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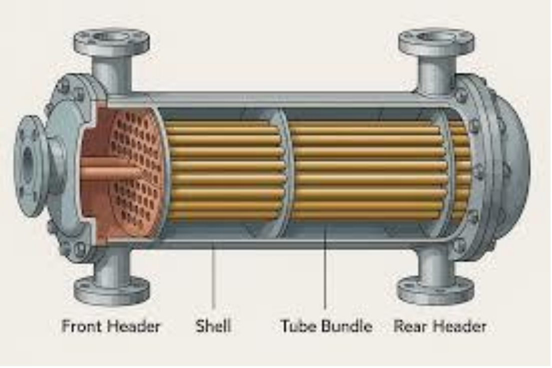
Mohamed Seddik Benyahia University of Jijel

Faculty of Science and Technology

Electrical Engineering Department

Course Handout

HEAT EXCHANGERS



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PREFACE

This course, *Heat Exchangers*, is primarily designed for first-year Master's students in the *Electromechanics* program at the *Mohamed Seddik Benyahia University of Jijel*. It serves as an introductory learning resource that enables students to acquire the fundamental principles of heat exchangers, which are among the most critical components in industrial thermal and energy systems. The course is structured to provide both theoretical knowledge and practical calculation skills, preparing students for more advanced studies in thermal engineering and for professional applications.

The first part of the course focuses on the classification of heat exchangers according to multiple criteria, including transfer process, flow arrangement, compactness, and construction type. A detailed understanding of these categories is essential, as the choice of a particular heat exchanger for a given application depends on many parameters such as the temperature and pressure range of the fluids, their physical properties and aggressiveness, maintenance requirements, and economic considerations. This Section also presents the most common types of heat exchangers encountered in industry, including tubular exchangers, plate exchangers, and other specialized designs.

The second part of the course is dedicated to the two most widely used methods for calculating the thermal power and sizing of heat exchangers: *the Logarithmic Mean Temperature Difference* (LMTD) method and the *Number of Transfer Units* (NTU) method. The LMTD method is particularly suitable when inlet and outlet temperatures are known, whereas the NTU method offers a more convenient approach when only inlet temperatures are specified. Both methods are presented in detail, including the derivation of the governing equations, the introduction of the global heat transfer coefficient, and the integration of fouling resistances that accumulate over time.

The third chapter provides a concise overview of the design and optimization of heat exchangers. It outlines the main steps involved in determining the required exchange surface, selecting appropriate flow velocities, calculating pressure drops, and iterating toward a feasible and cost-effective design. The chapter also introduces the fundamental concepts of static and dynamic optimization, emphasizing the necessary compromise between maximizing heat transfer and minimizing pumping power, as well as a simplified case study that demonstrates the usefulness of these concepts in a real-world engineering context.

Finally, the last chapter provides a comprehensive set of resolved exercises, covering all the cases studied. These exercises are designed to help students develop confidence in applying the LMTD and NTU methods, handling different flow configurations, and interpreting results from realistic industrial data. By working through these problems, students will be well prepared to tackle more complex thermal design challenges in their subsequent studies or professional careers.

ABBREVIATIONS

Symbol	Description	Unit
T	Temperature	K
$\dot{m}$	Mass Flow Rate	kg/s
C_p	Specific Heat Capacity at Constant Pressure	$J/(kg.K)$
C	Heat Capacity Rate	W/K
ϕ	Heat Transfer Rate	W
K	Overall Heat Transfer Coefficient	$W/(m^2.K)$
ε	Effectiveness of Heat Exchanger	/
NTU	Number of Transfer Units	/
Z	Heat Capacity Rate Ratio	/
S	Heat Transfer Surface Area	m^2
h	Convection Heat Transfer Coefficient	$W/(m^2.K)$
λ	Thermal Conductivity	$W/(m.K)$
e	Wall Thickness	m
R_f	Fouling Resistance	$m^2.K/W$
ΔT_{LMTD}	Logarithmic Mean Temperature Difference	K
F	Correction Factor	/
ΔP	Pressure Drop	Pa
ρ	Density	kg/m^3
μ	Dynamic Viscosity	$kg/(m.s)$

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GENERAL INTRODUCTION

In modern industrial facilities, it is frequently necessary to supply or extract significant amounts of thermal energy from a given part of a system. In the vast majority of cases, this heat transfer is accomplished using a device known as a heat exchanger. Estimates indicate that heat exchangers are responsible for the largest share of energy transfers in industry, making them indispensable components in sectors ranging from power generation and chemical processing to food production, refrigeration, and transportation. Among the many types of heat exchangers available, the most common are coaxial (or double-pipe) exchangers, shell-and-tube exchangers, and plate exchangers, each offering distinct advantages depending on the application.

Heat exchangers are thermal devices of paramount importance in virtually all thermal and energy installations. It is no exaggeration to state that at least one heat exchanger can be found in every thermal system, whether it is a simple heating circuit, a complex petrochemical plant, or a vehicle cooling system. The diversity of currently used technologies includes coaxial tube bundles, multi-tube shell-and-tube exchangers, brazed or gasketed plate exchangers, finned-tube exchangers, and many other specialized designs. This wide variety reflects the need to adapt to different operating conditions, fluid properties, temperature and pressure ranges, and economic constraints.

The main purpose of designing and calculating a heat exchanger is to resolve the fundamental duality that exists between heat transfer performance and pressure drop. In other words, the engineer must find an optimal compromise between maximizing the rate of heat exchange, which tends to require larger surfaces and higher flow velocities, and minimizing the pressure losses across the exchanger, which increase with velocity and surface complexity. This trade-off is directly linked to investment and operating costs: a larger exchanger increases initial capital expenditure, while higher pressure drops raise pumping energy consumption. Therefore, the ultimate goal of heat exchanger design is to achieve the required thermal duty at the lowest possible total cost over the equipment's lifetime, respecting all technical and safety constraints.

CHAPTER I

INTRODUCTION TO HEAT EXCHANGERS

1. INTRODUCTION TO HEAT EXCHANGERS

1.1. Introduction

The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications. The device designed to accomplish this transfer is called "*Heat Exchanger*". Common applications can be found in space heating and air conditioning systems, power production, waste heat recovery, chemical processing, and many other industrial systems.

Over the past quarter-century, the importance of heat exchangers has grown enormously, driven by the need for energy conservation, efficient energy conversion, waste heat recovery, and the successful implementation of new and renewable energy sources. Their significance has also increased due to growing environmental concerns, including thermal pollution, air and water pollution, and waste disposal. Today, heat exchangers are used across a wide spectrum of industries: process industries (chemicals, petrochemicals, steel, agri-food), energy production, transportation (automotive, aeronautics, marine), air conditioning and refrigeration, cryogenics, heat recovery, alternative fuels, and manufacturing. They are also key components in many commercial products available on the market.

In industrial companies, heat exchangers are an essential part of any energy management policy. A large proportion (i.e., 90%) of the thermal energy used in industrial processes passes through a heat exchanger at least once, both in the processes themselves and in the systems used to recover thermal energy from these processes. They are mainly used in industry (e.g., chemicals, petrochemicals, steel, agri-food, energy production, etc.), transportation (e.g., automotive, aeronautics), but also in the residential and tertiary sectors (e.g., heating, air conditioning, etc.).

The selection of a heat exchanger for a given application depends on many parameters, including the following ones:

- The temperature and pressure ranges of the fluids.
- The physical properties of the fluids (density, viscosity, thermal conductivity, specific heat).
- The chemical aggressiveness or corrosive nature of the fluids.

- Maintenance requirements and accessibility.
- Size, weight, and space constraints.
- Economic considerations.

A heat exchanger that is well-adapted to its duty, properly sized, correctly manufactured, and appropriately operated, contributes directly to improved process efficiency and energy savings. Conversely, a poorly designed or incorrectly selected exchanger can lead to excessive energy consumption, high maintenance costs, and premature failure.

1.2. Definitions

A heat exchanger is a thermal device designed to transfer internal thermal energy (heat) between two or more fluids at different temperatures. In most heat exchangers, the fluids are separated by a solid wall and, ideally, they do not mix with each other. The heat transfer occurs by convection from the hot fluid to the wall, by conduction through the wall, and then by convection from the wall to the cold fluid.

Heat exchangers are found in a wide range of industrial and domestic applications, including energy production, petroleum refining, transportation, air conditioning, refrigeration, cryogenics, heat recovery systems, and alternative fuel technologies. They are essential components in many thermal systems, enabling efficient energy use and process control.

Common examples of heat exchangers that we are all familiar with, include the following ones:

- *Car Radiators*: transfer heat from the engine coolant to the ambient air.
- *Condensers*: convert vapor into liquid by removing latent heat (e.g., in air conditioners and refrigeration cycles).
- *Evaporators*: convert liquid into vapor by adding heat (e.g., boilers).
- *Air Heaters*: warm air for space heating or industrial drying processes.
- *Oil Coolers*: remove excess heat from lubricating oil in engines and machines.

These examples illustrate the diversity of heat exchanger applications, ranging from small-scale automotive components to large industrial units.

1.3. General Operating Principle

The most fundamental operating principle of a heat exchanger consists of circulating two fluids at different temperatures through channels that bring them into thermal contact. Typically, the two fluids are separated by a solid wall, most often made of metal, which promotes efficient heat transfer while preventing direct mixing. As illustrated in Figure 1.1, the *hot fluid* transfers heat through the wall to the *cold fluid*, thereby raising the temperature of the cold fluid and lowering that of the hot fluid.

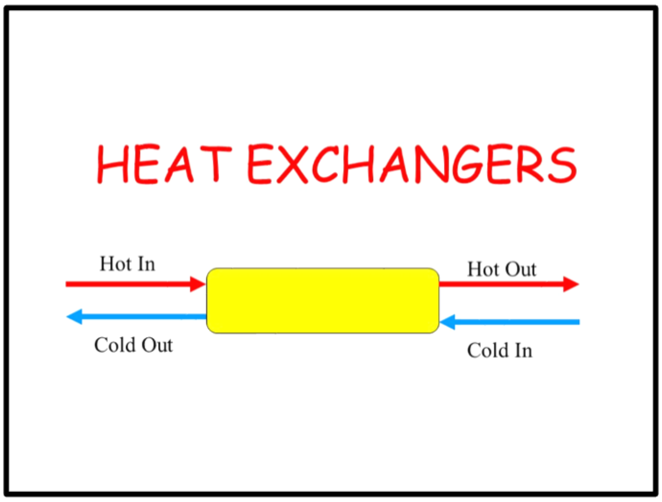


Figure 1.1: General Operating Principle of a Heat Exchanger

The primary design challenge is to determine the *sufficient heat transfer surface area* required to transfer the desired quantity of heat between the two fluids for a given flow configuration. The amount of heat transferred depends on several factors, such as the following ones:

- The inlet temperatures of both fluids.
- The thermal characteristics of the fluids, including their specific heat capacities and thermal conductivities.
- The convection heat transfer coefficients on each side of the wall.

- The surface area available for heat exchange.

1.4. Composition

A heat exchanger is composed of several essential elements: (*i*) *Heat exchange elements*, such as a core or matrix that contains the heat transfer surface; (*ii*) *Fluid distribution elements*, such as headers, reservoirs that direct the fluids to and from the exchange surface; (*iii*) *Inlet and outlet nozzles or pipes*: for connecting the exchanger to the external fluid circuits.

In most heat exchangers, there are no moving parts. However, exceptions exist, such as the *rotary recuperator*, in which the matrix rotates at a set speed to alternately expose it to hot and cold fluids. The heat transfer surface is in direct contact with the fluids, and heat is transferred by *conduction* through this surface. The part of the surface that physically separates the two fluids is called the *primary surface* (or direct contact surface). To increase the heat transfer area without significantly increasing the exchanger's volume, secondary surfaces known as *fins* can be attached to the primary surface. Fins are commonly used in air-cooled heat exchangers and finned-tube radiators.

Indeed, a comprehensive analysis of a heat exchanger typically includes three interconnected studies:

- *Thermal Study*: it determines the required heat exchange surface area, the actual heat transfer rate (thermal power), and the temperature distribution of both fluids from inlet to outlet.

- *Hydraulic Study*: this study calculates the pressure losses (pressure drops) experienced by each fluid as they flow through the exchanger. Pressure drop is a critical parameter because it determines the pumping power required and influences operating costs.

- *Mechanical Study*: it evaluates the mechanical forces and stresses acting on the exchanger components under operating conditions, considering operating temperatures, pressures, material properties, and safety margins.

These three studies are interdependent. For example, increasing the heat transfer surface area (thermal study) may increase pressure drops (hydraulic study) and require thicker walls (mechanical study). Therefore, an optimal design balances all three aspects.

1.5. Classification Criteria for Heat Exchangers

Heat exchangers can be classified according to a variety of criteria, each highlighting different aspects of their design, operation, or application. The most important classification criteria are presented in the subsequent Sections.

1.5.1. Classification According to Heat Transfer Process

Based on whether heat is stored temporarily or transferred directly, heat exchangers are divided into two categories:

- *Recuperators*: the two fluids flow simultaneously through the exchanger, separated by a solid wall. Heat is transferred continuously from the hot fluid to the cold fluid through the wall. This is the most common type.
- *Regenerators*: single matrix alternately comes into contact with the hot and cold fluids. During the hot period, the matrix stores heat; during the cold period, it releases heat to the cold fluid. Regenerators are often used in high-temperature applications (e.g., glass furnaces, gas turbines).

Specifically, Figure 1.2 shows a schematic representation of heat exchangers' classification according to the heat transfer process.

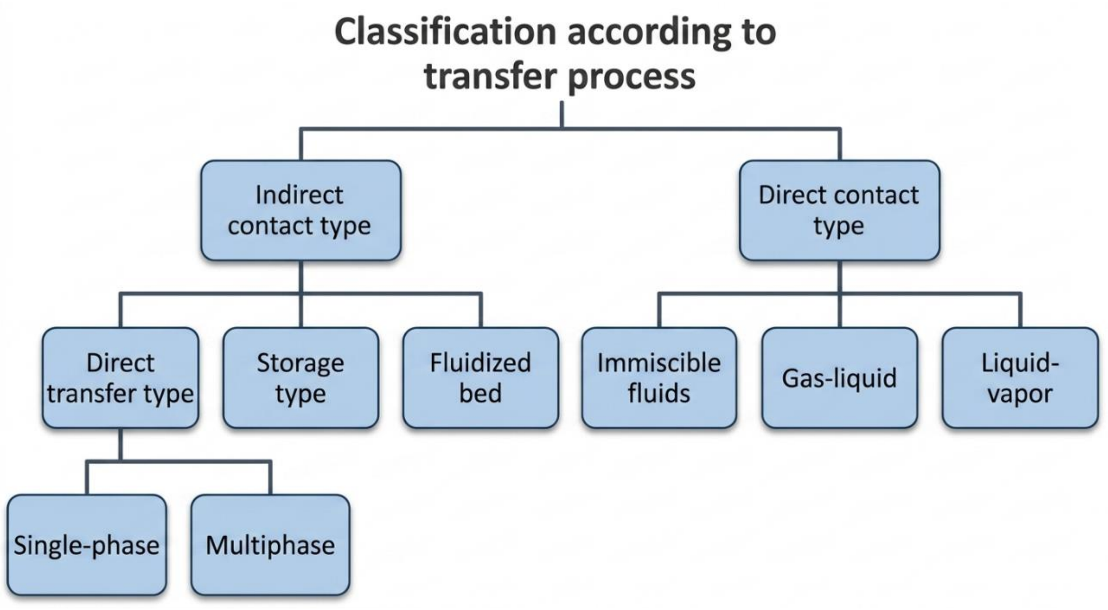


Figure 1.2: Schematic Representation of Classification According to Heat Transfer Process

1.5.2. Classification According to Number of Fluids

Most heat exchangers handle two fluids (one hot and one cold). However, some applications require three fluids (e.g., in cryogenic processes) or even *N fluids* (multiple streams). Multi-fluid exchangers are more complex to design but can offer significant energy savings by recovering heat from several sources simultaneously.

1.5.3. Classification According to Surface Compactness

Compactness is defined as the ratio of the heat transfer surface area to the volume of the exchanger. A high compactness means a large surface area is packed into a small volume. Compact heat exchangers (e.g., plate-fin and tube-fin types) are essential when space or weight is limited, such as in aerospace, automotive radiators, and electronic cooling. The degree of compactness is often expressed in square meters of surface area per cubic meter of volume (m^2/m^3). An illustration of compact versus non-compact heat exchanger surfaces is shown in Figure 1.3.

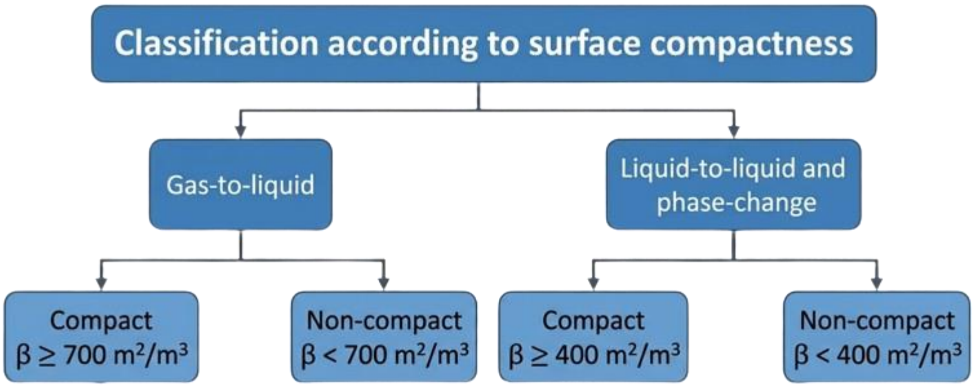


Figure 1.3: Illustration of Compact Versus Non-Compact Heat Exchanger Surfaces

1.5.4. Classification According to Design or Type

This criterion groups heat exchangers by their mechanical construction (see Figure 1.4). Common designs include the following types:

- *Tubular Exchangers.*
- *Plate Exchangers.*
- *Extended Surface Exchangers.*

- *Regenerative Exchangers.*

Each design has specific advantages in terms of cost, maintenance, pressure and temperature limits, and fouling resistance.

