

**People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
Mohamed Seddik Benyahia University – Jijel**



**Faculty of Science and Technology
Department of Architecture**

Course Handbook

Descriptive geometry of the space 1

1st year architecture

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GDE Module Overview:

Descriptive geometry is a fundamental discipline that enables the graphical representation of three-dimensional objects on two-dimensional surfaces. It is an essential tool for students of architecture, urban planning and technical disciplines, helping them to develop their spatial visualisation skills, their ability to interpret graphical representations, and their capacity to depict complex forms.

Semester	1
Teaching Unit	EA 1
Course	Descriptive Geometry of Space 1
Coefficient	2
Minimum Passing Grade	Mark below < 05/20

Objectives of the descriptive geometry module

The course 'SPATIAL GEOMETRY 1' introduces students to and prepares them for learning how to represent buildings graphically in space. This course helps students become familiar with the transition from 2D to 3D and vice versa, thereby developing their ability to visualise in space.

Course content:

The program will cover the following topics:

- Projection methods;
- Representation of points, lines and planes;
- Intersections of lines, planes and solids;
- Folding and affinity;
- Representation of circles, cones, cylinders and spheres;
- Methods of perspective representation;
- Methods of axonometric representation;
- Shadows.

Assessment method:

Nature of the assessment	Weighting (%)
Exam	60 %
Continuous assessment	40 %
Total	100 %

General introduction:

Descriptive Geometry is a fundamental discipline in architecture, engineering, industrial design, and other technical fields. Developed by Gaspard Monge in the eighteenth century, it provides a rigorous graphical method for representing three-dimensional objects on two-dimensional surfaces while preserving their geometric properties. It serves as the foundation of technical drawing and develops the spatial reasoning skills required for the analysis, visualization, and communication of complex geometric forms.

The primary objective of descriptive geometry is to enable the accurate representation of points, lines, planes, and solids through standardized projection systems. By mastering these graphical techniques, students learn to solve spatial problems involving distances, intersections, parallelism, perpendicularity, visibility, and true dimensions. These competencies are essential not only for architectural and engineering drawing but also for computer-aided design (CAD), manufacturing, construction, and digital modeling.

This textbook is organized into eight complementary chapters that progressively introduce the fundamental principles and advanced applications of descriptive geometry. The first chapter presents the principal projection methods, including orthogonal, axonometric, and conical perspective projections. The second chapter introduces the graphical representation of points, lines, and planes, which constitute the basic elements of spatial geometry. The third chapter examines the geometric relationships between these elements through the study of intersections, parallelism, and orthogonality.

The fourth chapter focuses on geometric transformations such as plane changes, rotations, and folding techniques, allowing students to determine true lengths, true shapes, and hidden geometric relationships. The fifth chapter extends these concepts to the representation and development of elementary curved solids, including circles, cylinders, cones, and spheres. The sixth and seventh chapters explore perspective and axonometric representation methods, emphasizing their applications in architectural visualization and technical communication. Finally, the eighth chapter introduces the theory and construction of shadows, an essential component for accurately representing volumes, lighting conditions, and spatial perception.

Throughout this book, theoretical concepts are systematically accompanied by graphical constructions and practical examples to facilitate progressive learning and strengthen students' visualization abilities. By the end of the course, students will have acquired the knowledge and technical skills necessary to interpret, construct, and communicate complex three-dimensional forms using internationally recognized methods of descriptive geometry. These competencies provide a solid foundation for advanced studies in architectural representation, engineering graphics, digital modeling, and professional design practice.

Chapter I Projection methods

Introduction:

Projection methods form the foundation of descriptive geometry and technical drawing, as they provide systematic techniques for representing three-dimensional objects on two-dimensional surfaces. These methods are essential for understanding spatial relationships, communicating design ideas, and solving geometric problems in architecture, engineering, and related fields.

This chapter introduces the three principal categories of projection: orthogonal projection, axonometric projection, and conic perspective projection. Each method is based on specific geometric principles and offers a different way of representing space depending on the required level of accuracy, realism, and purpose of representation.

Orthogonal projection is first presented as the most precise method, based on perpendicular projection rays and multiple projection planes. The concept of two-plane projection and the folding technique are also explained to facilitate the interpretation of spatial forms.

The chapter then explores axonometric projection, including its definition and main types such as military axonometry, front axonometry, isometry, and dimetry. These methods allow a three-dimensional visualization while preserving measurable proportions.

Finally, conic perspective projection is introduced as a more realistic representation system, based on visual perception. Different types of perspective, including central and oblique conical perspectives, are discussed to show how depth and realism are achieved in graphical representation.

1. Orthogonal projection

1.1. Definition

The orthogonal projection of a point (P) onto a plane is denoted as (p), obtained by drawing the perpendicular (Pp) from the point to the plane.

All the points located along a line perpendicular to the projection plan are projected onto a single point. Consequently, an orthogonal projection onto a single plan is not sufficient to define the position of the point in space.

