mu}$

Where, Re=Reynold's Number.

ρ = Density of fluid, v = Velocity of fluid, D = Diameter of pipe, μ= Viscosity of fluid.

For flow through noncircular pipes, the Reynolds number is based on the hydraulic diameter D_h defined as: $D_h = \frac{4S}{P}$

Where S is the cross-sectional area of the pipe and p is its wetted perimeter.

The flow in a circular pipe is laminar for Re < 2000, turbulent for Re > 3000, and transitional in between. That is, Re ≳3000 (4000) turbulent flow.

2000 ≲ Re ≲ 3000 transitional flow.

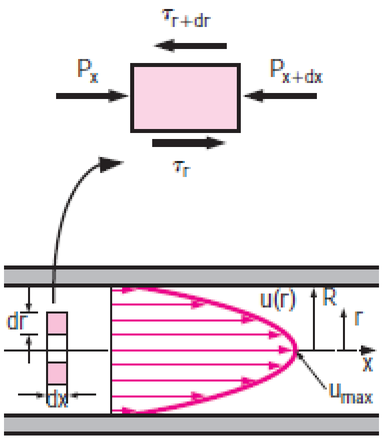
Re ≲ 2000 (2300) laminar flow.

4/ Laminar flow in pipes:

We mentioned that flow in pipes is laminar for Re ≲ 2000, and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length) so that the entrance effects are negligible.

We consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe.

The velocity profile is rewritten as:



$$v(r) = v_{max}(1 - \frac{r^2}{R^2})$$

$$F = \pi . r^2 \Delta P$$

$$F = 2\pi . r . L . \mu . \frac{dv}{dr}$$

$$\pi . r^2 \Delta P = 2\pi . r . L . \mu . \frac{dv}{dr}$$

$$dv = \frac{\Delta P . r . dr}{2 . L . \mu}$$

$$v(r) = \int_r^R \frac{\Delta P.r.dr}{2.L.\mu} = \frac{\Delta P}{4.L.\mu}\,(R^2 - r^2) = \frac{\Delta P.R^2}{4.L.\mu}(1 - \frac{r^2}{R^2})$$

So we have: $v_{max} = \frac{\Delta P.R^2}{4.L.\mu} = 2v$

a- Pressure Drop:

A quantity of interest in the analysis of pipe flow is the pressure drop ΔP.

Laminar flow: $\Delta P = \frac{4.L.\mu.v_{max}}{R^2} = \frac{16.L.\mu.v_{max}}{D^2} = \lambda\frac{L}{D}\,P_{dyn}$

$P_{dyn} = \frac{1}{2}\,\rho v^2$ or: $\lambda = \frac{32.L.\mu.v_{max}}{D^2}.\frac{D}{L.\rho v^2} = \frac{32.\mu.2}{\rho.v.D} = \frac{64}{R_e}$

b- Head Loss:

In the analysis of piping systems, pressure losses are commonly expressed in terms of the equivalent fluid column height, called the head loss hL. Noting from fluid statics that $\Delta P = \rho gh$ and thus a pressure difference of ΔP corresponds to a fluid height of $h = \Delta P/\rho g$, the pipe head loss is obtained by dividing ΔP by ρg to give:

Head loss: $h_L = \lambda\frac{L}{D}.\frac{v^2}{2g}$

The head loss h_L represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.

The average velocity for laminar flow in a horizontal pipe is:

$$v = \frac{\Delta P.R^2}{8.L.\mu} = \frac{\Delta P.D^2}{32.L.\mu}$$

Then the volume flow rate for laminar flow through a horizontal pipe of diameter *D* and length *L* becomes:

$$Q_V = v.S = \frac{\Delta P.R^2}{8.L.\mu}.\pi R^2 = \frac{\Delta P.\pi R^4}{8.L.\mu} = \frac{\Delta P.\pi D^4}{128.L.\mu}$$

This equation is known as Poiseuille's law.

5/ Turbulent Flow in pipes:

Colebrook equation: $\frac{1}{\sqrt{\lambda}} = -2.0 \log(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{\lambda}})$ for: $Re > 10^5$

$\frac{\varepsilon}{D}$: relative roughness or (roughness/ diameter).

an alternate form which is easier to use, was given by S. E. Haaland in 1983 as:

$$\frac{1}{\sqrt{\lambda}} = -1.8 \log(\frac{6.9}{Re} + (\frac{\varepsilon/D}{3.7})^{1.11}$$

The Blasius equation: $\lambda = 0{,}316Re^{-0.25}$ for: $Re < 10^5$

6/ Piping Systems with Pumps and Turbines:

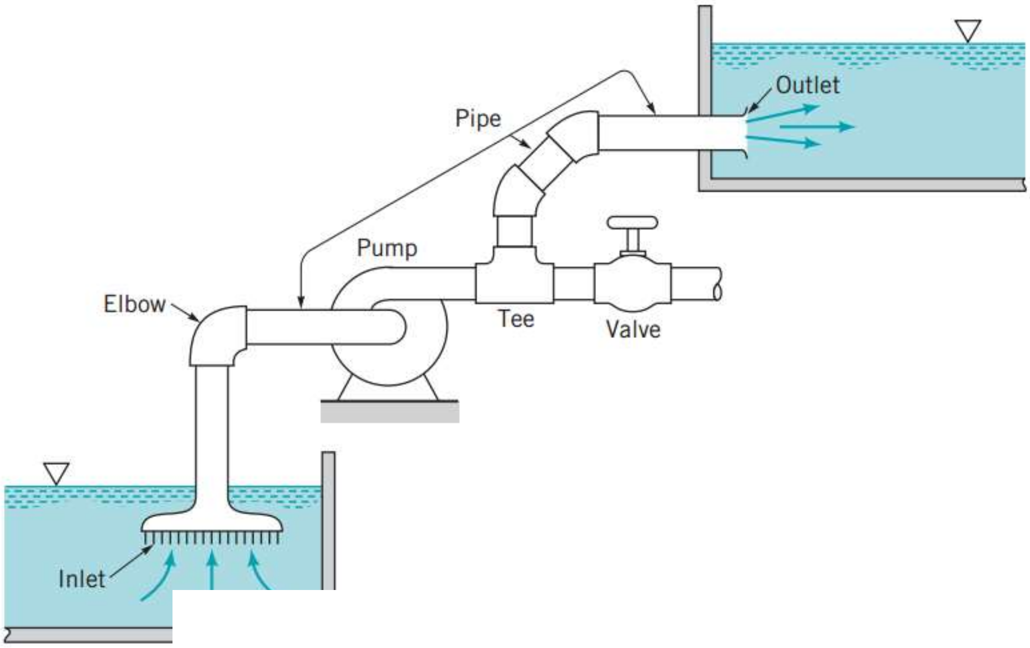
When a piping system involves a pump and/or turbine, the steady-flow energy equation on a unit-mass basis can be expressed as:

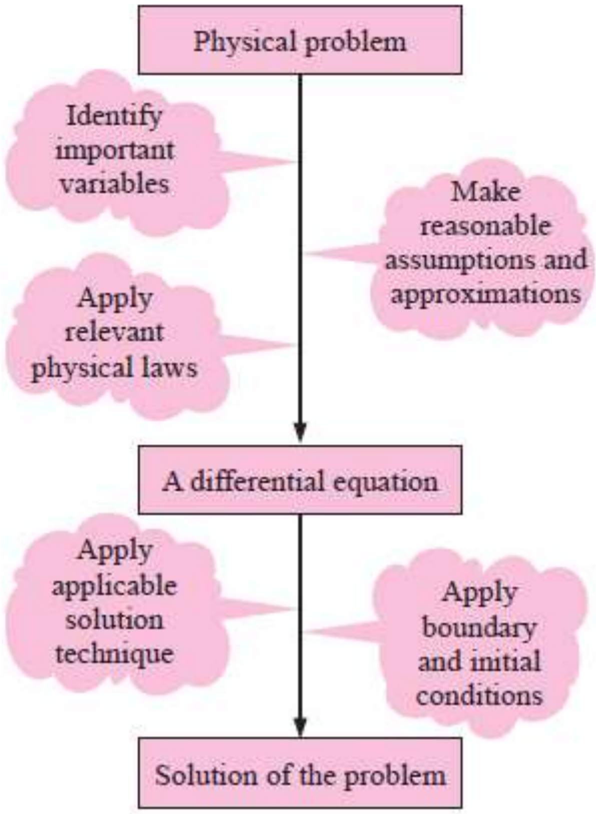
$$\frac{P_1}{\rho} + \alpha_1 \frac{v_1^2}{2} + gz_1 + w_{pump,u} = \frac{P_2}{\rho} + \alpha_2 \frac{v_2^2}{2} + gz_2 + w_{turbine,e} + gh_L$$

$\frac{w_{pump,u}}{g}$: is the useful pump head delivered to the fluid.

$\frac{w_{turbine,e}}{g}$: is the turbine head extracted from the fluid.

α is the kinetic energy correction factor whose value is nearly 1 for most (turbulent) flows encountered in practice, and h_L is the total head loss in piping.





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