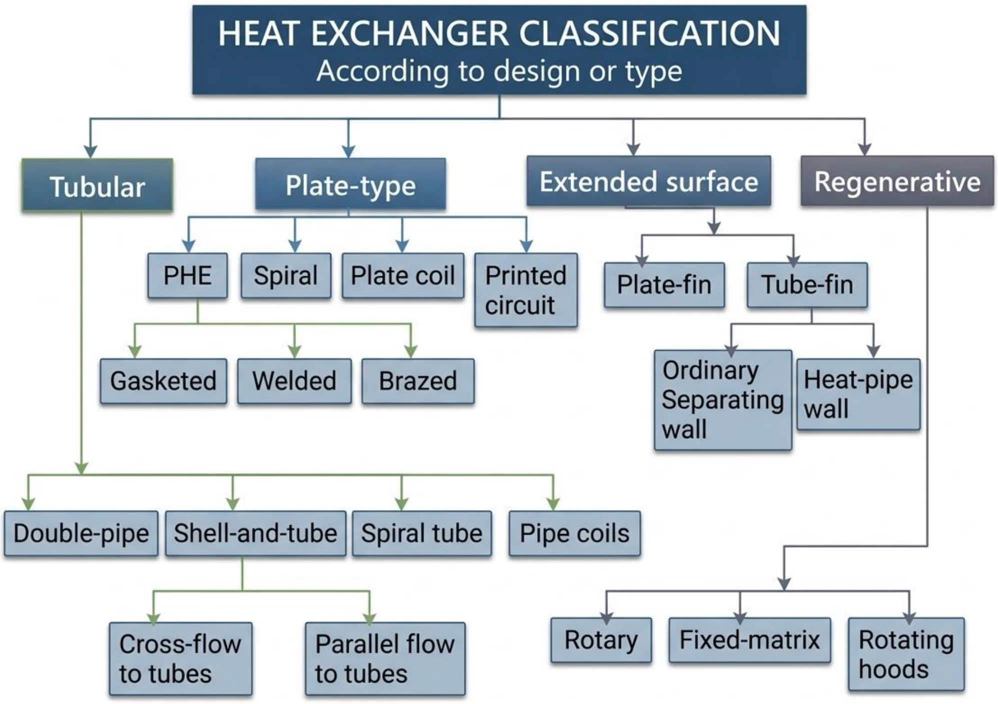


Figure 1.4: Overview of Major Heat Exchanger Design Types

1.5.5. Classification According to Flow Arrangements

The relative direction of fluid flow within the exchanger significantly affects heat transfer performance. Moreover, the main flow arrangements are the following ones:

- *Co-Current* (*Parallel Flow*), both fluids flow in the same direction.
- *Counter-current Flow*, fluids flow in opposite directions.
- *Crossflow*, fluids flow perpendicular to each other.
- *Mixed Flow Arrangements*, combinations of the above (e.g., cross-counter-current, multi-pass shell-and-tube).

Figure 1.5 presents the classification of heat exchangers according to the flow arrangements.

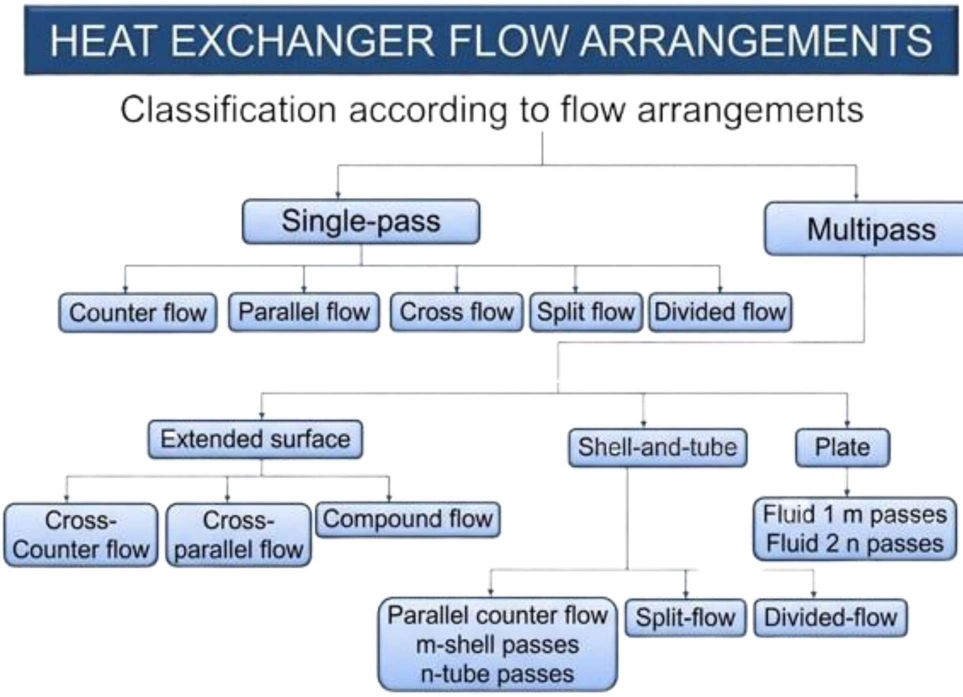


Figure 1.5: Overview of Heat Exchangers Classification According to Flow Arrangements

1.5.6. Classification According to Heat Transfer Mechanisms

This classification considers the phase state of the fluids and the dominant mode of heat transfer:

- Single-phase convection on both sides.
- Single-phase convection on one side, two-phase convection on the other side.
- Two-phase convection on both sides.
- Combined convection and radiative heat transfer.

1.5.7. Classification According to Process Function

Heat exchangers are often named by their role in a process, such as:

- *Condensers*, which convert vapor to liquid by removing latent heat.
- *Heaters*, which raise the temperature of a fluid.
- *Coolers*, which lower the temperature of a fluid.

- *Evaporators/Vaporizers*, which convert liquid to vapor.
- *Chillers,* which cool a fluid for air conditioning.

1.5.8. Classification According to Surface Material Nature

The choice of material affects thermal performance, corrosion resistance, weight, and cost. Common materials include the following ones:

- *Metallic,* such as steel, copper, aluminum, superalloys, and refractory metals.
- *Non-Metallic*, such as plastic (e.g., polypropylene, PTFE), ceramic, graphite, glass. These are used when corrosion resistance or electrical insulation is critical.

1.6. Different Types of Heat Exchangers

The selection of a heat exchanger for a given industrial application depends on multiple parameters, including the physical properties and aggressiveness of the fluids, the required temperature and pressure ranges, space and maintenance constraints, and economic considerations. Most heat exchangers fall into two broad categories based on their construction: tubular (tube) or plate exchangers. Each type has its own advantages and disadvantages, which are summarized in the following Sections.

1.6.1. Tubular Heat Exchangers

Tubular heat exchangers are the most basic and most common type, accounting for nearly half of all heat exchangers sold worldwide. Their main advantages include ease of manufacture, relatively low cost, simple maintenance, and, most importantly, the ability to operate at high pressures and temperatures. Their primary disadvantage is their size: they are robust and reliable but tend to be bulky and heavy. Tubular heat exchangers are generally divided into three categories:

- *Single-tube heat exchangers*: a single tube is placed in a tank (see Figure 1.6);
- *Double-pipe heat exchangers* (*Coaxial exchangers*): two concentric tubes, with one fluid flowing in the inner tube and the other in the annular space (see Figure 1.7). They can operate in co-current, counter-current, crossflow, or cross-counter-current arrangements.
- *Multi-tube heat exchangers*: several tubes inside a common shell. Subtypes include separate-tube exchangers (see Figure 1.8 (*a*)), close-tube exchangers (see Figure 1.8

(*b*)), finned-tube exchangers, with longitudinal or transverse fins (see Figure 1.9), and the widely used shell-and-tube heat exchangers (see Figure 1.10).

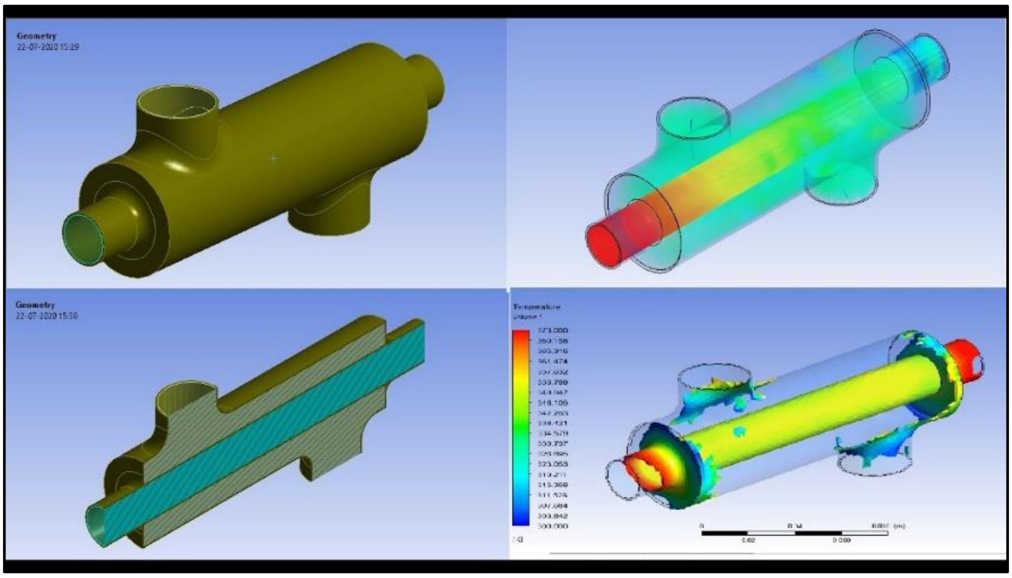
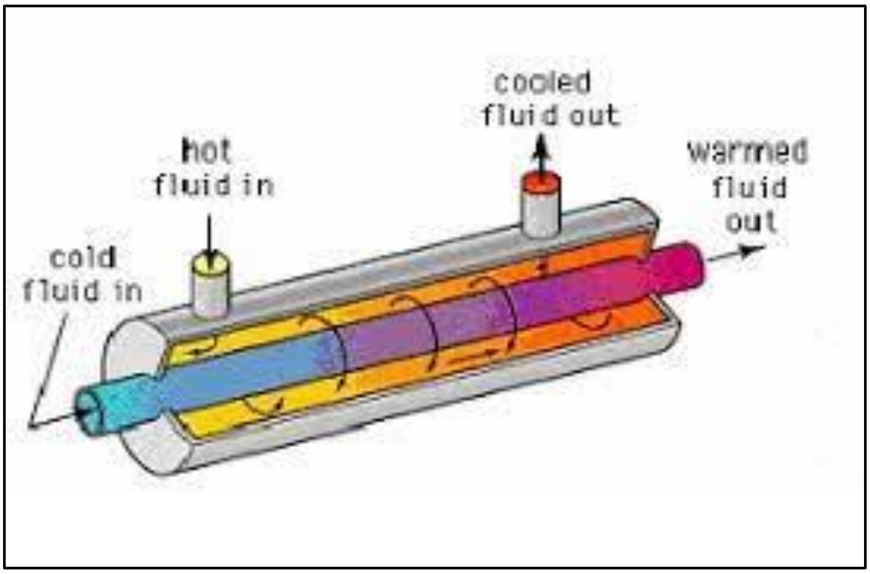
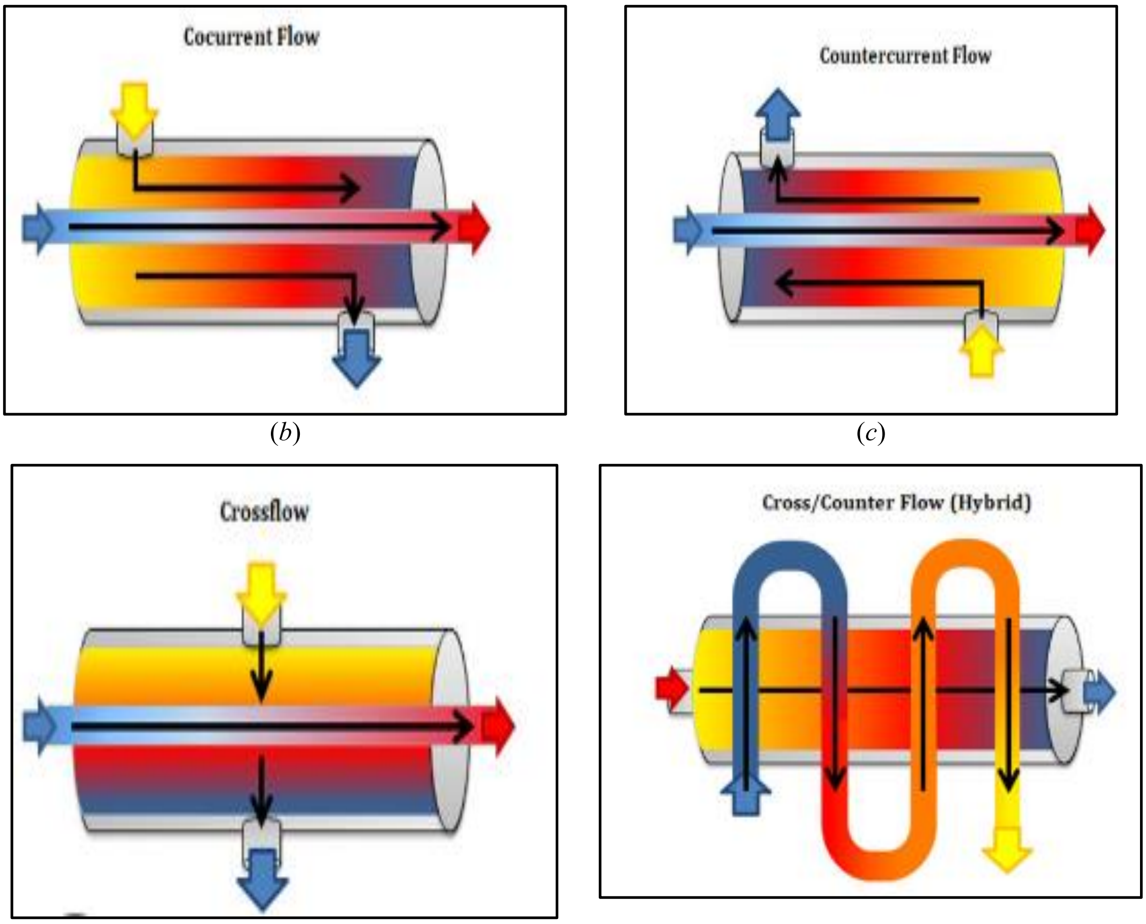


Figure 1.6: Single-Tube Heat Exchangers



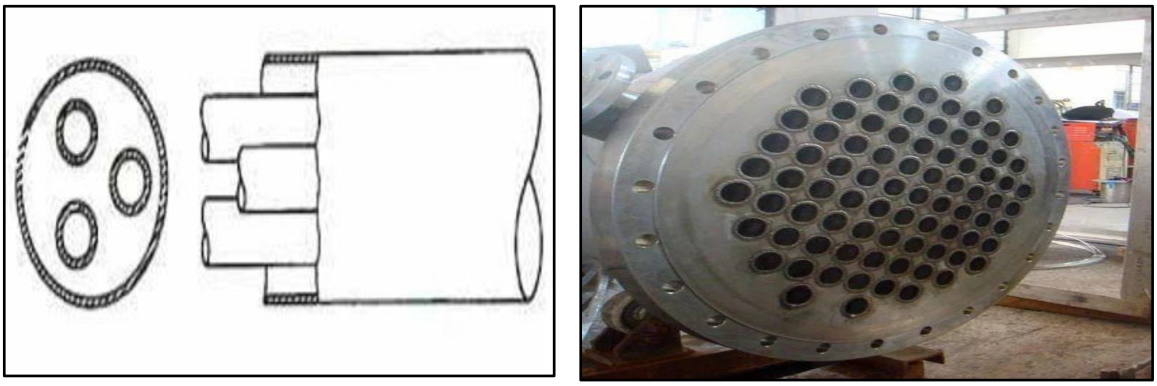
(*a*)



(*b*) (*c*)

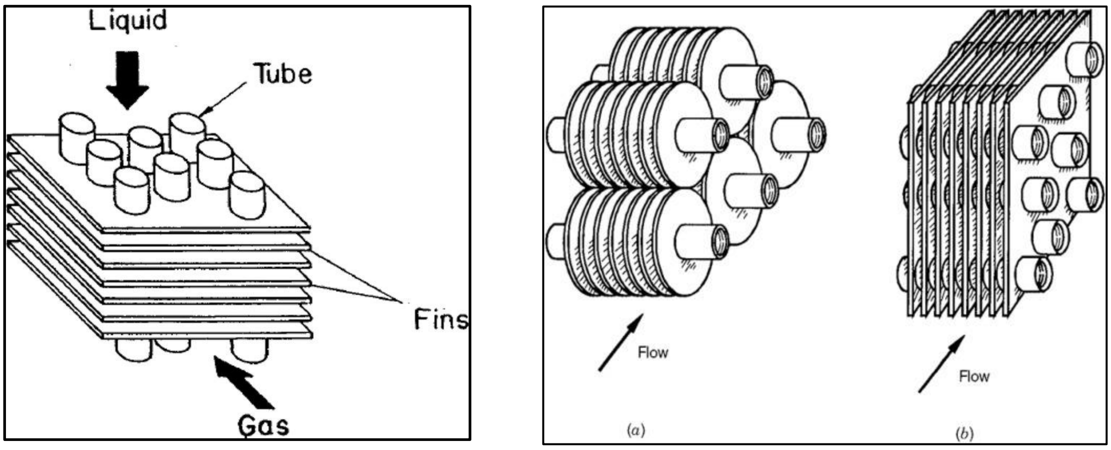
(*d*) (*e*)

Figure 1.7: Double-Pipe (Coaxial) Heat Exchanger: (a) Construction of Concentric Tubes; (b) Co-Current Flow; (c) Counter-Current Flow; (d) Crossflow; (e) Cross-Counter-Current Flow



(*a*) (*b*)

Figure 1.8: Multi-Tube Heat Exchangers: (a) Separate-Tube Heat Exchangers; (b) Close-Tube Heat Exchangers



(*a*) (*b*)

Figure 1.9: Finned Tube Heat Exchangers: (a) Longitudinal Fins; (b) Transverse Fins Continuous or Independent Fins

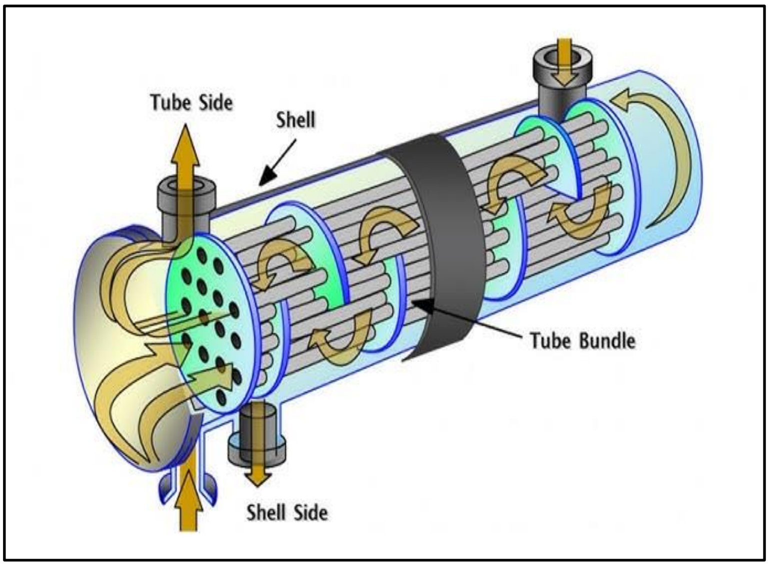


Figure 1.10: Shell-and-Tube Heat Exchangers

1.6.2. Plate Heat Exchangers

Plate heat exchangers consist of a stack of pressed metal plates (see Figure 1.11 and Figure 1.12). Heat is transferred between two fluids flowing through alternate channels formed by adjacent plates. Although they cannot always be used at very high temperatures and pressures, plate heat exchangers offer significant advantages in terms of higher thermal efficiency and compactness (i.e., large heat transfer surface per unit volume).