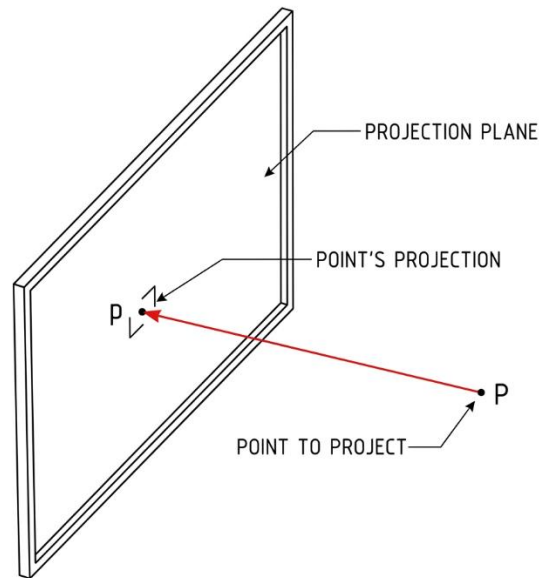


Figure I. 1: Orthogonal projection of a point

More generally, constructing the orthogonal projection of a solid involves finding the projections of its characteristic points.

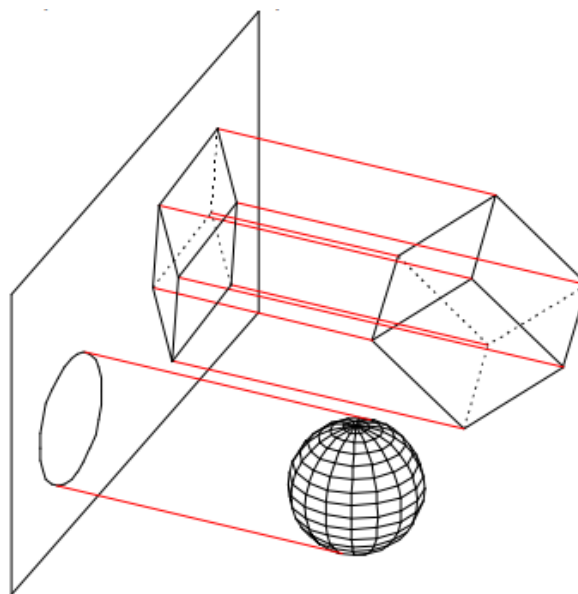


Figure I. 2: Orthogonal projection of a volume

1.2. Main characteristics:

- Multiple projection: orthogonal projection uses several simultaneous projections or views to represent an object.
- Perpendicular projection: perpendicular lines are drawn from points on the object to the projection plan.
- Orthogonal views: The various projections give rise to orthogonal views of the object.

- Dimensions and details: Orthogonal projection enables the accurate representation of an object's dimensions and details.
- Cartesian coordinates: Orthogonal projection uses Cartesian coordinates to locate object points on the projection plan.
- Projection planes: These are generally aligned perpendicular to each other.

1.3. Projection planes:

Consider the three-dimensional coordinate system (X, Y, Z) which determines three mutually orthogonal planes, namely (F), (H) and (P).

- The (F) plane is the frontal plane, where all points have a zero value on the Y-axis.
- The (H) plane is the horizontal plane, where all the points have a zero value for the Z axis.
- Plan (P) is the profile plane, where all points have a zero value for the X axis.

The OX axis is perpendicular to the OY axis, which is itself perpendicular to the OZ axis. Plan (F) is perpendicular to plan (H), which is perpendicular to plan (P).

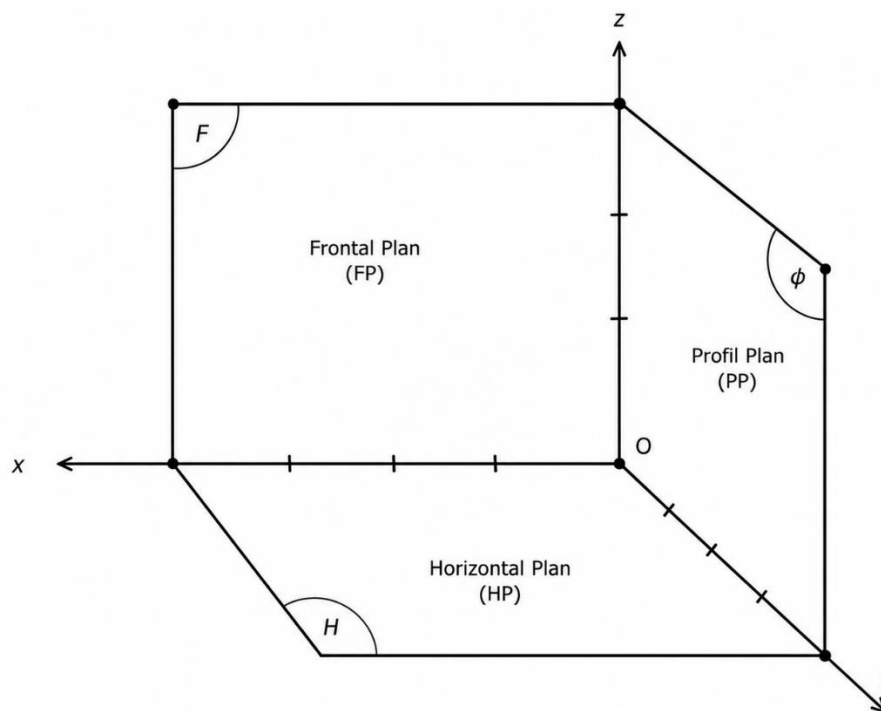


Figure I. 3: Projection planes

1.4. Two-plan projection:

The two-plan projection system consists of the front (F) and horizontal (H) planes, while the profile plan (P) acts as an auxiliary plan.

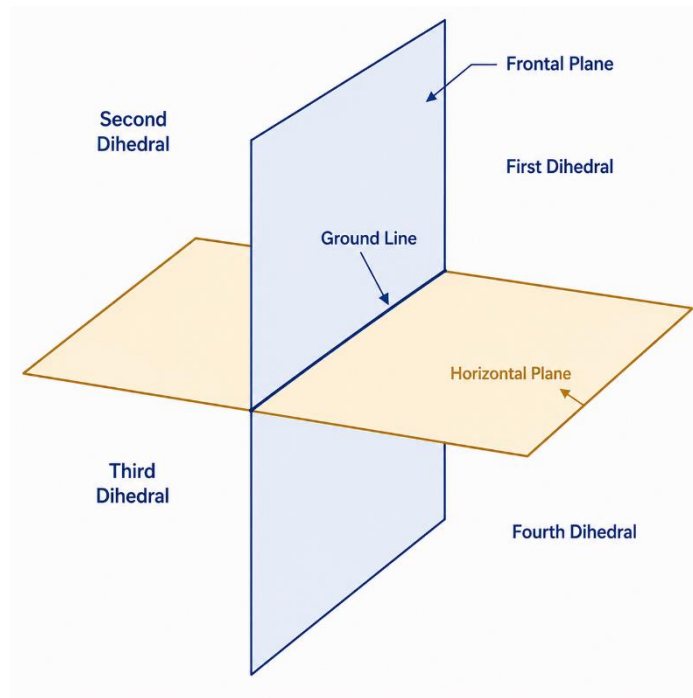


Figure I . 4: Projection of two planes and dihedrals

The line where the planes (F) and (H) intersect, corresponding to the OX axis, is called the ground line (LT).

Since planes are infinite, planes (F) and (H) divide space into four distinct sectors, called dihedral.

- The number assigned to each dihedral is determined by the Y and Z coordinates.
- The first dihedral: Y is positive and Z is positive.
- The second dihedral: Y is negative and Z is positive.
- The third dihedron: Y is negative and Z is negative.
- The fourth dihedron: Y is positive and Z is negative.

1.5. Folding the frontal plane:

Regardless of the position of a three-dimensional object (V) in space, the representation (v) is the horizontal projection of (V), while (v1) is the frontal projection of (V).

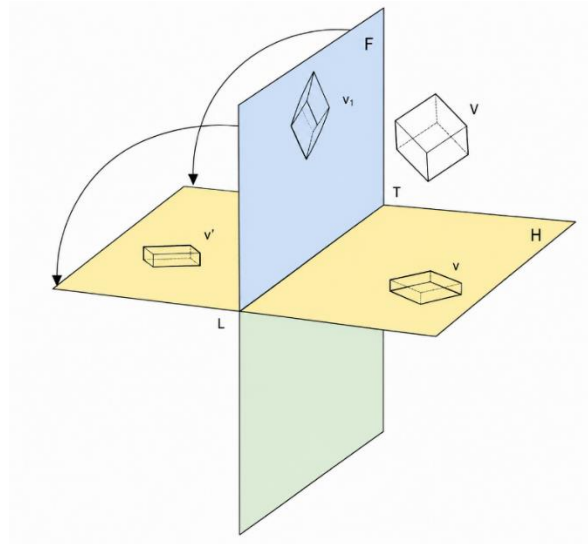


Figure I. 5: Folding of frontal plane

To obtain two two-dimensional projections on the same plane surface (e.g., a sheet of paper) and superimpose them, the frontal plane (F) is rotated around the ground line axis (LT), folding it onto the horizontal plane (H). This places the front projection (v_1) where (v) is, so that the two projections are superimposed.

NB: In descriptive geometry, we work with folding the horizontal plan.

2. Axonometric projection

2.1. Definition

The axonometric system offers the ability to represent an architectural volume using a single view. It involves projecting this volume from an external viewpoint, so that the receding lines remain parallel to each other.

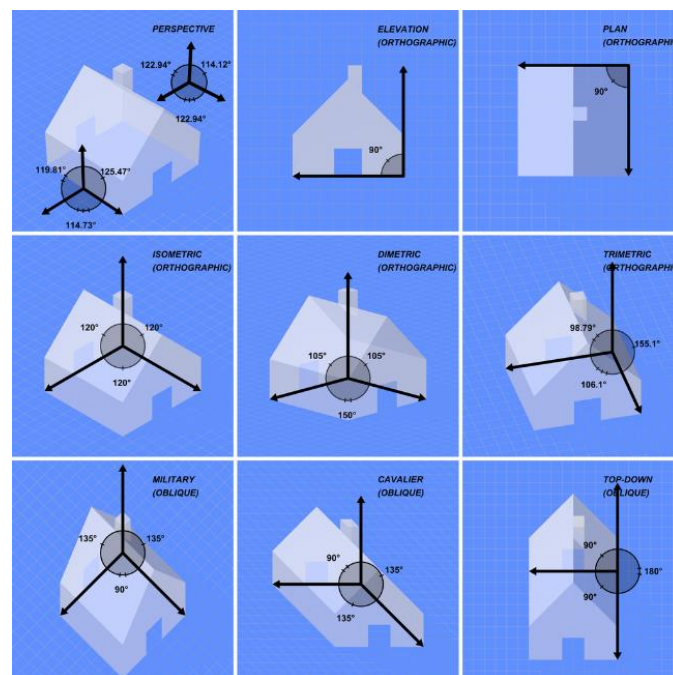


Figure I. 6: Different types of axonometry

Axonometry presents both top and corner views using oblique and parallel projection within an orthogonal spatial coordinate system. This method enables the complexity of a whole to be understood more quickly. In the context of architecture, it can be used to represent a building with its roofs, to present certain interior aspects by means of cross-sections, and to improve overall understanding by incorporating transparent walls in perspective.