Several competing technologies exist within the plate heat exchanger family, such as the following ones:

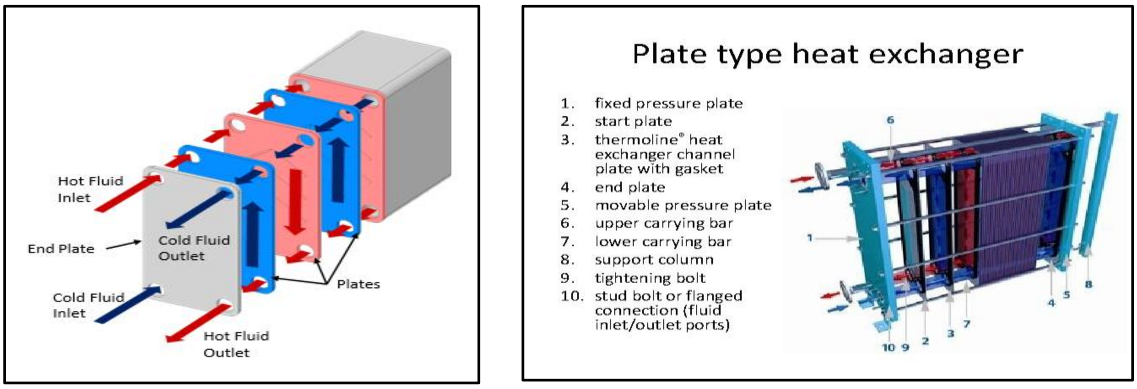


Figure 1.11: Plate Heat Exchangers Stack

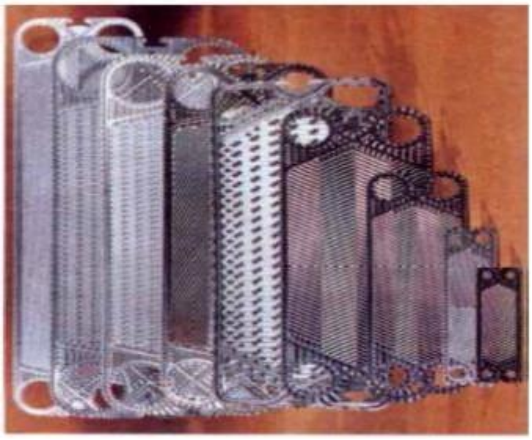


Figure 1.12: Primary Surface Heat Exchangers

- *Gasketed Plate Heat Exchangers*: this type is the oldest and most common type.

Elastomeric gaskets seal the exchanger and distribute the fluids into the channels (see Figure 1.13). They are easy to disassemble for cleaning.

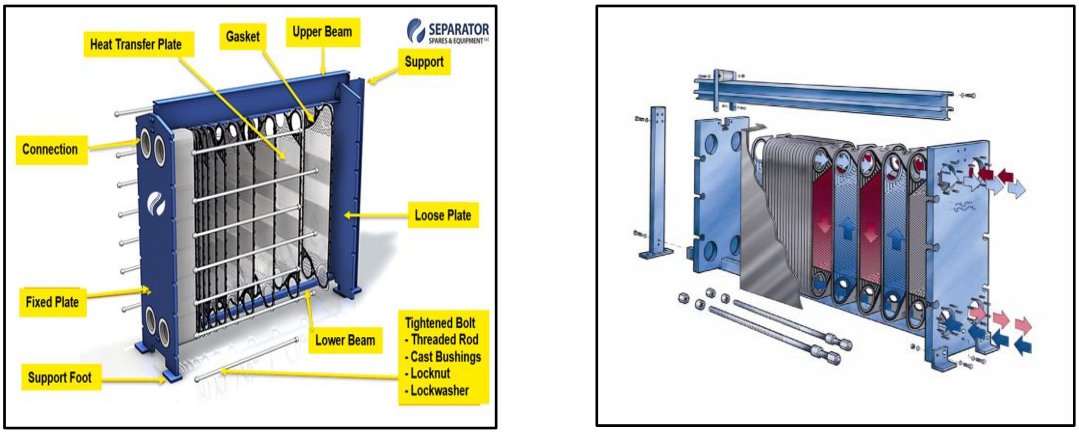


Figure 1.13: Gasketed Plate Heat Exchangers

- *Welded or Brazed Plate Heat Exchangers*: these overcome the limitations of gaskets (temperature, pressure, chemical compatibility). They are suitable for corrosive, hot, or pressurized fluids. Examples include: flat plate, lamellar, Compbloc, spiral, Pack inox, and brazed plate heat exchangers (see Figure 1.14).

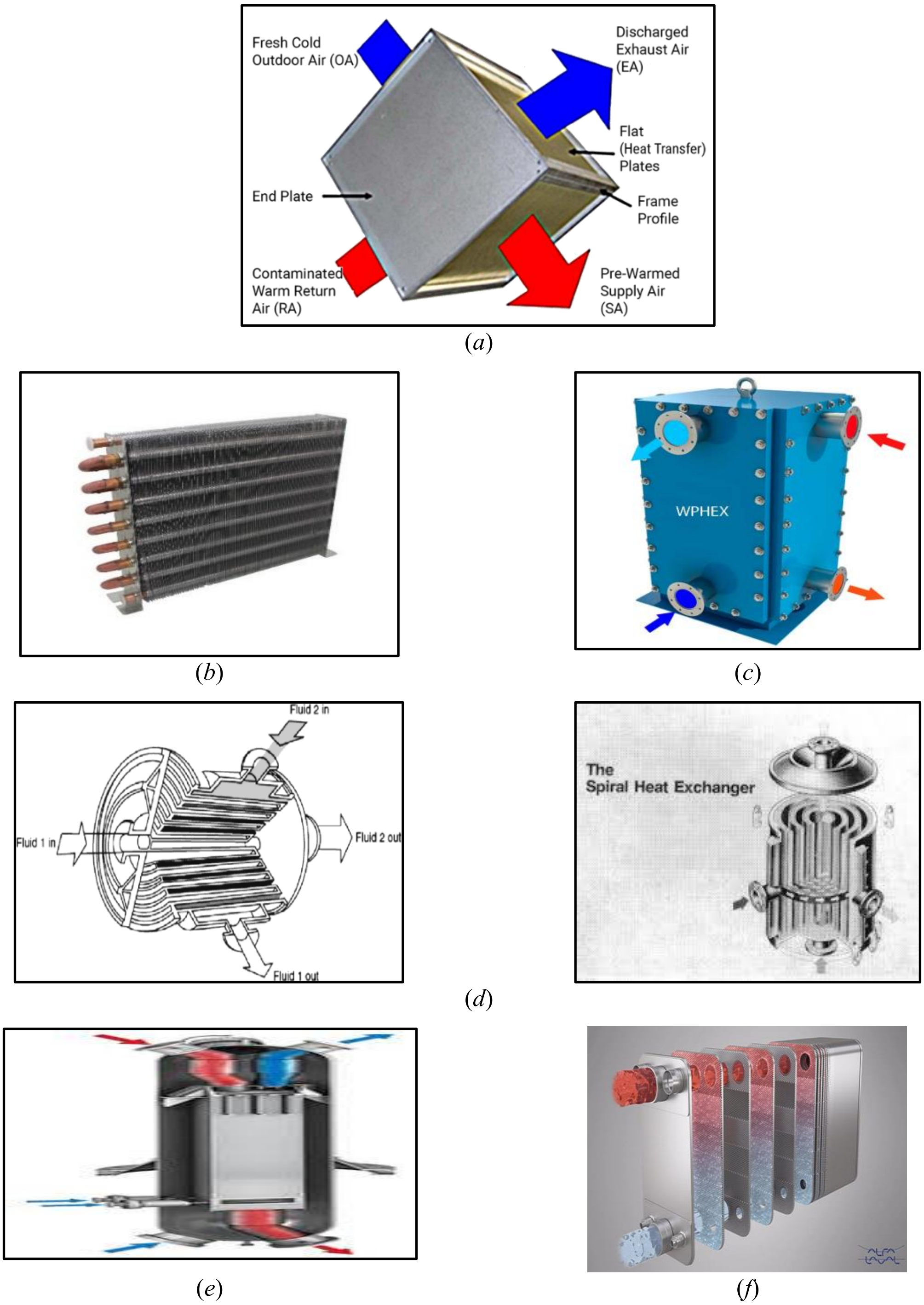


Figure 1.14: Welded or Brazed Heat Exchangers: (a) Flat plate; (b) Lamellar; (c) Compbloc; (d) Spiral; (e) Pack inox; (f) Brazed plate

1.6.3. Other Types of Heat Exchangers

Beyond tubular and plate designs, several specialized heat exchanger types exist for specific applications, such as the following ones:

- *Heat Pipe Exchanger*: a heat pipe acts as a thermal "superconductor" operating in a closed evaporation-condensation cycle. It can transfer large amounts of heat with very small temperature differences (see Figure I.15). Heat pipes are used in electronics cooling, heat recovery, and space applications.

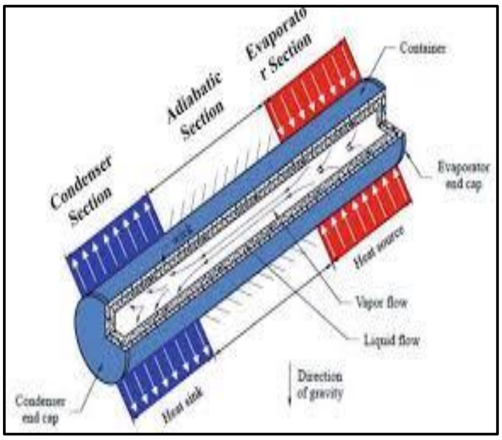
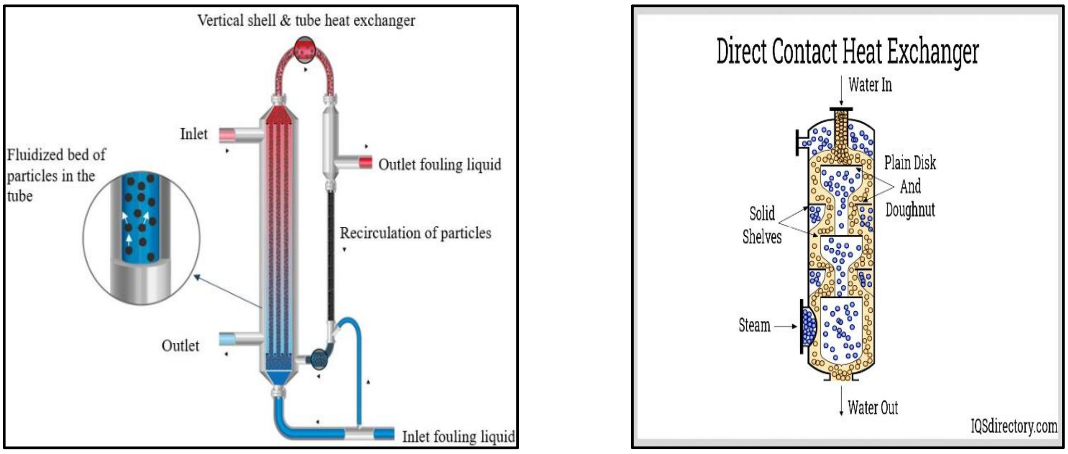


Figure 1.15: Heat Pipe Exchangers

- *Fluidized Bed Heat Exchanger*: a bed of solid particles is fluidized by a moving gas or liquid. The fluidized bed exchanges heat with an immersed heat transfer surface (see Figure 1.16(*a*)). These are used in drying, combustion, and chemical reactors.

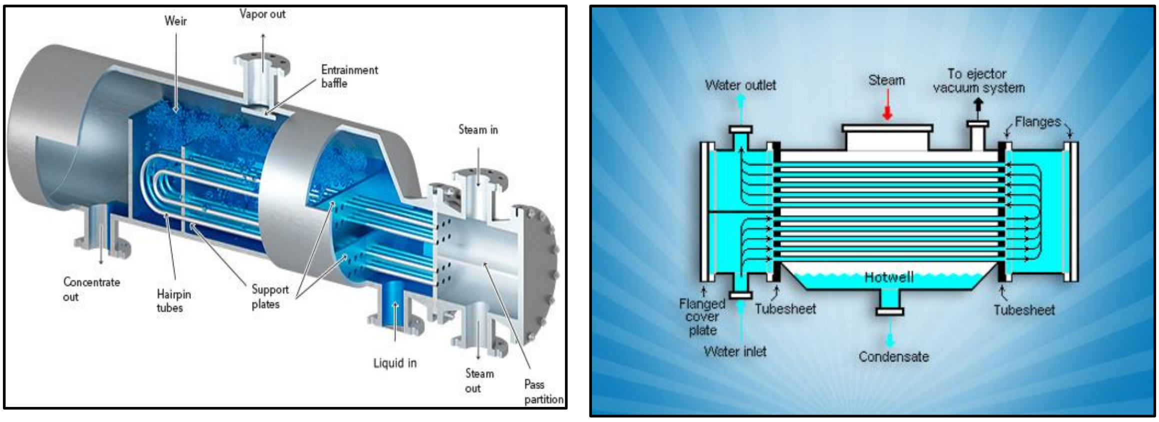
- *Direct Contact Heat Exchangers*: the two fluids are not separated by a wall; they come into direct contact, exchange heat, and are then separated. Because there is no wall, very close temperature approaches can be achieved. Often, mass transfer accompanies heat transfer (see Figure 1.16(*b*)). Examples include cooling towers and quench tanks.

- *Phase Change Heat Exchanger*: one of the fluids undergoes a phase change (boiling or condensation). Common examples are *evaporators* (cold fluid changes from liquid to vapor) and *condensers* (hot fluid changes from vapor to liquid). These are widely used in refrigeration, air conditioning, and power plants (see Figure 1.17).



(*a*) (*b*)

Figure 1.16: Specialized Exchangers: (a) Fluidized-Bed Heat Exchanger; (b) Direct Contact Heat Exchanger

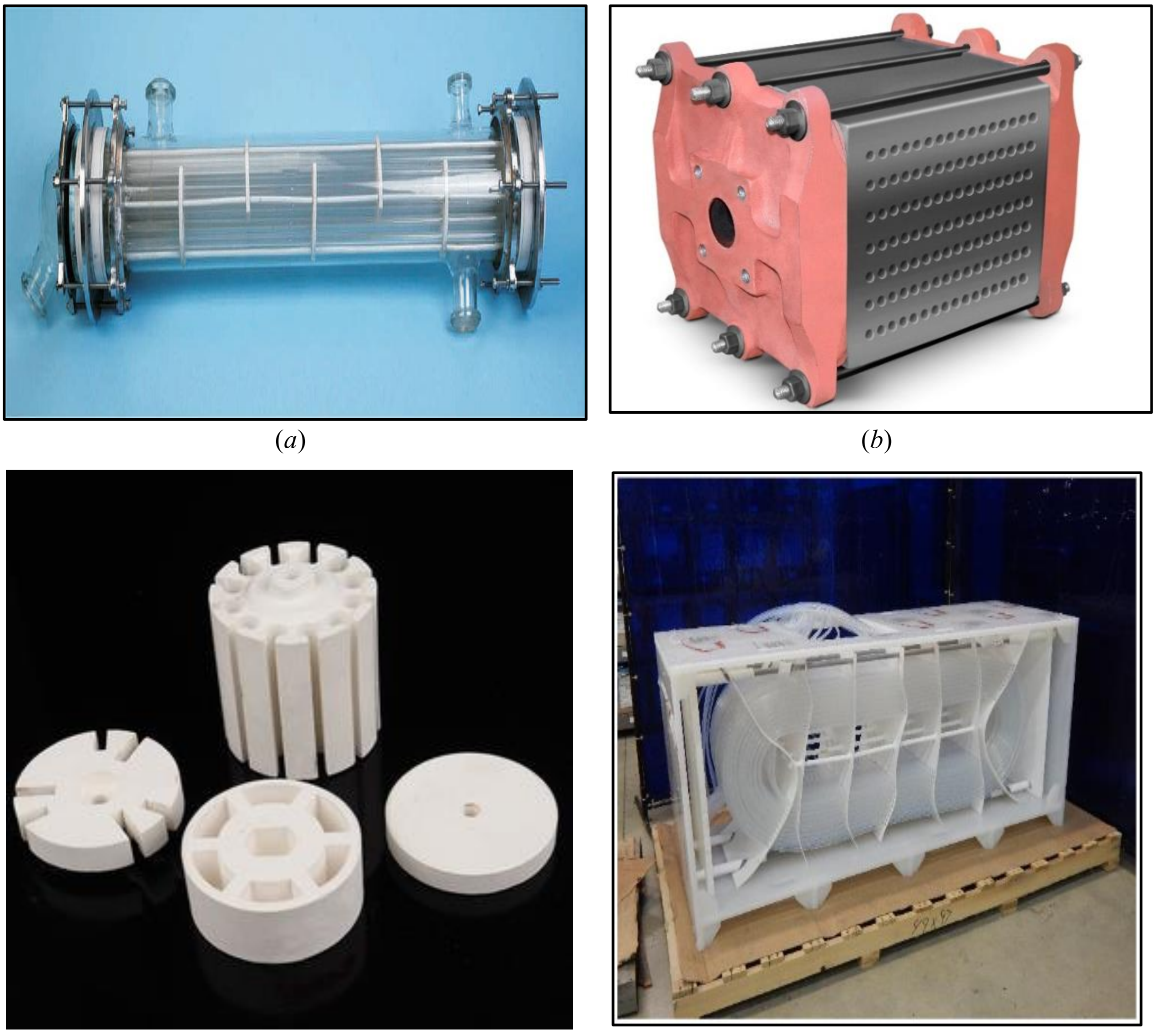


(*a*) (*b*)

*Figure 1.17: Phase-Change Heat Exchangers (*a*) Evaporators; (*b*) Condensers*

To solve their heat exchange problems, users are faced with a wide variety of products, the main categories of which are shell-and-tube and plate heat exchangers. However, the technology used in these types of heat exchangers focuses on the use of new materials and the conditions under which fluids are supplied. The materials most commonly used in heat exchangers are stainless steels and titanium, but other materials are also used, such as graphite, ceramics, glass, and plastics, to limit the effects of corrosion. The aim is to improve the strength of heat exchangers to make them more reliable when used in thermal cycles.

Used for over a century in the food industry, heat exchangers now have many applications, mainly in industry, but also in housing and transport. There is a wide variety of products, of different sizes and performances, which differ mainly in terms of their functions, conditions of use, technologies or materials used, as well as their price. Examples include *glass tube heat exchangers* (see Figure I.18 (a)), *graphite heat exchangers* (see Figure I.18 (b)), *ceramic heat exchangers* (see Figure I.18 (c)), and *plastic heat exchangers* (see Figure I.18 (d)).



(*a*) (*b*) (*c*) (*d*)

Figure 1.18: Non-Metallic Heat Exchangers (a) Glass Tube; (b) Graphite; (c) Ceramic; (d) Plastic

1.7. Conclusion

This chapter has introduced the fundamental concepts of heat exchangers, beginning with a general definition and an explanation of their operating principle. The essential components of a heat exchanger have been described, including the primary heat transfer surface, fluid distribution elements, and the importance of thermal, hydraulic, and mechanical studies in a complete design process.