Axonometric drawings are simple and quick to produce, as they can be constructed to scale from existing planes, elevations, or sections, depending on the importance of the element being represented. Different types of axonometry:

2.2.1. Plane Axonometry (military):

The plane retains its original form and the angle of view is arbitrary. Shortening the heights to $\frac{2}{3}$ improves the image.

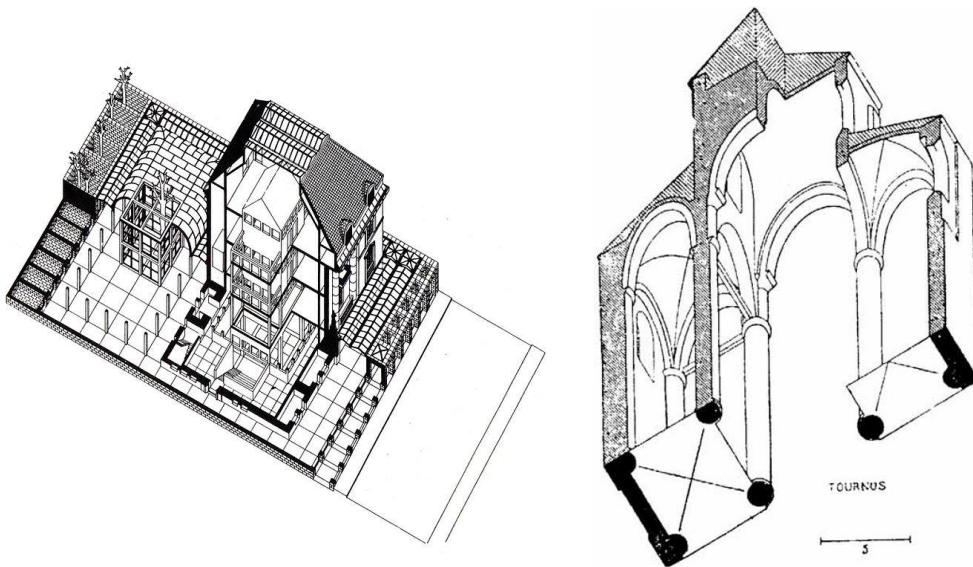


Figure I. 7: Military axonometry

2.2.2. Front axonometry:

The elevation retains its original form, but shortening the depths to $\frac{2}{3}$ improves the image.

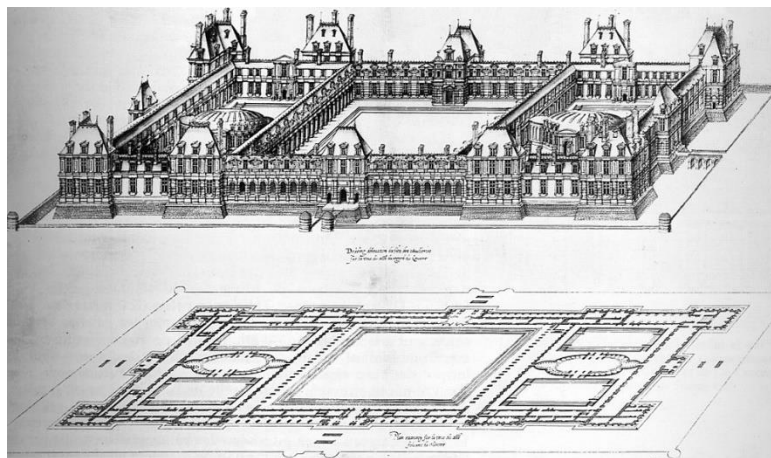


Figure I. 8: Front axonometry

2.2.3. Isometry:

All edges are represented in full size. All surfaces are deformed.

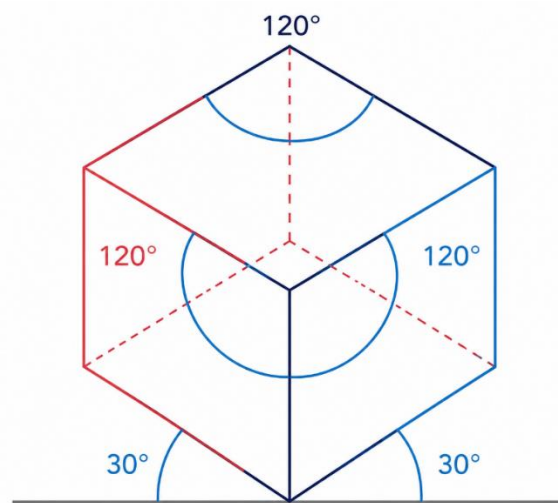


Figure I. 9: Isometric axonometry

2.2.4. Dimetry:

The elevation seen obliquely is shown full size, with the depths reduced to 2/3. All surfaces are distorted.