A detailed overview of the main classification criteria has been presented, allowing the reader to distinguish heat exchangers by their transfer process, number of fluids, compactness, mechanical design, flow arrangement, heat transfer mechanisms, process function, and material of construction. Understanding these classifications is crucial for selecting the most appropriate exchanger type for a given industrial application.

The next chapter will move from qualitative descriptions to quantitative analysis. It will present the fundamental assumptions underlying thermal calculations, describe the two main operating modes, and introduce the two most widely used design methods: The *Logarithmic Mean Temperature Difference* (LMTD) method and the *Number of Transfer Units* (NTU) method. These methods provide the engineer with the tools necessary to size a heat exchanger or predict its performance under given operating conditions.

CHAPTER II
CALCULATING THE PERFORMANCE OF HEAT EXCHANGERS

2. CALCULATING THE PERFORMANCE OF HEAT EXCHANGERS

2.1. Introduction

After providing a qualitative overview of heat exchangers, including their classification, components, and the main types encountered in industry. The present chapter moves from description to quantitative analysis. Its objective is to present the fundamental calculation methods used to determine the thermal performance of heat exchangers, namely the required heat transfer surface area, the actual heat transfer rate, and the outlet temperatures of the fluids.

Two primary methods are widely used in engineering practice: The *Logarithmic Mean Temperature Difference* (LMTD) method and the *Number of Transfer Units* (NTU) method. Both methods rely on a set of simplifying assumptions and require knowledge of the overall heat transfer coefficient and fouling resistances. The LMTD method is particularly convenient when all inlet and outlet temperatures are known, whereas the NTU method is more suitable when outlet temperatures are unknown. Each method will be derived, illustrated with practical examples, and applied to typical design problems.

2.2. Assumptions

The thermal calculation methods (LMTD and NTU) are based on a set of simplifying assumptions. These idealizations are widely accepted in standard heat exchanger design practice and are valid for most single-phase, steady-state applications. The user should be aware of these assumptions to apply the methods correctly and to recognize when more advanced analysis is required. Specifically, the following assumptions are applied:

- *Steady-State Operation*: all coefficients and variables are constant over time.
- *No Phase Change*: The fluids remain single-phase throughout the exchanger; boiling or condensation does not occur.
- *No Heat Loss*: exchanger is perfectly adiabatic; all heat transferred from the hot fluid is received by the cold fluid.
- *One-Dimensional Temperature Variation*: Temperatures change only along the flow direction; radial temperature gradients are neglected.

2.3. Operating Modes

In order to introduce the thermal calculation methods, we consider a simple tubular (double-pipe) heat exchanger formed by two concentric tubes. One fluid flows through the inner tube, while the other fluid flows through the annular space between the inner and outer tubes. The following quantities define the operating conditions:

- T_{hin} and T_{cin}: inlet temperatures of the hot and cold fluids, respectively $[K]$.
- T_{hout} and T_{cout}: Outlet temperatures of the hot and cold fluids, respectively $[K]$.
- $\dot{m}_h$ et $\dot{m}_c$: Mass flow rates [kg / S].

Two fundamental flow arrangements are possible, which are the following ones:

- *Parallel or Co-Current Flow*: The hot and cold fluids enter the exchanger at the same end and flow in the same direction (see Figure 2.1). The temperature difference between the two fluids decreases along the length of the exchanger.

- *Counter-Current Flow*: The hot and cold fluids enter at opposite ends and flow in opposite directions (see Figure 2.2). The temperature difference remains more uniform along the length, leading to a higher thermal efficiency for the same surface area.

These two flow arrangements exhibit different temperature distributions, which directly affect the driving force for heat transfer and, consequently, the required surface area for a given duty.

2.4. Logarithmic Mean Temperature Difference Method (LMTD)

The LMTD method is a classical approach for calculating the heat transfer rate and required surface area in a heat exchanger when the inlet and outlet temperatures of both fluids are known. It is based on the fact that the temperature difference between the hot and cold fluids varies along the length of the exchanger. The method introduces a logarithmic mean temperature difference ΔT_{LMTD} which, when multiplied by the overall heat transfer coefficient k and the surface area S, gives the total heat transfer rate ϕ.

2.4.1. The Parallel Flow Heat Exchangers

In a parallel flow arrangement, both the hot and cold fluids enter the heat exchanger at the same end and flow in the same direction. As heat is transferred from the hot fluid to the cold fluid, the temperature of the hot fluid decreases while that of the cold fluid increases. Consequently, the temperature difference between the two fluids, $\Delta T = T_h - T_c$, decreases exponentially along the length of the exchanger.

Figure 2.1 illustrates the typical temperature profiles for this configuration.

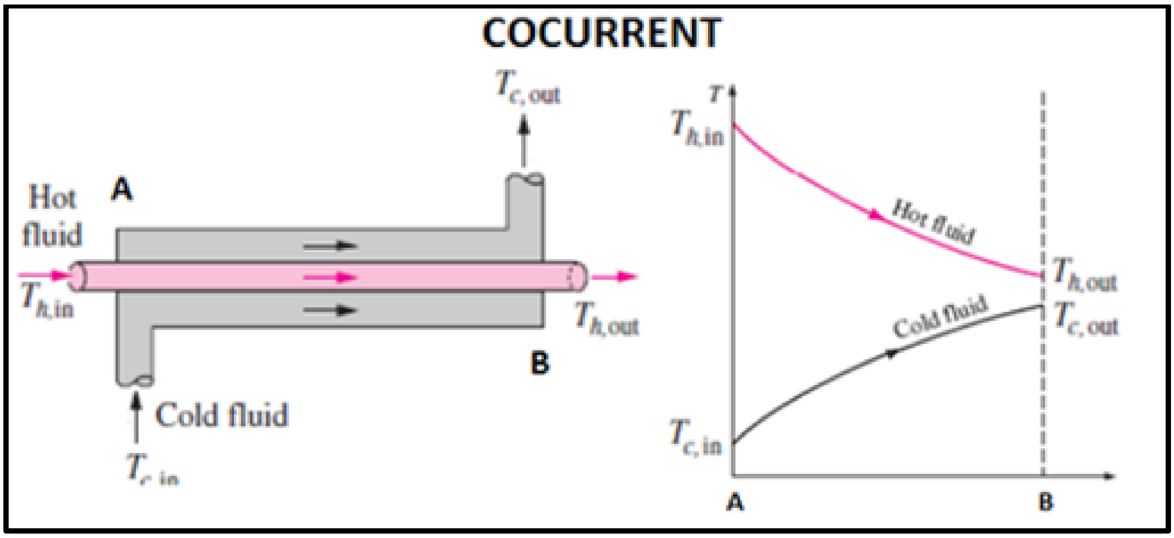


Figure 2.1: Temperature Distribution in a Parallel Flow (Co-Current) Heat Exchanger

We consider a differential surface element ds along the exchanger. The heat transfer rate $d\phi$ through this element can be expressed in three equivalent ways:

- From the overall heat transfer equation: $d\phi = k \times (T_h - T_c) \times ds$.
- From the energy balance on the hot fluid: $d\phi = -\dot{m}_h \times C_{ph} \times dT_h$.
- From the energy balance on the cold fluid: $d\phi = \dot{m}_c \times C_{pc} \times dT_c$

By combining the differential temperature changes, we have:

$$dT_h - dT_c = d(T_h - T_c) = -\left(\frac{1}{\dot{m}_h \times C_{ph}} + \frac{1}{\dot{m}_c \times C_{pc}}\right) \times d\phi \tag{2.1}$$

Moreover, by substituting $d\phi = k \times (T_h - T_c) \times ds$, we have:

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -\left(\frac{1}{\dot{m}_h \times C_{ph}} + \frac{1}{\dot{m}_c \times C_{pc}}\right) k \times ds = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) k \times ds \tag{2.2}$$

Integrating from the inlet ($x = 0$, where $T_h - T_c = T_{hin} - T_{cin}$) to an arbitrary position x, we obtain:

$$ln\,\frac{\left(T_h\,(x) - T_c\,(x)\right)}{(T_{hin} - T_{cin})} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) k \times s(x) \tag{2.3}$$

At the outlet ($x = L$), we have $T_h(L) = T_{hout}$, $T_c(L) = T_{cout}$, and $s(L) = S$. Thus, we have:

$$ln\,\frac{(T_{hout}\; - T_{cout}\,)}{(T_{hin} - T_{cin})} = -\left(\frac{1}{C_h} + \frac{1}{C_c}\right) k \times S \tag{2.4}$$

The total heat transfer rate ϕ can also be expressed from energy balances: $\phi = C_h(T_{hin} - T_{hout}) = C_c(T_{cin}\; - T_{cout}\,)$. Indeed, by using Equation (2.4), we obtain:

$$\phi = k \times S \times \left[\frac{(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})}{ln\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]}\right] \tag{2.5}$$

2.4.2. The Counter-Current Flow Heat Exchangers

In a counter-current flow arrangement, the hot and cold fluids enter at opposite ends of the heat exchanger and flow in opposite directions. The hot fluid enters at one end and exits at the other, while the cold fluid enters at the opposite end and flows toward the hot fluid inlet. This configuration maintains a more uniform temperature difference along the length of the exchanger, as illustrated in Figure 2.2. Consequently, for the same surface area, a counter-current exchanger achieves a higher heat transfer rate than a parallel-flow exchanger.

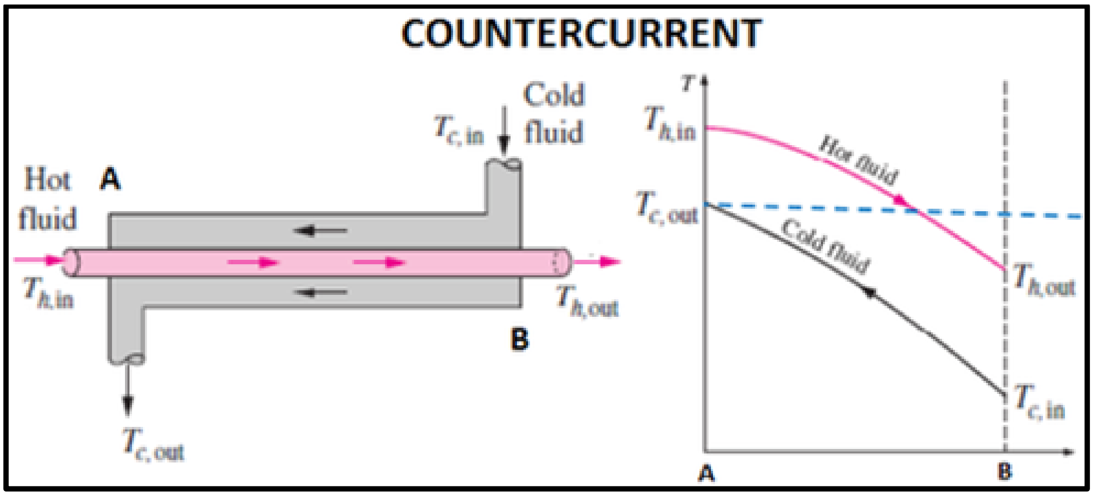


Figure 2.2: Temperature Distribution in Counter-Current Flow Heat Exchanger

Similarly, as formally provided in Section 2.4.1, we consider a differential surface element ds along the exchanger. For counter-current flow, the energy balances on the two fluids are the following ones:

- *Hot Fluid*: $d\phi = -\dot{m}_h \times C_{ph} \,.\, dT_h$.
- *Cold Fluid*: $d\phi = -\dot{m}_c \times C_{pc} \,.\, dT_c$.

Thus, the temperature difference $\Delta T = T_h - T_c$ varies along the exchanger. Following a similar integration procedure as for parallel flow, but accounting for the opposite flow directions, the resulting expression for the total heat transfer rate is:

$$\phi = k \times S \times \left[\frac{(T_{hout} - T_{cin}) - (T_{hin} - T_{cout})}{ln\left[\frac{(T_{hout} - T_{cin})}{(T_{hin} - T_{cout})}\right]} \right] \tag{2.6}$$

2.4.3. Generalisation

From the derivations for parallel flow and counter-current flow, a common expression emerges. For any flow arrangement where the temperature difference between the hot and cold fluids varies monotonically along the heat exchanger, the total heat transfer rate can be written in the following compact form:

$$\Phi = k\ S\ \Delta T_{LMTD} \tag{2.7}$$

where ΔT_{LMTD} is the *logarithmic mean temperature difference, defined generally as:*

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \tag{2.8}$$

Here, ΔT_1 and ΔT_2 represent the temperature differences between the hot and cold fluids at the two ends of the heat exchanger. The specific definitions of ΔT_1 and ΔT_2 depend on the flow arrangement, as summarized below.

- *Parallel* (*Co-Current*) *Flow*:

$$\Delta T_1 = (T_{hin} - T_{cin}) \qquad \Delta T_2 = (T_{hout} - T_{cout}) \tag{2.9}$$

i.e.,

$$\Delta T_{LMTD} = \left[\frac{(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})}{ln\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]} \right] \tag{2.10}$$

- *Counter-Current Flow*:

$$\Delta T_1 = (T_{hin} - T_{cout}) \qquad \Delta T_2 = (T_{hout} - T_{cin}) \tag{2.11}$$

i.e.,

$$\Delta T_{LMTD} = \left[\frac{(T_{hout} - T_{cin}) - (T_{hin} - T_{cout})}{ln\left[\frac{(T_{hout} - T_{cin})}{(T_{hin} - T_{cout})}\right]} \right] \tag{2.12}$$

2.4.4. Global Heat Transfer Coefficient

The overall heat transfer coefficient k (also called the global exchange coefficient) quantifies the ease with which heat is transferred from the hot fluid to the cold fluid through the separating wall. It combines the effects of convection on both sides and conduction through the wall. For a clean, unfouled heat exchanger with equal surface areas on both sides ($S_h = S_c = S$), the overall coefficient is given by:

$$k = \left[\frac{1}{\frac{1}{h_h} + \frac{e}{\lambda} + \frac{1}{h_c}} \right] \tag{2.13}$$

where h_h and h_c are the convection heat transfer coefficients on the hot side and cold side, respectively. e is the thickness of the solid wall and λ is the thermal conductivity of the wall.

In many heat exchangers, the heat transfer surface areas on the hot and cold sides are not equal. This is particularly true for finned surfaces, where one side has extended surfaces (fins) to enhance heat transfer. In such cases, the overall heat transfer coefficient must be referenced to a specific surface area. Two coefficients are defined:

- k_h: overall coefficient based on the hot-side surface area S_h.
- k_c: overall coefficient based on the cold-side surface area S_c.

Moreover, during operation, other materials accumulate on the heat transfer surfaces. These fouling layers have low thermal conductivity and add additional resistance. Let:

- R_{eh}: fouling resistance per unit area on the hot-side surface area S_h.
- R_{ec}: fouling resistance per unit area on the cold-side surface area S_c.

As a result, the overall coefficients, including fouling, become:

$$k_h = \frac{1}{\frac{1}{h_h} + Re_h + \frac{e}{\lambda}\frac{S_h}{S_m} + \left(R_{ec} + \frac{1}{h_c}\right)\frac{S_h}{S_c}} \tag{2.14}$$

$$k_c = \frac{1}{\frac{1}{h_c} + R_{ec} + \frac{e}{\lambda}\frac{S_c}{S_m} + \left(R_{eh} + \frac{1}{h_h}\right)\frac{S_c}{S_h}} \tag{2.15}$$

Typical values of the overall heat transfer coefficient k for various configurations are given in Table 2.1.

Table 2.1: Order of Magnitude Values of Global Heat Transfer Coefficients K

Configuration	Typical value	Typical range
Gas-to-gas at normal pressure	20	5-50
Gas-to-gas at high pressure	200	50-500
Liquid-to-gas or gas-to-liquid	50	10-100
Liquid-to-liquid tubular	1000	200-2000
Liquid-to-liquid plate	2500	500-5000
Condenser (gas side)	50	10-100
Condenser (liquid side)	3000	500-6000
Vaporiser (gas side)	50	10-100
Vaporiser (liquid side)	5000	500-10 000
Vaporiser (condensing gas side)	3000	600-6000

2.4.5. Fouling Resistances

During normal operation, heat exchanger surfaces gradually accumulate deposits such as scale, dirt, rust, or organic matter. These deposits, collectively referred to as *fouling*, introduce additional thermal resistances that reduce the overall heat transfer coefficient and degrade performance over time. Fouling resistances are typically determined by comparing the heat transfer performance of a clean (new) exchanger with that of an exchanger that has been in service for a period.

Typical fouling resistances range from approximately 1×10^{-4} to 70×10^{-4} $m^2 °C/W$, as shown in Table 2.2, depending on the fluid type, temperature, velocity, and surface material. Higher velocities tend to reduce fouling, while higher temperatures often increase it, especially for fluids containing scale-forming minerals.

Table 2.2: Some Fouling Resistance Values

Fluid	$\boldsymbol{R_f}$
Sea Water at $T < 50°C$	10^{-4}
Sea Water at $T > 50°C$	2×10^{-4}
City Water at $T < 50°C$	2×10^{-4}
City Water at $T > 50°C$	3.5×10^{-4}
River Water	$3.5 \times 10^{-4} - 7 \times 10^{-4}$
Non-Greasy Water Vapor	10^{-4}
Oily Water Vapor	2×10^{-4}
Fuel Oil	$4 \times 10^{-4} - 9 \times 10^{-4}$
Steam (Oil-Free)	10^{-4}
Refrigerants (Liquid)	1.8×10^{-4}
Refrigerants (Vapor)	4×10^{-4}
Alcohol Vapors	10^{-4}
Air	4×10^{-4}
Gasoline, Kerosene	2×10^{-4}
Lubricating Oil	1.8×10^{-4}
Non-Dusted Air	3.5×10^{-4}
Gaseous Combustion Products	$20 \times 10^{-4} - 70 \times 10^{-4}$

2.4.6. Applications

The following two applications illustrate the use of the LMTD method for calculating the logarithmic mean temperature difference and the required heat transfer surface area.

2.4.6.1. Application 1

In this application, we calculate ΔT_{LMTD} in both cases (co-current and counter-current), a heat exchanger operates under the following inlet and outlet temperatures:

$T_{hin} = 100°\text{C}$ $\quad T_{hout} = 45°\text{C}$ $\quad T_{cin} = 15°\text{C}$ $\quad T_{cout} = 30°\text{C}$

The logarithmic mean temperature difference is calculated using the general formula:

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

- *Co-Current Flow*:

$$\Delta T_{LMTD} = \frac{(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})}{\ln\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]} = \frac{(45-30)-(100-15)}{\ln\left[\frac{(45-30)}{(100-15)}\right]} = 40.35\ °\text{C} = 313.5K$$

- *Counter-Current Flow*:

$$\Delta T_{LMTD} = \frac{(T_{hout} - T_{cin}) - (T_{hin} - T_{cout})}{ln\frac{(T_{hout} - T_{cin})}{(T_{hin} - T_{cout})}} = \frac{(45-15)-(100-30)}{\ln\left[\frac{(45-15)}{(100-30)}\right]} = 47.20°\text{C} = 320.35\ K$$

2.4.6.2. Application 2

A heat exchanger has the following operating conditions:

$K = 300 W/m^2.°\text{C}$ $\quad T_{hin} = 110°\text{C}$ $\quad T_{hout} = 30°\text{C}$ $\quad \dot{m}_h = 5000\ kg/h$ $\quad C_{ph} = 2100\ J/kg\ .K$ $\quad T_{cin} = 12°\text{C}$ $\quad \dot{m}_c = 12000\ kg/h$ $\quad C_{pc} = 4180\ J/kg\ .K$

Step 1: Calculate the heat transfer rate ϕ: using the hot side energy balance, we have that:

$$\Phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout}) = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$$
$$= \frac{5000}{3600} \times 2100 \times (110 - 30) = 2.33 \times 10^5 W$$

Step 2: Determine the cold fluid outlet temperature T_{cout}: using the cold side energy balance, we have that:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) \quad \Rightarrow T_{cout} = T_{cin} + \frac{\Phi}{\dot{m}_c \times C_{pc}}$$

$$T_{cout} = T_{cin} + \frac{\Phi}{\dot{m}_c \times C_{pc}} = 12 + \frac{2.33 \times 10^5}{\frac{12000}{3600} \times 4180} = 28.52℃ = 301.67K$$

Step 3: Calculate the required surface area S:

- Co-Current Flow:

$$\Phi = K \times S \times \left[\frac{(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})}{ln\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]}\right]$$

i.e.,

$$S = \Phi \times \left[\frac{ln\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]}{K \times [(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})]}\right] =$$

$$= 2.33 \times 10^5 \times \left[\frac{ln\left[\frac{(30 - 28.52)}{(110 - 12)}\right]}{300 \times [(30 - 28.52) - (110 - 12)]}\right] = 33.74\, m^2$$

- Counter-Current Flow:

$$\Phi = K \times S \times \left[\frac{(T_{hout} - T_{cin}) - (T_{hin} - T_{cout})}{ln\left[\frac{(T_{hout} - T_{cin})}{(T_{hin} - T_{cout})}\right]}\right]$$

i.e.,

$$S = \Phi \times \left[\frac{ln\left[\frac{(T_{hout} - T_{cin})}{(T_{hin} - T_{cout})}\right]}{K \times [(T_{hout} - T_{cin}) - (T_{hin} - T_{cout})]}\right] =$$

$$= 2.33 \times 10^5 \times \left[\frac{ln\left[\frac{(30 - 12)}{(110 - 28.52)}\right]}{300 \times [(30 - 12) - (110 - 28.52)]}\right] = 18.7\, m^2$$

The counter-current arrangement requires significantly less surface area (18.7 m^2) compared to co-current (33.74 m^2), demonstrating the advantage of counter-current flow for heat exchanger design.

2.5. Number of Transfer Units Method

When the inlet or outlet temperatures of the fluid streams are not known, the LMTD method requires an iterative trial-and-error procedure in the thermal analysis of heat exchangers. To avoid this, the method of the *Number of Transfer Units* (NTU) is used by introducing the concept of heat exchanger effectiveness ε.

2.5.1. Definition of Heat Exchanger Efficiency

The efficiency of a heat exchanger is defined as the ratio of the actual heat transfer rate to the maximum theoretically possible heat transfer rate under the same inlet conditions:

$$\varepsilon = \frac{\Phi_{real}}{\Phi_{max}} \tag{2.16}$$

The maximum possible heat transfer rate Φ_{max} would be achieved if one of the fluids were to undergo a temperature change equal to the maximum temperature difference available in the exchanger, i.e., $T_{hin} - T_{cin}$. The actual heat transfer is limited by the fluid with the smaller heat capacity rate $C_{min} = \min(C_h, C_c)$. Therefore:

$$\Phi_{max} = C_{min}(T_{hin} - T_{cin}) \tag{2.17}$$

Depending on which fluid has the smaller heat capacity rate, the effectiveness takes two forms:

- $C_c > C_h$:

$$\varepsilon = \frac{(T_{hin} - T_{hout})}{(T_{hin} - T_{cin})} \tag{2.18}$$

- $C_c < C_h$:

$$\varepsilon = \frac{(T_{cout} - T_{cin})}{(T_{hin} - T_{cin})} \tag{2.19}$$

2.5.2. Efficiency Calculation

For the two basic flow arrangements.

- *Parallel (Co-Current) Flow*:

$$\varepsilon = \frac{1 - exp\left(-\left(\frac{1}{C_c} + \frac{1}{C_h}\right) k \times S\right)}{\left(1 + \frac{C_h}{C_c}\right)} \tag{2.20}$$

For $S \to \infty$, we have:

$$\varepsilon = \frac{C_c}{(C_h + C_c)} \tag{2.21}$$

- *Counter-Current Flow*:

$$\varepsilon = \frac{1 - exp\left(-\left(\frac{1}{C_c} - \frac{1}{C_h}\right) k \times S\right)}{\left(1 - \frac{C_h}{C_c}\left(exp\left(-\left(\frac{1}{C_c} - \frac{1}{C_h}\right) k \times S\right)\right)\right)} \tag{2.22}$$

For $S \to \infty$, we have:

$$\varepsilon = 1 \tag{2.23}$$

2.5.3. NTU Approach

The Number of Transfer Units (NTU) is a dimensionless parameter that represents the heat transfer size or capacity of the exchanger. It is defined as:

$$NTU = \frac{k \times S}{C_{min}} \tag{2.24}$$

where k is the overall heat transfer coefficient, S is the heat transfer surface area, and C_{min} is the smaller heat capacity rate. A larger NTU indicates a larger exchanger or better heat transfer capability.

By defining $Z = \frac{\boldsymbol{C_h}}{\boldsymbol{C_c}} < 1$, we have that $Z = \frac{C_{min}}{C_{max}}$. As a result, the effectiveness ε can be expressed as a function of NTU and Z. The relationships can also be inverted to obtain NTU given ε and Z:

- *Parallel (Co-Current) Flow*:

$$NTU_{max} = -\frac{Log[1-(1+Z)\times\varepsilon]}{1+Z} \tag{2.25}$$

We have:

$$\varepsilon = \frac{1-exp[-NTU_{max}(1+Z)]}{1+Z} \tag{2.26}$$

- *Counter-Current Flow*:

$$NTU_{max} = \frac{1}{Z-1}\ Log\left[\frac{(\varepsilon-1)}{(Z\times\varepsilon-1)}\right] \tag{2.27}$$

We have:

$$\varepsilon = \frac{1-exp[-NTU_{max}(1-Z)]}{1-Z\times exp[-NTU_{max}(1-Z)]} \tag{2.28}$$

- *Particular Case* $Z = 0$:

$$NTU_{max} = -\ Log[1-\varepsilon] \tag{2.29}$$

We have:

$$\varepsilon = 1-\exp[-NTU_{max}] \tag{2.30}$$

- *Particular Case* $Z = 1$:

$$NTU_{max} = \frac{\varepsilon}{1-\varepsilon} \tag{2.29}$$

We have:

$$\varepsilon = \frac{NTU_{max}}{NTU_{max}+1} \tag{2.30}$$

The NTU method is particularly useful for performance (rating) problems, where the exchanger geometry is known, and the outlet temperatures are to be found, and for sizing problems where one outlet temperature is specified.

2.5.4. Applications

2.5.4.1. Application 1

A counter-current heat exchanger operates under the following conditions:

$$T_{hin} = 350℃ \quad T_{hout} = 200℃$$

$$T_{cin} = 120℃ \quad T_{cout} = 290℃$$

$$C_{min} = C_c \qquad \phi = 415kW$$

Step 1: Calculate the heat transfer rate in the co-current case $\phi_{co-current}$:

Since $C_{min} = C_c$, we have that:

$$\varepsilon = \frac{T_{cout} - T_{cin}}{T_{hin} - T_{cin}} = \frac{290 - 120}{350 - 120} = 0.74$$

i.e.,

$$Z = \frac{C_{min}}{C_{max}} = \frac{T_{hin} - T_{hout}}{T_{cout} - T_{cin}} = \frac{350 - 200}{290 - 120} = 0.882$$

Using the counter-current NTU equation (see Equation (2.27)), we have:

$$NTU = \frac{1}{Z-1}\, Log\left[\frac{(1 - Z \times \varepsilon)}{(1-\varepsilon)}\right] = \frac{1}{0.882 - 1}\, Log\left[\frac{(1 - 0.882 \times 0.74)}{(1 - 0.74)}\right] = 2.45$$

If the exchanger operates in co-current flow, with the same inlet temperatures and the same flow rates, i.e., $NTU_{co-current} = NTU_{conter-current}$, indeed, we have:

$$\varepsilon_{co-current} = \frac{1 - exp[-NTU \times (1 + Z)]}{1 + Z} = \frac{1 - exp[-2.45 \times (1 + 0.882)]}{1 + 0.882} = 0.526$$

Since the conditions are identical in both cases: $\frac{\phi_{co-current}}{\phi_{counter-current}} = \frac{\varepsilon_{co-current}}{\varepsilon_{counter-current}}$, we have:

$$\phi_{co-current} = \phi_{counter-current}\left(\frac{\varepsilon_{co-current}}{\varepsilon_{counter-current}}\right) = 415 \times 10^3 \times \left(\frac{0.526}{0.74}\right)$$
$$= 295 \times 10^3 W$$

Step 2: Calculation of the new outlet temperatures:

We have that $\varepsilon_{co-current} = \frac{T'_{cout} - T_{cin}}{T_{hin} - T_{cin}}$:

$$T'_{cout} = T_{cin} + \varepsilon_{co-current}(T_{hin} - T_{cin}) = 120 + 0.526 \times (350 - 120) = 241℃ = 514.15K$$

Moreover, we have that $Z = \frac{T_{hin} - T'_{hout}}{T_{cout} - T_{cin}}$:

$$T'_{hout} = T_{hin} - Z\left(T'_{cout} - T_{cin}\right) = 350 - 0.882 \times (241 - 120) = 243.3℃ = 516.45K$$

2.5.4.2. Application 2

A double pipe parallel flow heat exchanger uses oil ($c_{p,oil} = 1.88\ kJ/kg.K$) at an initial temperature of $205°C$ to heat water ($c_{p,water} = 4.18\ kJ/kg.K$), flowing at $225kg/h$ from $16°C$ to $44°C$. The oil flow rate is $270\ kg/h$. For an overall heat transfer coefficient of $K = 340\ W/m^2.K$. What is the required heat transfer area, NTU, and effectiveness ε.

Step 1: Calculation of Heat transfer rate ϕ

$$\phi = \dot{m}_c \times\ C_{pc}\ \times (T_{cout} - T_{cin}) = \frac{225}{3600} \times 4.18 \times 10^3 \times (44 - 16) = 7315W$$

Step 2: Calculation of Hot fluid outlet temperature

$$\phi = \dot{m}_h\,.\,C_{ph}\,.\,(T_{hin} - T_{hout})$$

i.e.,

$$T_{hout} = T_{hin} - \frac{\phi}{\dot{m}_h \times C_{ph}} = 205 - \frac{7315}{\left[\frac{270}{3600}\right] \times 1.88 \times 10^3} = 153.12°C = 426.27K$$

Step 3: Calculation of required surface area S

$$S = \frac{\phi}{K \times \Delta T_{LMTD}} = \frac{\phi}{K \times \dfrac{(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})}{Log\left[\dfrac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]}}$$

$$= \frac{7315}{340 \times \left[\dfrac{(153.12 - 44) - (205 - 16)}{Log\left[\dfrac{(153.12 - 44)}{(205 - 16)}\right]}\right]} = 0.148\, m^2$$

Step 4: Calculation of NTU

$$\begin{cases} C_c = \dot{m}_c \times C_{pc}\ = \dfrac{225}{3600} \times 4.18 \times 10^3\ = 261.25W/K \\ C_h = \dot{m}_h \times\ C_{ph} = \dfrac{270}{3600} \times 1.88 \times 10^3\ = 141W/K \end{cases}$$

Thus:

$$C_{min} = C_h$$

$$C_{max} = C_c$$

$$NTU = \frac{k \times S}{C_{min}} = \frac{340 \times 0.148}{141} = 0.36$$

Step 5: Calculation of Effectiveness

$$Z = \frac{C_{min}}{C_{max}} = \frac{141}{261.25} = 0.54$$

$$\varepsilon\ = \frac{1 - exp[-NTU \times (1 + Z)]}{1 + Z} = \frac{1 - exp[-0.36 \times (1 + 0.54)]}{1 + 0.54} = 0.275 = 27.5\%$$

2.6. Calculating a Heat Exchange

Once the overall heat transfer coefficient k has been determined (including fouling resistances if required), the engineer must calculate either the required surface area S (for a sizing problem) or the outlet temperatures (for a performance problem). The choice between the LMTD and NTU methods depends on which data are known. Two distinct cases are considered below.

2.6.1. 1st Case: Known Output Temperatures

When the inlet and outlet temperatures of both fluids are known, the heat transfer rate ϕ can be calculated directly from an energy balance on either fluid. The LMTD method is then straightforward and does not require iteration.

- *LMTD Method*
  - Calculate heat transfer rate:

    $$\phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout}) = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}).$$

  - Calculate ΔT_{LMTD} using the appropriate formula (parallel or counter-current).
  - Determine the required surface area: $S = \frac{\phi}{k\ .\Delta T_{LMTD}}$.
- *NTU Method*
  - Compute C_h, C_c, then C_{min}, C_{max}, and $Z = \frac{C_{min}}{max}$.
  - Calculate ε.
  - Calculate NTU_{max} depending on the case:

$$NTU_{max} = -\frac{Log[1-(1+Z)\times\varepsilon]}{1+Z} \text{ (co-current flow)}$$

$$NTU_{max} = \frac{1}{Z-1}\times\ Log\left[\frac{(\varepsilon-1)}{(Z\times\varepsilon-1)}\right] \text{ (Counter-current flow).}$$

- Compute: $S = NTU_{max} \times \left(\frac{C_{min}}{k}\right)$

Both methods yield the same surface area; the choice is a matter of convenience.

2.6.2. 2nd Case: Unknown Output Temperatures

When only the inlet temperatures are known, and the exchanger geometry (or a tentative surface area) is given, the outlet temperatures are unknown. In this situation, the LMTD method would require an iterative trial-and-error procedure because ΔT_{LMTD} depends on the unknown outlet temperatures. The NTU method provides a direct, non-iterative solution and is therefore preferred.

- *LMTD Method*

The application of the LMTD method requires the numerical solution of the equations:

$$\begin{cases} \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout}) = k \times S \times \Delta T_{LMTD} \\ \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout}) = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) \end{cases}$$

- *NTU Method*
  - Compute C_h, C_c, then C_{min}, C_{max}, and $Z = \frac{C_{min}}{max}$.
  - Calculate $NTU_{max} = \frac{k\times S}{C_{min}}$
  - Calculate ε.
  - Compute ϕ.
  - Determine one of the unknown output temperatures, T_{hout} or T_{cout} from the definition of ε.
  - Obtain the second unknown output temperature from the energy balance.

The NTU method eliminates iteration and is the standard approach for performance (rating) problems.

2.7. Conclusion

This chapter presented the fundamental thermal calculation methods for single-phase heat exchangers under steady-state conditions.

The LMTD method provides a direct way to determine the required heat transfer surface area when all four inlet and outlet temperatures are known, while the NTU method is better suited for performance problems where only the inlet temperatures are specified. The global heat transfer coefficient, incorporating convective, conductive, and fouling resistances, was introduced along with typical values for common fluids and flow configurations.

In the next chapter, these calculation methods are extended to more complex geometries such as shell-and-tube and crossflow exchangers, where correction factors are applied. The next chapter also addresses the iterative nature of thermal design, the calculation of pressure drops, and the optimization of heat exchangers to achieve the best compromise between heat transfer performance, pumping power, and total cost.

CHAPTER III
DESIGNING AND OPTIMIZING HEAT EXCHANGERS

3. DESIGNING AND OPTIMISING HEAT EXCHANGERS

3.1. Introduction

In any heat exchanger design, the primary objective is not merely to achieve a required heat transfer rate, but to do so under the most favorable economic conditions. This means that the engineer must find an optimal compromise between two competing cost categories: investment costs and operating costs. Investment costs include the purchase price of the exchanger, the cost of materials, manufacturing, installation, and associated auxiliary equipment. Operating costs, on the other hand, primarily consist of the energy required to pump or circulate the fluids through the exchanger as well as maintenance and cleaning expenses over the equipment's lifetime.

A heat exchanger that is oversized or made of expensive materials will have high investment costs but may offer low pressure drops and reduced pumping energy, leading to lower operating costs. Conversely, an undersized exchanger may be cheap to purchase but will cause high fluid velocities, large pressure losses, and consequently high pumping costs, and may also fail to meet the required thermal duty. The art of heat exchanger design, therefore, lies in selecting the geometry, flow arrangement, materials, and size that minimize the total cost (investment plus operating costs over a specified period, often expressed as annualized cost or net present value) while satisfying all thermal, hydraulic, mechanical, and safety constraints.

In addition to cost considerations, other factors such as available space, weight limitations, maintenance accessibility, fouling tendencies, and environmental regulations may influence the final design. Consequently, the design process is inherently iterative: the designer proposes a preliminary configuration, calculates its thermal and hydraulic performance, estimates costs, and then modifies the design until an acceptable or optimal solution is found. This chapter provides an overview of the main steps involved in this design and optimization process, from initial assumptions to the final selection of a heat exchanger for a given industrial application.

3.2. Assumptions

The thermal calculation methods presented in this course (i.e., LMTD and NTU) rely on a set of simplifying assumptions. These assumptions are widely accepted in standard heat exchanger design practice. The user should be aware of these idealizations to apply the methods correctly and to recognize when more advanced analysis is required. Specifically, the main assumptions are the following ones:

- *Steady State*: all variables are constant over time.
- *Constant fluid Properties*: Density (ρ), Viscosity (μ), Thermal Conductivity (λ), and Specific Heat (C_p) do not vary.
- *Temperature Variation:* Temperatures are variable in the exchanger.
- *Negligible Pressure Variation*: Pressure changes are small and do not significantly affect fluid properties.
- *Single-Phase Fluids*: No phase change occurs inside the exchanger.
- Heat is transferred by convection and conduction only.
- *Adiabatic Exchanger*: No heat exchange with the external environment.

3.3. Designing

The thermal sizing, or design, of a heat exchanger in an industrial context starts with the selection of the most appropriate type of exchanger for the given application. This choice depends on multiple factors, including the required heat duty, fluid properties, temperature and pressure levels, fouling tendencies, available space, and economic constraints. Once the type of exchanger is selected, the main thermal sizing step consists of determining the required heat transfer surface area that will allow the desired heat transfer rate to be achieved under the specified operating conditions.

Two broad categories of methods are available for calculating and sizing heat exchangers, which are the following ones:

- *Analytical Methods*: this category includes the LMTD method and the NTU method, which both are derived from fundamental energy balances and are widely used for preliminary design.
- *Numerical methods*: this category includes *finite volume*, *finite element*, and *finite difference techniques*. Numerical methods are required when the simplifying assumptions of analytical methods are no longer valid. They are also essential for detailed optimization and for simulating local temperature and flow fields.

In practice, most industrial heat exchanger designs are initially carried out using analytical methods, followed by verification and refinement using numerical simulations or specialized software.