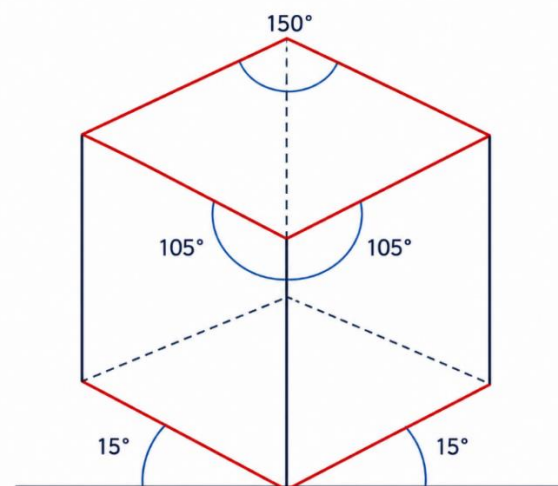


Figure I. 10: Dimetric axonometry

3. Perspective conic projection

3.1. Definition

In perspective drawing, the object is represented not by its true dimensions, but as it appears visually, similar to a photograph. To fully understand perspective drawing, you need to imagine the table plan drawn perpendicular to the ground plan.

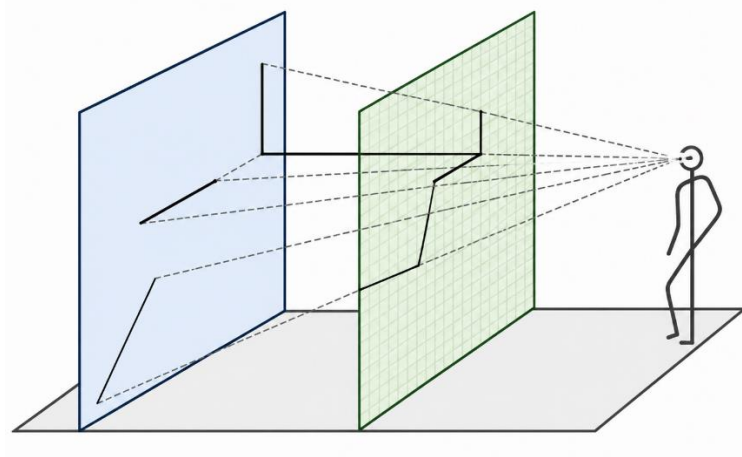


Figure I. 11: Perspective conic projection

In perspective, all the lines that are parallel in a given direction converge on a single point called the vanishing point. However, lines parallel to the table plan also remain parallel in perspective, even though their vanishing points are infinitely far apart. Vertical edges also remain vertical in perspective.

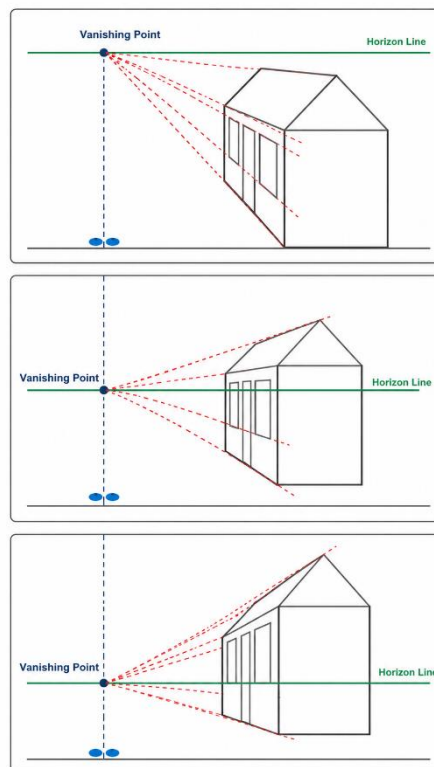


Figure I. 12: Perspective views

3.2. Different types of perspective

3.2.1. Central conic perspective:

This perspective is used when the front of a volume is parallel to the observer's forehead. (single vanishing point perspective).



Figure I. 13: Central conical perspective

3.2.2. Conical oblique perspective :

Oblique conical perspective is constructed from two vanishing points (and even with 3 vanishing points).



Figure I. 14: Conical oblique perspective

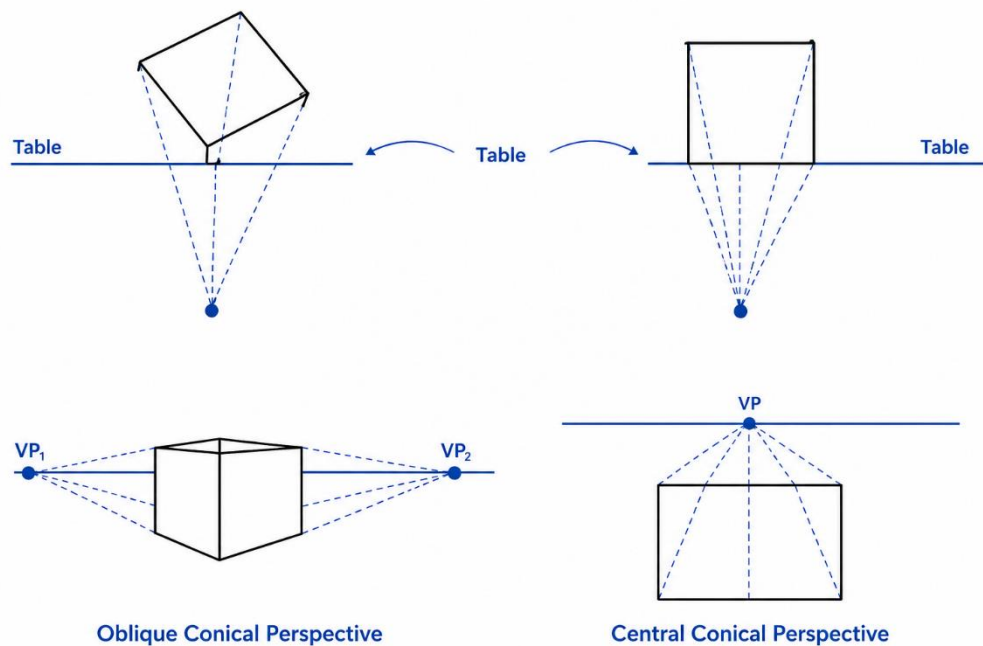


Figure I . 15: Types of perspective

Conclusion:

Descriptive geometry, through its projection methods—orthogonal, axonometric, and perspective—is an essential tool for representing and visualizing three-dimensional objects on two-dimensional surfaces. These methods allow complex ideas to be communicated clearly and accurately, playing a crucial role in fields such as architecture, engineering, design and many others. Orthogonal projection, using perpendicular projections, is fundamental to accurately representing the shapes and dimensions of objects. It is particularly useful for studying the geometric properties and cross-sections of objects. Axonometric projection, on the other hand, provides a three-dimensional view of the object, using parallel lines to help visualise objects from different angles. It is often used to present complex concepts in a more intuitive way.

Finally, perspective creates realistic representations of objects by adding vanishing lines, simulating the visual appearance as we perceive it in the real world. It is a powerful technique for creating realistic drawings that resemble what the human eye perceives. In conclusion, descriptive geometry and its projection methods play a central role in the way we represent and understand three-dimensional space. Each method has its own advantages and is used according to context and purpose, providing invaluable tools for visual communication and artistic creation.

Chapter II Representation of the point, line and plan

Introduction

The representation of points, lines, and planes constitutes a fundamental stage in the study of descriptive geometry, as these three elements form the basic building blocks of all spatial constructions. A clear understanding of their graphical representation is essential for analyzing geometric relationships and solving spatial problems in architecture, engineering, and technical drawing.

This chapter introduces the principles governing the representation of the point as the simplest geometric entity, including its projection on one, two, and three reference planes, as well as its position within different spatial configurations such as dihedrals and bisector planes. The technique of folding planes is also presented as a key tool for interpreting spatial positions in a two-dimensional representation.

The study then extends to the line, defined as a set of points, and explores its different types, traces, and special positions in space. Particular attention is given to characteristic lines such as vertical, horizontal, frontal, and profile lines, as well as methods for locating points on a line and determining its true length using rotation and Pythagoras-based techniques.

Finally, the chapter focuses on the plane as a fundamental geometric surface. Its definition, representation, traces, and special positions are examined, along with principal lines such as horizontal, frontal, and lines of greatest slope. Construction methods for locating points within a plane are also introduced, providing essential tools for spatial analysis and geometric construction.

1. The point

1.1. Definition

In descriptive geometry and orthogonal projection, a point is projected from its location in space onto the projection planes along perpendicular lines. These projections create two-dimensional coordinates on the projection planes, which help to locate the point in the context of a coordinate system.

1.2. Representation of a point on three planes

A point in space is called (A):

- Its horizontal projection is (a).
- Its frontal projection is (à).
- Its profile projection is called (a'').

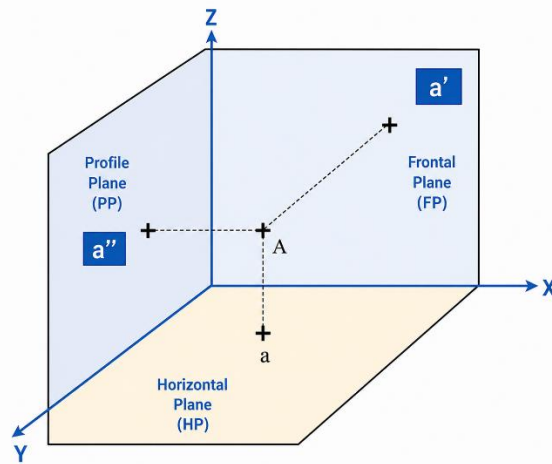


Figure II. 1: Representation of a point on 3 projection planes

- ❖ The horizontal projection (a) is obtained at the intersection of the line projecting from point (A) onto the horizontal plane. This line is perpendicular to the ground line (GL) (LT) and to the horizontal plane (PH).
- ❖ The frontal projection (a') is obtained at the intersection of the line projecting from point (A) onto the frontal plane. This line is perpendicular to the ground line (LT) and the frontal plane (FP).
- ❖ The profile projection (a'') is obtained at the intersection of the line projecting from point (A) onto the profile plane. It is perpendicular to this plane.

1.3. Representation of a point on two planes

To switch from 3d to 2d representation, the frontal plane must be aligned with the horizontal plane by means of a geometric transformation (rotation around the ground line).