3.3.1. Principles of Calculation

In heat exchanger thermal analysis, two fundamental types of calculation problems are encountered, depending on what information is known and what must be determined. Both problems can be solved using either the LMTD method or the NTU method, but one approach is often more convenient than the other depending on the specific situation.

- *Type 1 – Sizing problem* (*determination of exchange surface area S*): In this case, the required heat transfer power is known, and the inlet and outlet temperatures of both fluids are specified. The objective is to calculate the necessary heat transfer surface area (S) that will achieve this duty under the given flow rates and fluid properties.

- *Type 2 – Performance problem* (*determination of outlet temperatures*): In this case, the heat exchanger already exists or has a fixed surface area (S), and the inlet temperatures of both fluids are known. The objective is to determine the outlet temperatures of the two fluids (and consequently the actual heat transfer power) under the given flow conditions.

3.3.2. LMTD Method

The LMTD method is the traditional approach for sizing heat exchangers when the inlet and outlet temperatures of both fluid streams are known. The fundamental relationship governing heat transfer in a heat exchanger is given by:

$$\phi = k \times S \times \Delta T_{LMTD} \tag{3.1}$$

Where k is the overall heat transfer coefficient, S is the heat transfer surface area, and ΔT_{LMTD} is the logarithmic mean temperature difference defined as:

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{Ln\left[\frac{\Delta T_2}{\Delta T_1}\right]} \tag{3.2}$$

Here, ΔT_1 and ΔT_2 represent the temperature differences between the hot and cold fluids at the two ends of the exchanger.

From Equation (3.1), the required heat transfer surface area, S, can be directly calculated as:

$$S = \frac{\phi}{k \times \Delta T_{LMTD}} \tag{3.3}$$

However, for more complex flow configurations, the simple LMTD calculated for a pure counter-current flow must be corrected. In practice, the following form is used:

$$S = \frac{\phi}{k \times F \times \Delta T_{LMTD}} \tag{3.4}$$

This correction factor F is 1 in the case of a counter-current exchanger.

The typical calculation procedure using the LMTD method consists of several steps, as shown in Figure 3.1.

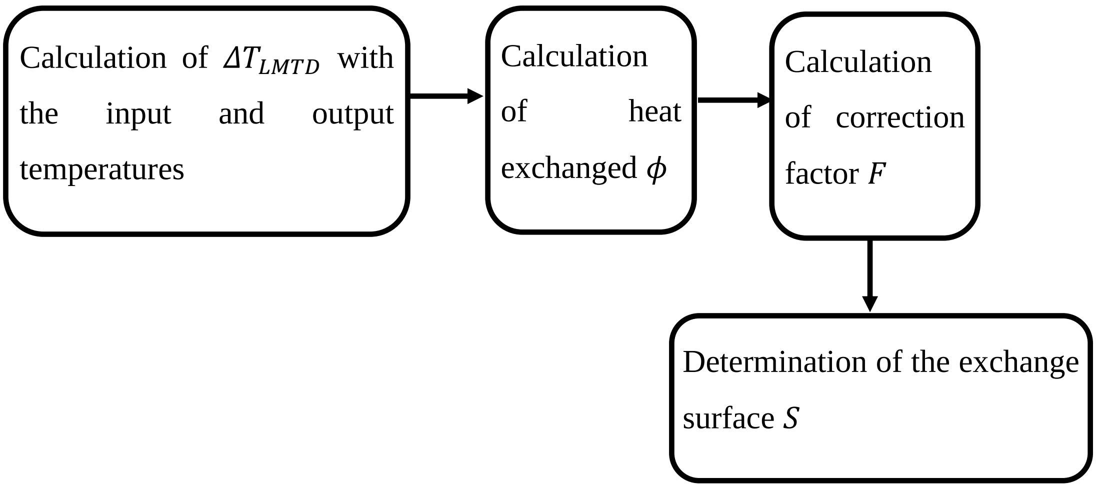


Figure 3.1: Calculation Procedure Using LMTD Method

3.3.3. NTU Method

The NTU method, also known as the effectiveness-NTU method, provides an elegant and efficient alternative to the LMTD method, particularly when the outlet temperatures of the fluids are unknown. This situation commonly arises in performance problems, where the heat exchanger already exists (or its size is fixed), and the goal is to determine how much heat will be transferred and what the outlet temperatures will be for given inlet conditions.

Specifically, the NTU method is particularly well-suited for two main types of problems:

- *Design Problems*: The required heat transfer rate and the inlet temperatures are known, and one of the outlet temperatures is imposed. From this, the effectiveness can be directly calculated, and then the required NTU is obtained using the appropriate NTU relationship for the chosen flow configuration.

- *Performance Problems*: The heat exchanger geometry is known, as well as the inlet temperatures and flow rates. The NTU and heat capacity are calculated from the data, and then the effectiveness is obtained from the appropriate correlation.

Figure 3.2 shows the calculation steps via the NTU method.

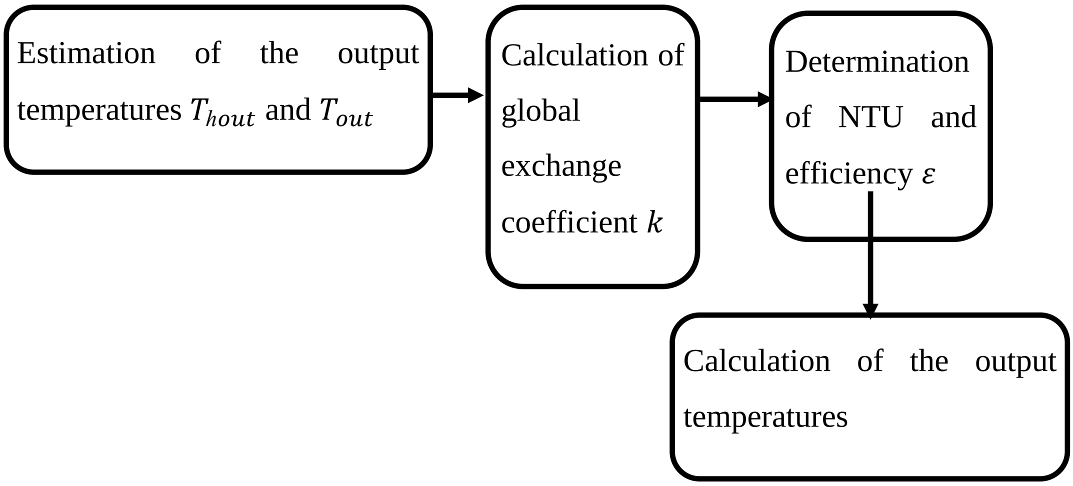


Figure 3.2: Calculation Procedure Using NTU Method

3.3.4. Calculation Steps

The practical design or rating of a heat exchanger typically follows a systematic sequence of steps. These steps combine thermal, hydraulic, and geometric calculations and often require iterations. The general procedure is outlined below.

- *Step 1 – Determine Unknown Temperatures*: using the energy balance equation, we solve for any unknown inlet or outlet temperature if sufficient data are available.

$$\phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout}) = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) \qquad (3.5)$$

- *Step 2 – Calculate the Logarithmic Mean Temperature Difference*: computing ΔT_{LMTD} based on the flow arrangement.

- *Step 3 – Estimate NTU Number of Transfer Units*: depending on the problem type, determine the required NTU and exchanger effectiveness using the appropriate correlations.

- *Step 4 – Choose Characteristic Flow Velocity*: Based on the exchanger geometry, select an initial flow velocity.

- *Step 5 – Calculate Heat Transfer Coefficients*: Using empirical correlations, compute the individual heat transfer coefficients on each side of the fluids.

- *Step 6 – Evaluate Overall Heat Transfer Coefficient k*: First, compute the clean exchange coefficient. Then, repeat the calculation, including the appropriate fouling resistances, to obtain the fouled exchange coefficient, which is used for realistic design.

- *Step 7 – Calculate Heat Transfer Surface Area S*: Use either the LMTD method or the NTU method, depending on which is more convenient.

- *Step 8 – Compare Calculated S with Initially Assumed Geometry*: If the calculated S differs significantly from the surface area corresponding to the chosen geometry, adjust the geometric parameters and return to *Step 4*.

- *Step 9 – Calculate Pressure Drops*: Using appropriate correlations, compute the pressure losses on each fluid circuit.

- *Step 10 – Compare Pressure Drops with Permissible Limits*: If the calculated pressure drops exceed the maximum allowable values, modify the design and repeat the calculations from Step 4.

3.4. Optimizing Heat Exchangers

Heat exchangers are employed in a vast range of industrial applications, which has led to an enormous variety of configurations. These configurations differ not only in geometry and flow arrangement but also in the materials of construction. The choice of material, whether metal (e.g., Steel, Copper, Aluminum, Titanium, superalloys), glass, plastic, ceramic, or graphite, is dictated by multiple constraints, including:

- *Thermomechanical Constraints*: such as operating temperature and pressure levels, as well as the aggressive nature of the fluids (chemical attack, corrosion). For example, ammonia service typically requires steel pipes.

- *Fluid Properties and Flow Characteristics*: including whether the fluid is a liquid, gas, two-phase mixture (liquid-gas), or contains solid particles (liquid-particles, gas-particles, or powder flows).

Given this wide variety of possible solutions, the design of a heat exchanger for a specific situation naturally leads to an optimization problem. Optimization can be considered at two successive levels:

- *Static Optimization*: this optimization concerns the optimal sizing of the heat exchanger for a given steady operating condition (design optimization), or the optimal operation of an existing exchanger under varying steady loads. This is sometimes called design or parametric optimization.

- *Dynamic Optimization*: this approach addresses the transient behavior of the heat exchanger over time. As operating conditions change, the goal is to optimize performance dynamically, for example, by controlling bypass flows or cleaning schedules.

These two types of optimization are complementary and are discussed separately below.

3.4.1. Static Optimization of Heat Exchangers

Static optimization, viewed from the perspective of the first law of thermodynamics, typically pursues one or more of three practical objectives:

- Increase thermal performance (higher heat transfer rate).
- Reduce associated costs.
- Reduce overall dimensions.

It is important to note that these three objectives often conflict with each other and are subject to different constraints. For instance, increasing performance by adding fins or increasing flow velocity raises pressure drop and thus operating cost. Reducing dimensions may require higher velocities, again increasing pressure drop. Therefore, the optimization problem consists of finding the best compromise.

A common observation is that the search for maximum thermodynamic efficiency generally coincides with the minimization of operating costs (mainly pumping power), but this may come at the expense of higher initial investment.

Among the global criteria used to evaluate exchanger performance in static optimization, the following are frequently considered:

- Minimization of heat transfer surface area S, which directly reduces manufacturing cost.
- Minimization of mechanical costs.
- Maximization of effectiveness for a given NTU.

These objective functions are always associated with constraints, such as maximum allowable pressure drop, minimum acceptable outlet temperature, geometric limits (maximum tube length or shell diameter), and material strength requirements.

3.4.2. Dynamic Optimization of Heat Exchangers

Dynamic optimization deals with the behavior of heat exchangers during transient (time-varying) conditions. Transients can be classified by their duration, leading to very different system responses.

- *Long Transients* – These occur over extended periods, typically hours or days. A typical example is the progressive fouling or clogging of the heat exchanger surfaces, which gradually degrades performance. The time constant for such transients is of the order of a day or more.
- *Short Transients* – These occur over very brief periods, from fractions of a second to a few minutes. An example is the sudden opening or closing of an on-off fluid supply valve. The time constant depends on whether mechanical (pressure wave propagation) or thermal (temperature change diffusion) aspects dominate, ranging from seconds to minutes.

Moreover, dynamic optimization can focus on different output variables:

- Temperature T – The most common approach in the literature is to study how the outlet temperatures respond to changes in inlet conditions. This is critical for control system design.
- Heat transfer rate ϕ – Less common but important when the total energy exchanged over a transient period matters, for example, in batch processes or energy storage systems.

In practice, dynamic optimization of heat exchangers is essential for designing effective control strategies, predicting off-design performance, and scheduling maintenance (e.g., cleaning cycles) to maximize long-term efficiency.

3.5. Applications

The following two applications illustrate the use of the LMTD method for sizing heat exchangers in co-current and counter-current configurations.

3.5.1. Application 1: Tubular Heat Exchanger

A tubular heat exchanger is used to heat a fluid flow of $1000\ kg/h$ from $13°C$ to $22°C$ using hot water at $60°C$ flowing at $5900\ kg/h$. The thermophysical properties and the overall heat transfer coefficient are given below. The required heat transfer surface area is to be determined for both co-current and counter-current flow arrangements.

Let:

$$Cp_{fluid} = 0{,}9\ kcal/kg\ °C \qquad \rho_{fluid} = 1071\ \ kg/m^3$$

$$Cp_{water} = 1\ kcal/kg\ °C \qquad \rho_{water} = 1000\ \ kg/m^3$$

$$K = 6300\ \ kcal/h\ m^2°C$$

Where: K is the Overall heat transfer coefficient.

Step 1: Calculate the heat transfer rate ϕ: using the cold side energy balance (see Equation (3.5)), we have that:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$$

i.e.:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = \left(\frac{1000}{3600}\right) \times 0.9 \times 10^3 \times 4.180 \times (22 - 13) = 9405\ W$$

Step 2: Determine the hot fluid outlet temperatures T_{hout}, using the hot side energy balance (see Equation (3.5)), we have that:

$$\Phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout})$$

As a result:

$$T_{hout} = T_{hin} - \left(\frac{\Phi}{\dot{m}_h \, . C_{ph}}\right) = 60 - \frac{9405}{\left(\frac{5900}{3600}\right) \times 4180} = 58.6\ °C$$

Step 3: Calculate the required surface area S, The general formula (see Equation (3.3)) is $S = \frac{\phi}{K \times \Delta T_{LMTD}}$, with ΔT_{LMTD} depending on the flow arrangement.

- *Co-Current Flow*:

$$S = \frac{\Phi}{K} \times \left[\frac{ln\left[\frac{(T_{hout} - T_{cout})}{(T_{hin} - T_{cin})}\right]}{[(T_{hout} - T_{cout}) - (T_{hin} - T_{cin})]}\right] = \frac{9405}{6300 \times 10^3 \times \left(\frac{4.18}{3600}\right)} \times \left[\frac{ln\left[\frac{(58.6-22)}{(60-13)}\right]}{[(58.6-22)-(60-13)]}\right] = 0.031\ m^2$$

- *Counter-Current Flow*:

$$S = \Phi \times \left[\frac{ln\frac{(T_{hout} - T_{cin})}{(T_{hin} - T_{cout})}}{K \times (T_{hout} - T_{cin}) - (T_{hin} - T_{cout})}\right] = \frac{9405}{6300 \times 10^3 \times \left(\frac{4.18}{3600}\right)} \times \left[\frac{ln\left[\frac{(58.6-13)}{(60-22)}\right]}{[(58.6-13)-(60-22)]}\right] = 0.03\ m^2$$

Both arrangements yield very similar surface areas in this case because the temperature differences are nearly balanced. In practice, counter-current flow generally gives a smaller required area.

3.5.2. Application 2: Tube Heater – Determination of Tube Length

Given an Engine oil with $C_{pc} = 2100 J/kg°C$, it must be heated from $20°C$ to $60°C$ inside a thin-walled tube of diameter $d = 20cm$. The oil flow rate is $\dot{m}_c = 0{,}3\ kg/s$ with specific heat of $C_{pc} = 2100 J/kg°C$. Heating is provided by a hot fluid (outside the tube) that enters at $T_{hin} = 130°C$ and comes out at $T_{hout} = 55°C$. Moreover, the overall heat transfer coefficient is $K = 650\ W/m^2 . °C$. The tube length L is to be determined

Step 1: Calculate the heat transfer rate ϕ: using the cold side energy balance (see Equation (3.5)).

We have that:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$$

i.e.:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = 0.3 \times 2100 \times (60 - 20) = 25.2\ kW$$

Step 2: Determine the required surface area S .

We have that:

$$S = \frac{\Phi}{K \times \Delta T_{LMTD}}$$

i.e.,

$$S = \frac{\Phi}{K \times \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{Ln\left[\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right]}\right]} = \frac{25200}{650 \times \left[\frac{(130 - 60) - (55 - 20)}{Ln\left(\frac{130 - 60}{55 - 20}\right)}\right]} = 0.768\ m^2$$

Step 3: Calculate the tube length L - For a cylindrical tube, the heat transfer surface area can be formulated as is $S = \pi \times D \times L$ as a result, we have:

$$L = \frac{S}{\pi \times d} = \frac{0.77}{\pi \times 0.2} = 1.22\ m$$

Thus, a tube length of approximately $1.22\ m$ is required.

3.6. Example of Optimization of a Multi-Tube Exchanger

To illustrate the principles of heat exchanger optimization, we consider the following design problem.

A heat exchanger must be designed to cool a flow of benzene from $75°C$ to $50°C$. The benzene flow rate is $1\ kg/s$. Cooling is provided by a stream of water available at $10°C$. The objective is to select the design variables that minimize the total cost C, defined as the sum of the following costs: (*i*) *Investment Cost C_i*. (*ii*) *Operating Cost C_o*. (*iii*) *Pumping Cost C_p*. The optimization problem can be written as:

$$min\ C = C_i + C_o + C_p$$

This design must satisfy the following constraints:

- *Thermal Constraint*: The heat transfer rate Φ required to cool the benzene must equal the heat transferred through the wall:

$$\Phi = \dot{m}_c \times C_{pc}\ (T_{cout} - T_{cin}) = \dot{m}_h \times C_{ph}(T_{hin} - T_{hout}) = K \times S \times F \times \Delta T_{LMTD}$$

- Overall Transfer Coefficient: K is a function of the convection coefficients h_h, h_c, wall conduction, and fouling resistances, formulated as:

$$k = \frac{1}{\left(\frac{1}{h_h} + R_{eh}\right) + \left(\frac{e}{\lambda}\right) \times \left(\frac{S_h}{S_m}\right) + \left(\frac{1}{h_c} + R_{ec}\right) \times \left(\frac{S_h}{S_c}\right)}$$

- *Convection Correlations*: h_h, h_c are obtained from *Nusselt number correlations*:

$$N_u = \frac{h \times D_{eq}}{\lambda} = 0.02 \times Re_{eq}^{0.2} \times Pr^{0.33}$$

- *Hydraulic Pressure Drop*: The pressure drop on each side is given by:

$$\Delta P = \frac{1}{2} \times f \times \left(\frac{L}{r_H}\right) \times \rho \times V^2$$

- *Mechanical Resistance*: The tube wall thickness e must be sufficient to withstand the pressure difference:

$$e \geq \left(\frac{d_i}{2}\right) \times \left(\sqrt{\frac{R_F + \Delta P_h}{R_F + \Delta P_c}} - 1\right)$$

- *Non-Negativity and Upper Bounds*: All variables are non-negative, and T_{cout} must not exceed the hot fluid inlet temperature ($T_{cout} \leq 75°C$).