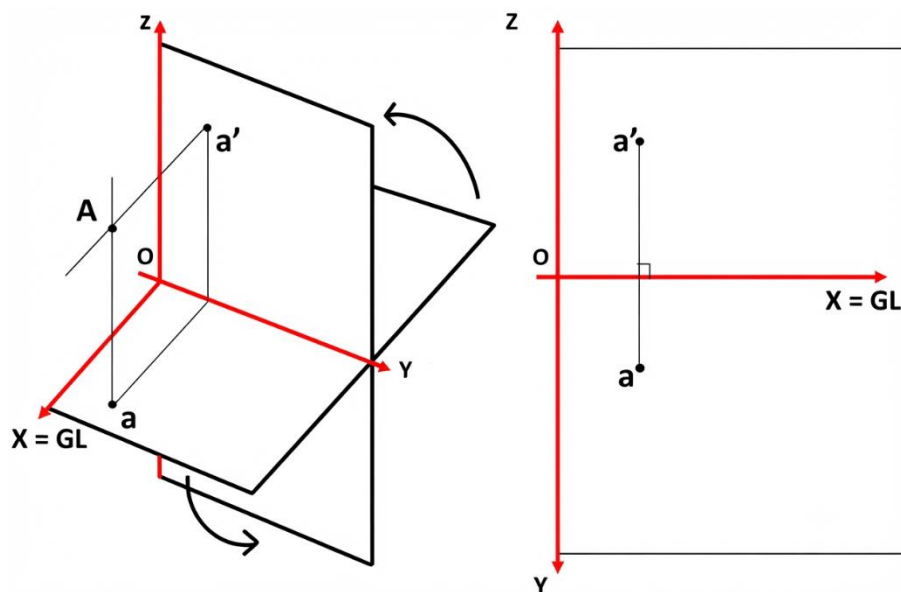


Figure II. 2: Representation of a point on two planes

The point (a) traces a quarter circle with (O) as its centre. The connection between points (a) and (a') is called the projection line of point (A), which is necessarily perpendicular to the ground line (LT).

- Distancing of (A):

The distance from a point to the frontal plane, which is the same distance from (a) to the ground line (LT), is called the distance of a point.

- Side of (A):

The distance from this point to the horizontal plane, which is the same distance from (a') to the ground line (LT), is called the side of a point.

1.4. Points belonging to (PH, PF et LT):

- $A \in (PH)$, $a=0$, $a'=A$, so (a') is on (LT).
- $B \in (PF)$, $b=0$, $b'=B$, so (b) is on (LT).
- $C \in (LT)$, $c=0$, $c'=0$, so (c)=(c') are on (LT).

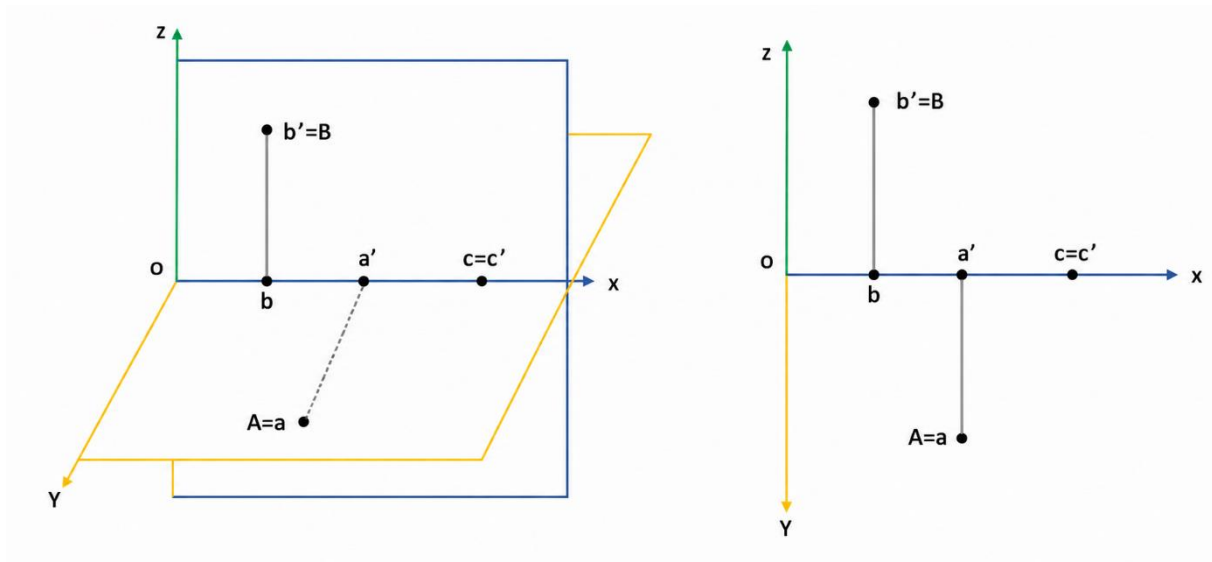


Figure II. 3: Points belonging to (PH, PF et LT)

1.5. Points belonging to different dihedrals :

- $A \in$ 1st quadrant, (a) and (a') are positive, (a) is below the (LT), and (a') is above the (LT).
- $B \in$ 2nd quadrant, (b') is positive, (b) is negative, so (b) and (b') are above the (LT).
- $C \in$ 3rd quadrant, (c) and (c') are negative, (c) is above (LT), and (c') is below (LT).
- $E \in$ 4th quadrant, (e') is negative, (e) is positive, so (e) and (e') are below the (LT).