Moreover, all costs are expressed as follows:

- *Investment Cost*:

$$C_i = \left[\left(\left(\frac{\pi}{4}\right) \times (d_i + 2e)^2 - \left(\frac{\pi}{4}\right) \times {d_i}^2\right) + \left(\left(\frac{\pi}{4}\right) \times (D_i + 2e)^2 - \left(\frac{\pi}{4}\right) \times {D_i}^2\right)\right] \times L \times \rho_m \times C_M$$

- *Pumping Cost*:

$$C_p = C_{ele} \times \left[\left(\frac{\dot{m}_c}{\rho_c}\right) \Delta P_c + \left(\frac{\dot{m}_h}{\rho_h}\right) \Delta P_h\right]$$

- *Operating Cost*:

$$C_o = \dot{m}_o \times C_e$$

where C_e is the price per $1kg$ of cooling water; C_{ele} is the price per $1KW/h$; C_M is the cost per hour for $1kg$ investment.

Using a suitable numerical optimization algorithm, Table 3.1 shows the obtained optimal design.

Table 3.1: Optimal Design of a Heat Exchanger

Variable	Value	Unit
d_i	0.0296	m
e	0.001	m
D_i	0.0358	m
L	15.4261	m
$\dot{m}_c$	0.2143	kg/s
T_{cout}	58	$°C$

As a result, the minimum total cost is $C = 771.54$. This result represents the best compromise between capital investment (larger exchanger) and operating expenses (water consumption and pumping power). The optimal design uses a relatively low cooling water flow rate ($0.214\ kg/$s instead of a higher value) and a long tube length ($15.4\ m$) to achieve the required cooling with acceptable pressure drops.

3.7. Conclusion

This chapter moved from the basic thermal calculation methods to the practical aspects of designing and optimising heat exchangers. It introduced the iterative nature of thermal sizing, where the engineer must select an appropriate exchanger type, estimate flow velocities, calculate heat transfer coefficients and pressure drops, and compare the required surface area with the assumed geometry.

The LMTD and NTU methods were revisited in the context of design, with emphasis on the correction factor FF for complex flow configurations and the importance of accounting for fouling resistances. The chapter also outlined the systematic calculation steps that combine thermal and hydraulic analyses to arrive at a feasible design.

The second part of the chapter addressed the optimisation of heat exchangers, distinguishing between static optimisation (choosing the best design for steady operation) and dynamic optimisation (managing transient behaviour over time). A detailed case study on the optimisation of a multi-tube exchanger illustrated how investment, operating, and pumping costs are balanced against thermal and mechanical constraints. The optimal solution demonstrated the trade-off between heat transfer performance and pressure drop. The next chapter provides a comprehensive set of resolved exercises that apply all the concepts covered in previous Chapters, allowing students to practice sizing and optimisation problems on realistic industrial cases.

CHAPTER IV
APPLICATIONS AND EXERCISES

4. APPLICATION AND EXERCISES

This chapter presents a comprehensive set of exercises and their detailed solutions, covering all the main concepts introduced in previous Chapters. These exercises are designed to help students develop confidence in applying the LMTD and NTU methods, handling different flow configurations (co-current, counter-current, crossflow, shell-and-tube with multiple passes), and interpreting results from realistic industrial data. Problems range from basic sizing of double-pipe exchangers to more advanced topics such as optimisation, fouling, phase change, and comparison of multiple exchanger configurations. By working through these exercises, students will be well prepared to tackle complex thermal design challenges in their subsequent studies or professional careers.

4.1. Exercises

4.1.1. Exercise 1

A heat sink is placed against a computer microprocessor to cool it. The microprocessor generates a thermal power of $200\ W$. The cooling air is supplied by a fan, with the following operating conditions:

- overall heat transfer coefficient: $K = 40\ W/m^2.K$
- Airflow rate: $\dot{V} = 50\ m^3/h$
- Air properties: $C_p = 1006\ K/kg.K$ and $\rho = 1.29\ kg/m^3$
- Air inlet temperature: $T_{cin} = 21°C$

Questions:

1. Calculate the outlet temperature of the air.
2. For heat exchanger effectiveness values of 30%, 40%, and 50%, calculate the corresponding microprocessor temperature.
3. Using the NTU method, determine the required heat-exchange surface area.

After determining the surface area, consider two extreme ambient conditions at the computer location: (*i*) Winter: $T_{cin} = -2°C$; (*ii*) Summer: $T_{cin} = 39°C$. For these two cases:

4. Calculate the outlet temperature of the air.
5. Calculate the temperature of the microprocessor.

4.1.2. Exercise 2

A counter-current heat exchanger with a parallel tube assembly and two tube passes is used to heat a water flow. The following data are provided:

- Cold water: $\dot{m}_c = 500\ kg/h$, $T_{cin} = 20°C$ and $T_{cout} = 50°C$.
- Hot water: $\dot{m}_h = 200\ kg/h$, $T_{hin} = 200°C$ and $r = 10mm$.
- Reynolds number: $R_e = 5000$.
- Global heat transfer coefficient: $K = 400\ W/m^2.K$.

Thermophysical properties of hot water at 200°C:

- Density: $\rho = 864.58\ kg/m^3$.
- Dynamic Viscosity: $\mu = 0.134 \times 10^{-3} kg/m.s$
- Specific Heat: $C_p = 4401\ J/kg.K$.

Questions:

1. Calculate the power exchanged and the output temperature of the cold fluid.
2. Determine the exchange surface.
3. Determine tube velocity.
4. Calculate the number of tubes and the total Beam length.

4.1.3. Exercise 3

A counter-current shell-and-tube heat exchanger is used to heat a water flow. The following data are provided:

- Cold water: $\dot{m}_c = 4\ kg/s$, $T_{cin} = 30°C$ and $T_{cout} = 50°C$.
- Hot water: $\dot{m}_h = 2\ kg/s$, and $T_{hin} = 100°C$.

Questions: Design the heat exchanger under the given constraints.

4.1.4. Exercise 4

Let’s consider a shell-and-tube heat exchanger used to heat water. The following data are provided:

- Cold water: $\dot{m}_c = 3.783\ kg/s$, $T_{cin} = 37.78°C$ and $T_{cout} = 54.44°C$.
- Hot water: $\dot{m}_h = 1.892\ kg/s$, and $T_{hin} = 93.33°C$
- Global heat transfer coefficient: $K = 1{,}419\ W/m^2.K$.
- Tube diameter $d = 1.9\ cm$, and velocity $v = 0.366\ m/s$.
- Maximum tube length: $L_{max} = 2.5m$.

Questions:

1. Calculate the number of tube passes.
2. Calculate the number of tubes per pass.
3. Calculate tube length.

4.1.5. Exercise 5

In a district heating substation, a shell-and-tube heat exchanger is to be installed to heat a water flow. The following data are provided:

- Cold water: $\dot{m}_c = 20{,}000\ kg/h$, $T_{cin} = 40°C$ and $T_{cout} = 60°C$.
- Hot water: $\dot{m}_h = 10{,}000\ kg/h$, and $T_{hin} = 180°C$
- Global heat transfer coefficient: $K = 450\ W/m^2.K$.
- Tube diameter $d = 20\ mm$, and velocity $v = 0.366\ m/s$.
- Reynolds number: $R_e = 10{,}000$.

Thermophysical properties of hot water at 200°C:

- Density: $\rho = 920\ kg/m^3$.
- Dynamic Viscosity: $\mu = 19 \times 10^{-5} kg/m.s$
- Specific Heat: $C_p = 4315\ J/kg.K$.

Questions:

1. Calculate the exchanged heat power ϕ and the outlet temperature of the hot fluid.
2. The exchanger operates in counter-current, with a single pass for each fluid, the tubes being mounted in parallel. Determine:

- ✓ The required heat exchange surface S.
- ✓ The velocity inside the tubes.
- ✓ The total cross-sectional area of the tubes.
- ✓ The number of tubes and the length of the tube bundle.

4.1.6. Exercise 6

A condenser is used to condense steam at a constant temperature of $T_{hin} = 30°C$ (since condensation occurs isothermally). Cooling water flows through the tubes, entering at $T_{cin} = 14°C$ and leaving at $T_{cout} = 22°C$.

The heat exchanger has a heat transfer surface area $S = 45\ m^2$, and the overall heat transfer coefficient is $K = 2100\ W/m^2.°C$. Moreover, the latent heat of vaporization of water at $30°C$ is $h_{fg} = 2431\ KJ/kg$, and the specific heat of cooling water is $c_p = 4184\ J/kg.°C$.

Questions:

1. Determine the required mass flow rate of the cooling water.
2. Determine the rate of steam condensation in the condenser.

4.1.7. Exercise 7

A perfectly insulated coaxial heat exchanger is used to heat a fluid. The fluid ($c_p = 3.8\ kJ/kg.°C$ and $\rho = 950\ kg/m^3$) circulates inside the inner tube, while saturated steam at $T_{SS} = 130°C$ circulates in the annular jacket and condenses on the outer surface of the tube. The steam remains saturated at the exit (no subcooling).

Test 1:

The fluid enters the exchanger at the temperature $T_{cin} = 20°C$ and at a volume flow rate $Q_{V1} = 0.7\ m^3/h$. Its outlet temperature is $T_{cout1} = 100°C$.

Test 2:

The flow rate of the fluid is increased to a value $Q_{V2} = 1.2\ m^3/h$, corresponding to an overall conductance K_2. The outlet temperature of the fluid becomes, under these conditions $T_{cout2} = 95°C$, the inlet temperature remaining fixed at $T_{cin} = 20°C$.

Assumptions:

Since the flow regimes in the tubes are assumed to be turbulent, it can be assumed, a priori, a dependence of the internal fluid conductance/wall h_i, of the type: $h_i = K_s \times {Q_v}^{0,8}$, where K_s is a constant to be determined.

Also, the external conductance in condensation will be assumed to remain constant and equal to $h_e = 3000\ W/m^2°C$. In addition, the conductive resistance of the tubes can be neglected.

Questions:

1. Calculate the powers Q_1 and Q_2 exchanged during each test.
2. Calculate the logarithmic mean temperatures for the two tests.
3. Deduce the K_1/K_2 ratio.
4. Determine the value of the constant K_s.
5. Calculate the overall heat transfer coefficient K_1 corresponding to the flow rate Q_{V1}.
6. Deduct the total exchange area S required.
7. Repeat the problem using the NUT method.

4.1.8. Exercise 8

A counter-current tubular heat exchanger is used to heat water by cooling oil. The following data are provided:

- Water: $\dot{m}_c = 1.25\ kg/s$, $T_{cin} = 35°C$ and $T_{cout} = 90°C$.
- Oil: $C_{ph} = 2\ kJ/kg.°C$, $T_{hin} = 150°C$ and $T_{hout} = 85°C$.
- Global heat transfer coefficient: $K = 850\ W/m^2.°C$.

The objective is to compare the performance and economics of the single large heat exchanger with an alternative configuration consisting of two smaller counter-current heat exchangers (see Figure 4.1), where the water circuits are arranged in series, and the oil circuits are arranged in parallel. If the overall heat transfer coefficient K remains the same for the small exchangers. The cost per square meter of a small heat exchanger is 20% higher than that of the large exchanger.

Questions: Which solution is more economical: one large exchanger or two small ones?

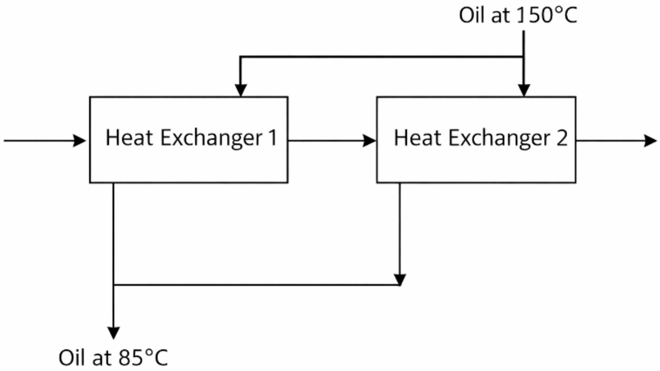


Figure 4.1: Configuration of Two Small Counter-Current Heat Exchangers

4.1.9. Exercise 9

A hot solution must be cooled from $66°C$ to $39°C$ using cooling water. The following data are provided:

- Water: $\dot{m}_c = 29500\ kg/h$ and $T_{cin} = 12°C$.
- Hot solution: $\dot{m}_h = 30000\ kg/h$
- Specific heat of the solution: $C_{psol} = 0.9\ kcal/kg.°C$
- Global heat transfer coefficient: $K = 2100\ kcal/m^2.°C$.

Questions: Determine the required heat transfer surface area S for each of the following exchanger configurations:

1. Single-pass tubular heat exchanger in co-current flow.
2. Single-pass tubular heat exchanger in counter-current flow.
3. 1–2 shell-and-tube heat exchanger.
4. 2–4 shell-and-tube heat exchanger.
5. Crossflow heat exchanger with two unmixed fluids.

4.1.10. Exercise 10

We consider a shell-and-tube heat exchanger designed to heat a water flow. The following data are provided:

- Cold water: $\dot{m}_c = 7000\ kg/h$, $T_{cin} = 40°C$ and $T_{cout} = 60°C$.

- Hot fluid: $\dot{m}_h = 400\ kg/h$, and $T_{hin} = 180°C$
- Global heat transfer coefficient: $K = 2440\ W/m^2.K$.
- Tube diameter $d = 0.01\ m$.

Thermophysical properties of hot water:

- Density: $\rho = 1000\ kg/m^3$.
- Specific Heat: $C_p = 4180\ J/kg.K$.

Questions:

1. Calculate the heat transfer rate ϕ and the outlet temperature of the hot fluid.
2. The exchanger operates in counter-current, with a single pass for each fluid, the tubes being mounted in parallel. Determine:
   - ✓ The required heat exchange surface S.
   - ✓ The velocity inside the tubes.
   - ✓ The total cross-sectional area of the tubes.
   - ✓ The number of tubes and the length of the tube bundle.

4.2. Solutions

4.2.1. Solution 1

1. Calculation of the outlet temperature of the air:

$$\dot{m}_c = \frac{\dot{V} \times \rho}{3600} = \frac{50 \times 1.29}{3600} = 0.018\ \ kg/s$$

We have that: $\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$, as a result:

$$T_{cout} = T_{cin} + \left(\frac{\Phi}{\dot{m}_c \times C_{pc}}\right) = 21 + \left(\frac{200}{0.018 \times 1006}\right) = 32.1°C = 305.25\ K$$

2. Calculation of the microprocessor temperature:

The effectiveness ε is given by $\varepsilon\ = \frac{T_{cout} - T_{cin}}{T_{hin} - T_{cin}}$, indeed, we have:

$$T_{hin} = \frac{T_{cout} - T_{cin}}{\varepsilon} + T_{cin}$$

- For $\varepsilon = 0.3$:

$$T_{hin} = \frac{32.1 - 21}{0.3} + 21 = 58\,°C = 331.15\,K$$

- For $\varepsilon = 0.4$:

$$T_{hin} = \frac{32.1 - 21}{0.4} + 21 = 48.75\,°C = 321.89\,K$$

- For $\varepsilon = 0.5$:

$$T_{hin} = \frac{32.1 - 21}{0.5} + 21 = 43.2\,°C = 316.35\,K$$

3. Determination of the required heat-exchange surface area:

For this NTU method, we have that: $\varepsilon = \frac{1 - exp[-NTU(1+Z)]}{1+Z}$ with $Z = \frac{C_{min}}{C_{max}}$. In this case, we have:

$$\varepsilon = 1 - exp[-NTU]$$

i.e.,

$$NTU = -ln(1 - \varepsilon)$$

Also, we have that: $NTU = \frac{k \times S}{C_{min}}$, the previous equation becomes:

$$S = \frac{NTU \times C_{min}}{k} = \frac{-ln(1 - \varepsilon) \times \dot{m}_c \times C_p}{k}$$

- For $\varepsilon = 0.3$:

$$S = \frac{-ln(1 - 0.3) \times 0.18 \times 1006}{40} = 0.16\,m^2$$

- For $\varepsilon = 0.4$:

$$S = \frac{-ln(1 - 0.4) \times 0.18 \times 1006}{40} = 0.23\,m^2$$

- For $\varepsilon = 0.5$:

$$S = \frac{-ln(1 - 0.5) \times 0.18 \times 1006}{40} = 0.32\,m^2$$

4. Calculation of the outlet temperature of the air in winter and summer:

For each configuration, we have that $T_{cout} = T_{cin} + \left(\frac{\Phi}{\dot{m}_c \times C_{pc}}\right)$

- **Winter:** $T_{cin} = -2°C$

$$T_{cout} = -2 + \left(\frac{200}{0.018 \times 1006}\right) = 9.1°C = 282.25\ K$$

- **Summer:** $T_{cin} = 39°C$

$$T_{cout} = 39 + \left(\frac{200}{0.018 \times 1006}\right) = 50.1°C = 323.25\ K$$

5. Calculation of the microprocessor temperature:

Now the microprocessor temperature T_{hin} for each effectiveness and each season, we have that: $T_{hin} = \frac{T_{cout}-T_{cin}}{\varepsilon} + T_{cin}$

- **Winter:** $T_{cin} = -2°C$
  - For $\varepsilon = 0.3$:

$$T_{hin} = \frac{9.1-(-2)}{0.3} - 2 = 35°C = 308.15\ K$$

  - For $\varepsilon = 0.4$:

$$T_{hin} = \frac{9.1-(-2)}{0.4} - 2 = 25.75°C = 298.9\ K$$

  - For $\varepsilon = 0.5$:

$$T_{hin} = \frac{9.1-(-2)}{0.5} - 2 = 20.2°C = 293.35\ K$$

- **Summer:** $T_{cin} = 39°C$
  - For $\varepsilon = 0.3$:

$$T_{hin} = \frac{50.1-39}{0.3} + 39 = 76°C = 349.15\ K$$

  - For $\varepsilon = 0.4$:

$$T_{hin} = \frac{50.1-39}{0.4} + 39 = 66.75°C = 339.9\ K$$

  - For $\varepsilon = 0.5$:

$$T_{hin} = \frac{50.1-39}{0.5} + 39 = 76°C = 334.35\ K$$

4.2.2. Solution 2

1. Calculation of the power exchanged and the output temperature of the cold fluid:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = \left(\frac{500}{3600}\right) \times 4180 \times (50 - 20) = 17416.67\ W$$

2. Determination of the exchange surface:

- Calculating the output temperature of hot fluid: since we have that $\Phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout})$:

$$T_{hout} = T_{hin} - \left(\frac{\Phi}{\dot{m}_h \times C_{ph}}\right) = 200 - \left(\frac{17416.67}{\frac{200}{3600} \times 4401}\right) = 128.77°C$$

- Calculating the exchange surface:

$$S = \frac{\Phi}{K \times \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right]} = \frac{17416.67}{400 \times \left[\frac{(200 - 50) - (128.77 - 20)}{ln\left(\frac{200 - 50}{128.77 - 20}\right)}\right]} = 0.34m^2$$

3. Determination of tube velocity:

We have that $R_e = \frac{\rho \times V \times d}{\mu}$ as a result:

$$V = \frac{\mu \times R_e}{\rho \times d} = \frac{(0.134 \times 10^{-3}) \times 5000}{864 \times 58 \times 0{,}02} = 0.04\ m/s$$

4. Calculating the number of tubes and Beam length:

- The tube section:

$$S_{ec} = \frac{\dot{m}_c}{\rho \times V} = \frac{\left(\frac{200}{3600}\right)}{864.58 \times 0.04} = 1.6 \times 10^{-3}\ m^2$$

- The number of tubes:

$$N = \frac{S_{ec}}{\pi \times r^2} = \frac{1.6 \times 10^{-3}}{\pi \times (0.01)^2} = 5.1 \Rightarrow N = 6$$

- Tube length: we have $S = N \times (2 \times \pi \times r \times L)$

$$L = \frac{S}{2 \times \pi \times r \times N} = \frac{0.34}{2 \times \pi \times 0.01 \times 6} = 0.9\ m$$

- Beam length:

$$L_T = 2 \times 6 \times L = 10.8\ m$$

4.2.3. Solution 3

Let's consider one pass and check whether the conditions are satisfied.

- Flow calculation: we have that $\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$

$$\Phi = 4 \times 4182 \times (50 - 30) = 334560\ W$$

- Calculation of hot water temperature: we have that $\Phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout})$

$$T_{hout} = T_{hin} - \frac{\Phi}{\dot{m}_h \times C_{ph}} = 100 - \frac{334560}{2\ \times 4182} = 60°C = 333.15K$$

- Surface area calculation for a counter-current heat exchanger:

$$S = \frac{\Phi}{K \times \Delta T_{LM}} = \frac{\Phi}{K \times \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right]} = \frac{334560}{1420 \times \left[\frac{(100-50)-(60-30)}{ln\left(\frac{100-50}{60-30}\right)}\right]} = 6.02\ m^2$$

- Calculation of the total flow area: we have that $\dot{m}_c = \rho \times s \times v$, as result

$$s = \frac{\dot{m}_c}{\rho \times v} = 0.01\ m^2$$

- Calculating the number of tubes: we have $s = N_{tube}\left(\frac{\pi \times d^2}{4}\right)$

$$N_{tube} = s \times \left(\frac{4}{\pi \times d^2}\right) = 31.8\ \Rightarrow N_{tube} = 32$$

- Calculation of surface area per tube per meter :

$$\pi \times d = \pi \times 0.02 = 0.063\ m^2/tube\ .m$$

- Calculation of tube length: by giving that $S = N_{tube} \times \pi \times d \times L$, we have:

$$L = \frac{S}{N_{tube} \times \pi \times d} = \frac{6.02}{32 \times 0.063} = 2.9\ m \simeq 3m$$

$$N = 1pass$$

4.2.4. Solution 4

4.2.4.1. Tube Side

- Calculating heat exchanger power:

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = 3.783 \times 4182 \times (54.44 - 37.78\,) = 263600\ W$$
$$= 263.6\ kW$$

$$T_{hout} = T_{hin} - \frac{\Phi}{\dot{m}_h \times C_{ph}} = 93.33 - \frac{263600}{1.892 \times 4182} = 60\ °C$$

- For a counter-current exchanger, calculation of logarithmic mean temperature difference:

$$\Delta T_{LM} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(93.33 - 54.44) - (60 - 37.78)}{ln\left(\frac{93.33 - 54.44}{60 - 37.78}\right)}\right] = 29.78°C$$

- Calculating the exchange surface :

$$S = \frac{\Phi}{K\ \times \Delta T_{LM}} = \frac{263600}{1419 \times 29.78} = 6.24\ m^2$$

- Using the average velocity of the water in the tubes and the flow rate, we calculate the total surface area of the flow:

$$s_f = \frac{\dot{m}_c}{\rho \times v} = \frac{3.783}{1000 \times 0.366} = 0.0104\ m^2$$

- Calculating the number of tubes:

$$N_{tubes} = s_f \times \left(\frac{4}{\pi \times d^2}\right) = 0.0104 \times \left(\frac{4}{\pi \times (0.019)^2}\right) = 36.7 \Rightarrow N_{tube} = 37$$

- Calculating tube lengths: by letting $S = N_{tubes} \times \pi \times d \times L$

$$L = \frac{S}{N_{tubes} \times \pi \times d} = \frac{6.24}{37 \times \pi \times 0.019} = 2.82\ m$$

Tube lengths must not exceed 2.5 m. So, more than one tube-side pass must be used.

- Using two passes, the correction factor $F = 0.88$

$$S = \frac{\Phi}{K \times F \times \Delta T_{LM}} = \frac{263600}{1419 \times 0.88 \times 29.78} = 7.1\ m^2$$

The number of tubes per pass is always 36 because of the velocity condition. For a two-pass exchanger on the tube side, all surface area is related to tube length:

$$L = \frac{S}{2 \times N_{tubes} \times \pi \times d} = \frac{7.1}{2 \times 37 \times \pi \times 0.019} = 1.41\ m$$

4.2.4.2. Shell Side

The metal envelope surrounding the tube bundle is the calender, generally made of carbon steel. The shell must be able to contain the volume of all the tubes and the space between each tube, as well as the volume between the tubes and the shell.

4.2.5. Solution 5

1. Heat Power Exchanged Φ:

$$\dot{m}_c = \frac{20000}{3600} = 5.56\ kg/s$$

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = 5.556 \times 4180 \times (60 - 40) = 464000\ W = 464\ kW$$

- Outlet Temperature of Hot Fluid:

$$\dot{m}_h = \frac{10000}{3600} = 2.78\ kg/s$$

$$T_{hout} = T_{hin} - \frac{\Phi}{\dot{m}_h \times C_{ph}} = 180 - \frac{464000}{2.78 \times 4315} = 141°C$$

2. Heat Transfer Surface S:

$$\Delta T_{LM} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(180 - 60) - (141 - 40)}{ln\left(\frac{180 - 60}{141 - 40}\right)}\right] = 108.5°C$$

$$S = \frac{\Phi}{K \times \Delta T_{LM}} = \frac{464000}{450 \times 108.5} = 9.5\ m^2$$

- Velocity in Tubes: we have that $R_e = \frac{\rho \times v \times d}{\mu}$

$$v = \frac{\mu \times R_e}{\rho \times d} = \frac{19 \times 10^{-5} \times 10000}{920 \times 0.02} = 0.103\ m/s$$

- Total Tube Cross-Section:

$$s_f = \frac{\dot{m}_c}{\rho \times v} = \frac{2.78}{920 \times 0.103} = 0.0293\ \ m^2$$

- Number of Tubes:

$$N_{tubes} = \ s_f \times \left(\frac{4}{\pi \times d^2}\right) = 0.0293 \times \left(\frac{4}{\pi \times (0.02)^2}\right) = 94$$

- Tube Length: we have $S = N_{tubes} \times \pi \times d \times L$

$$L = \frac{S}{N_{tubes}\ \times \pi \times d} = \frac{9.5}{94 \times \pi \times 0.02} = 1.6\ m$$

4.2.6. Solution 6

1. Calculation of the required mass flow rate of the cooling water:

$$\Delta T_{LM} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(30 - 22) - (30 - 14)}{ln\left(\frac{30 - 22}{30 - 14}\right)}\right] = 11.55°C$$

$$\Phi = K \times S \times F \times \Delta T_{LM} = 2100 \times 45 \times (11{,}55 + 273{,}15) = 1087\ kW$$

Moreover, by using $\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$, we have

$$\dot{m}_c = \frac{\Phi}{C_{pc} \times (T_{cout} - T_{cin}\ )} = \frac{1087}{4184(22 - 14\ )} = 32.5\ kg/s$$

2. Calculating the rate of steam condensation in the condenser:

$$\dot{m}_{stream} = \frac{\Phi}{h_{fg}} = \frac{1087}{2431} = 0.45\ kg/s$$

4.2.7. Solution 7

1. The powers Φ_1 and Φ_2 exchanged during each test:

- Test 1:

$$\Phi_1 = \dot{m}_1 \times C_{p1} \times (T_{out1} - T_{in}) = \rho \times Q_{V1} \times C_p \times (T_{cout1} - T_{cin})$$

$$\Phi_1 = 950 \times 1.944 \times 10^{-4} \times 3800 \times (100 - 20) = 56100\ W$$

- Test 2:

$$\Phi_2 = \dot{m}_1 \times C_{p1} \times (T_{out1} - T_{in}) = \rho \times Q_{V2} \times C_p \times (T_{cout2} - T_{cin})$$

$$\Phi_2 = 950 \times \frac{1.2}{3600} \times 3800 \times (95 - 20) = 90250\, W$$

2. Calculate the logarithmic mean temperatures for the two tests.

- Test 1:

$$\Delta T_{LM1} = \left[\frac{(T_{hin} - T_{cin}) - (T_{hout} - T_{cout1})}{ln\left(\frac{T_{hin} - T_{cin}}{T_{hout} - T_{cout1}}\right)}\right] = \left[\frac{(130 - 20) - (130 - 100)}{ln\left(\frac{130 - 20}{130 - 100}\right)}\right] = 61.6°C$$

- Test 2:

$$\Delta T_{LM2} = \left[\frac{(T_{hin} - T_{cin}) - (T_{hout} - T_{cout2})}{ln\left(\frac{T_{hin} - T_{cin}}{T_{hout} - T_{cout2}}\right)}\right] = \left[\frac{(130 - 20) - (130 - 95)}{ln\left(\frac{130 - 20}{130 - 95}\right)}\right] = 65.5°C$$

3. Deduce the $\frac{K_1}{K_2}$ ratio.

Using $\Phi = K \times S \times F \times \Delta T_{LM}$, thus we have: $\Phi_1 = K_1 \times S \times F \times \Delta T_{LM1}$ and $\Phi_2 = K_2 \times S \times F \times \Delta T_{LM2}$, so:

$$\frac{K_1}{K_2} = \frac{\Delta T_{LM1}}{\Delta T_{LM2}} = 0.66$$

4. Determine the value of the constant K_s: We have that $h_i = K_s \times Q_v^{0.8}$ and $\frac{1}{\mathrm{K}} = \frac{1}{h_i} + \frac{1}{h_e}$.

$$\frac{1}{K_1} = \frac{1}{K_s \times Q_{v1}^{0.8}} + \frac{1}{h_e}$$

$$\frac{1}{K_2} = \frac{1}{K_s \times Q_{v2}^{0.8}} + \frac{1}{h_e}$$

From the ratio $\frac{K_1}{K_2} = 0.66$, we can solve iteratively, and we have:

$$K_s = 14000$$

5. Calculate the overall heat transfer coefficient K_1 corresponding to the flow rate Q_{V1}

$$h_{i1} = K_s \times Q_{v1}^{0.8} = 1400 \times \left(\frac{1.2}{3600}\right)^{0.8} = 14.8\, W/m^2{}^\circ C$$

$$\frac{1}{K_1} = \frac{1}{h_{i1}} + \frac{1}{h_e} = \frac{1}{14.8} + \frac{1}{3000} \Rightarrow \ K_1 = 14.73\ \mathrm{W/m^2{}^\circ C}$$

6. Deduct the total exchange area S: by using $\Phi_1 = K_1 \,.\, S \,.\, \Delta T_{LM1}$ we have

$$S = \frac{\Phi_1}{K_1 \times \Delta T_{LM1}} = \frac{56100}{14.73 \times 61.6} = 61.8\, m^2$$

4.2.8. Solution 8

1. Heat power of the exchanger

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = 1.25 \times 4180 \times (90 - 35) = 287375\, W = 287\, kW$$

2. Oil flow rate

From the energy balance on the oil side $\Phi_h = \dot{m}_h \times C_{ph} \times (T_{hout} - T_{hin})$, we have:

$$\dot{m}_h = \frac{\Phi_h}{C_{ph} \times (T_{hout} - T_{hin})} = \frac{287000}{2000 \times (150 - 85)} = 2.21\, kg/s$$

3. Logarithmic mean temperatures difference

$$\Delta T_{LM} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(150 - 90) - (85 - 35)}{ln\left(\frac{150 - 90}{85 - 35}\right)}\right] = 54.8\, ^\circ C$$

4. Area of the large exchanger

Using $\Phi = K \times S \times \Delta T_{LM}$, we have:

$$S = \frac{\Phi}{K \times \Delta T_{LM}} = \frac{287000}{850 \times 54.8} = 6.15\, m^2$$

5. Two small exchangers

Water is in series, oil is in parallel

a. Oil flow per exchanger

$$\dot{m}_{h1} = \dot{m}_{h2} = \frac{2.21}{2} = 1.105\, kg/s$$

b. Heat removed per exchanger

$$\Phi_{h1} = \dot{m}_{h1} \times C_{ph} \times (T_{hin} - T_{hout}) = 1.105 \times 2000 \times (150 - 85) = 143650\ W$$
$$= 143.65\ kW$$

c. Water temperature between exchangers: using $\Phi_c = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$, we have:

$$T_{cout} = \frac{\Phi_c}{\dot{m}_c \times C_{pc}} + T_{cin} = \frac{143650}{1.25 \times 4180} + 35 = 62.5\ °C$$

6. LMTD for each exchanger

a. Exchanger 1

$$\Delta T_{LM1} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(150 - 62{,}5) - (85 - 35)}{ln\left(\frac{150 - 62{,}5}{85 - 35}\right)}\right] = 66.9\ °C$$

b. Exchanger 2

$$\Delta T_{LM2} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(150 - 90) - (85 - 62{,}5)}{ln\left(\frac{150 - 90}{85 - 62{,}5}\right)}\right] = 38.8\ °C$$

7. Areas of the two exchangers

a. Exchanger 1

Using $\Phi_1 = K \times S_1 \times \Delta T_{LM1}$, we have:

$$S_1 = \frac{\Phi_1}{K \times \Delta T_{LM1}} = \frac{143650}{850 \times 66.9} = 2.53\ m^2$$

b. Exchanger 2

Using $\Phi_2 = K \times S_2 \times \Delta T_{LM2}$, we have:

$$S_2 = \frac{\Phi_2}{K \times \Delta T_{LM2}} = \frac{143650}{850 \times 38.8} = 4.36\ m^2$$

c. Total area

$$S = S_1 + S_2 = 2.53 + 4.36 = 6.89\ m^2$$

8. Cost Comparison

a. The cost of a large exchange is $Cost_L = 6.15\ C$

b. The cost of the two small exchangers is $Cost_s = 6.89 \times 1.2\ C = 8.27\ C$

9. Conclusion

$$Cost_s > Cost_L$$

The single large heat exchanger is more economical

4.2.9. Solution 9

1. Heat transfer power

$$\Phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout}) = 30000 \times 0.9 \times \left(\frac{4180}{3600}\right) \times (66 - 39) = 846450\ W$$

2. Outlet temperature of cooling water

Using $= \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin})$, we have that:

$$T_{cout} = \frac{\Phi}{\dot{m}_c \times C_{pc}} + T_{cin} = \frac{846450}{29500 \times \left(\frac{4180}{3600}\right)} + 12 = 36.7\ °C$$

3. Temperature difference

a. Co-current flow

$$\Delta T_{LMco} = \left[\frac{(T_{hin} - T_{cin}) - (T_{hout} - T_{cout})}{ln\left(\frac{T_{hin} - T_{cin}}{T_{hout} - T_{cout}}\right)}\right] = \left[\frac{(66 - 12) - (39 - 36.7)}{ln\left(\frac{66 - 12}{39 - 36.7}\right)}\right] = 16.39\ °C$$

b. Counter–current flow

$$\Delta T_{LMcount} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(66 - 36{,}7) - (39 - 12)}{ln\left(\frac{66 - 36.7}{39 - 12}\right)}\right] = 28.13°C$$

4. Heat transfer area

Using $S = \frac{\Phi}{K \times F \times \Delta T_{LM}}$, we have:

a. Co-current flow

$$S = \frac{846450}{2100\left(\frac{4180}{3600}\right) \times 1 \times 16.39} = 21.18\ m^2$$

b. Counter-current flow

$$S = \frac{846450}{2100\left(\frac{4180}{3600}\right) \times 1 \times 28.13} = 12.34\, m^2$$

c. 1-2 shell-and-tube

$$S = \frac{846450}{2100\left(\frac{4180}{3600}\right) \times 0.93 \times 28.13} = 13.27\, m^2$$

d. 2-4 shell-and-tube

$$S = \frac{846450}{2100\left(\frac{4180}{3600}\right) \times 0.97 \times 28.13} = 12.73\, m^2$$

e. Crossflow, both fluids unmixed

$$S = \frac{846450}{2100\left(\frac{4180}{3600}\right) \times 0.85 \times 28.13} = 14.52\, m^2$$

Table 4.1: Required Heat Transfer Surface Area for each Exchanger Configuration

Configuration	***F***	$\boldsymbol{S\ (m^2)}$
Co-current	1	21.18
Counter-current	1	12.34
1-2 shell-and-tube	0.93	13.27
2-4 shell-and-tube	0.97	12.73
Crossflow (unmixed)	0.85	14.52

4.2.10. Solution 10

1. Heat transfer power

$$\Phi = \dot{m}_c \times C_{pc} \times (T_{cout} - T_{cin}) = 7000 \times \left(\frac{4180}{3600}\right) \times (60 - 40) = 162.6\, kW$$

- Hot fluid outlet temperature

Using $\Phi = \dot{m}_h \times C_{ph} \times (T_{hin} - T_{hout})$, we have:

$$T_{hout} = T_{hin} - \frac{\Phi}{\dot{m}_h \times C_{ph}} = 180 - \frac{162600}{400 \times \left(\frac{4180}{3600}\right)} = -170.1\ °C$$

This is impossible because the temperature cannot drop below absolute zero, and in practice, the hot fluid cannot be cooled below the cold fluid outlet temperature ($T_{hout} = 60°C$). The required heat duty exceeds the energy available from the hot fluid, given its flow rate and inlet temperature. The design is thermodynamically infeasible.

2. Heat Transfer Surface S:

$$\Delta T_{LM} = \left[\frac{(T_{hin} - T_{cout}) - (T_{hout} - T_{cin})}{ln\left(\frac{T_{hin} - T_{cout}}{T_{hout} - T_{cin}}\right)}\right] = \left[\frac{(180 - 60) - (60 - 40)}{ln\left(\frac{180 - 60}{60 - 40}\right)}\right] = 55.82\ °C$$

$$S = \frac{\Phi}{K \times \Delta T_{LM}} = \frac{162600}{2440 \times 55{,}82} = 1.2\ m^2$$

- Velocity inside tubes

$$\dot{v} = \frac{\dot{m}_h}{\rho} = \frac{400 \times \left(\frac{1}{3600}\right)}{1000} = 1.11 \times 10^{-4} m^3/s$$

- Tube cross-sectional area

$$S_{tube} = \frac{\pi \times d^2}{4} = \frac{\pi \times (0{,}01)^2}{4} = 7.85 \times 10^{-5}\ m^2$$

- Number of tubes

By assuming $v = \frac{1m}{s}$, we have:

$$N_{tubes} = \frac{\dot{v}}{v \times S_{tube}} = \frac{1.11 \times 10^{-4}}{1 \times 7.85 \times 10^{-5}} = 1.42 \quad \rightarrow \quad N_{tubes} = 2$$

- Tube length

Using $S = N_{tubes} \times \pi \times d \times L$, we have:

$$L = \frac{S}{N_{tubes} \times \pi \times d} = \frac{1.2}{2 \times \pi \times 0.01} = 19.1\ m$$

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