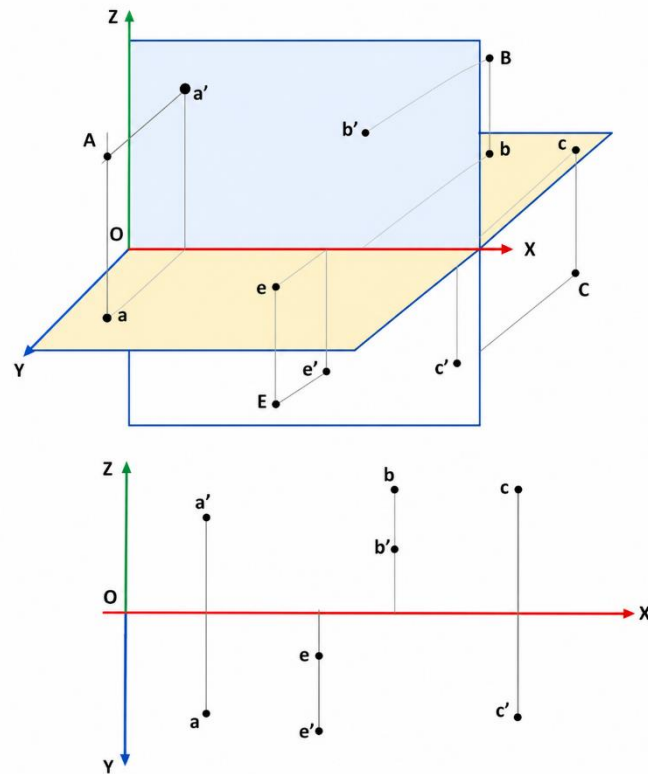


Figure II. 4: Points belonging to different dihedrals

1.6. Bisectors planes:

By convention, the four quadrants are divided into two by median planes called bisectors. These bisector planes are perpendicular and form a 45-degree angle with the projection planes. Points located on the bisector planes share the characteristic of being at an equal distance from the horizontal projection plane and the frontal projection plane. Consequently, the sides and distances of these points are equal in absolute value.

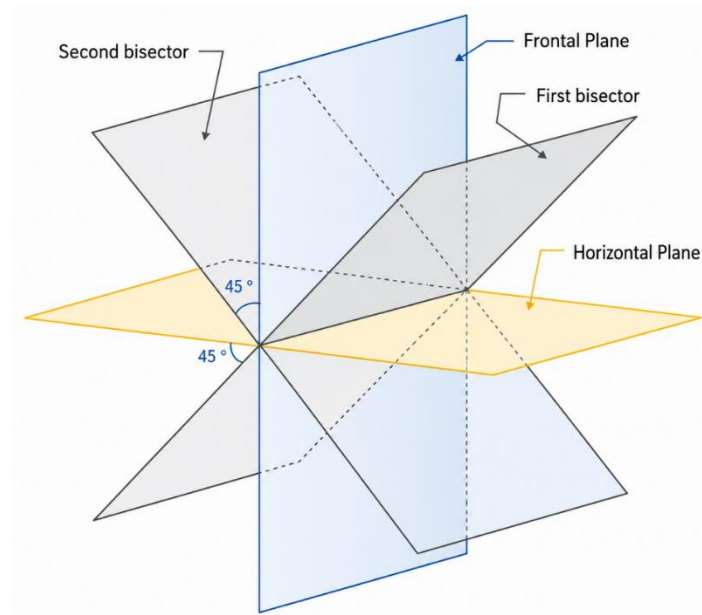


Figure II. 5: Bisectors planes

Consider a point (P) located in the first bisector (B1). This point (P) is equidistant from both the frontal plane and the horizontal plane. Consequently, it resides either in the first quadrant or in the third quadrant. In this case, the side and the distance of this point share the same sign. Thus, the distance of (P) is equal to its side meaning that these two values are numerically identical.

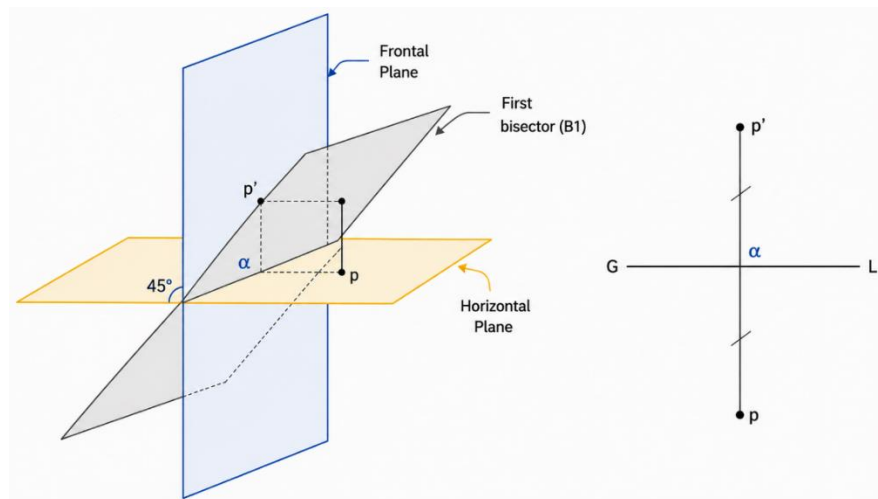


Figure II. 6: Point belonging to the 1st and 3rd dihedrals

Consider a point (Q) located in the second bisector (B2). This point (Q) is equidistant from both the frontal plane and the horizontal plane. Consequently, it is either in the second quadrant or in the fourth quadrant. In this scenario, the side and the distance of this point have opposite signs. Thus, the distance of (Q) is equal to the negation of its side, meaning these two values are numerically inverse of each other.

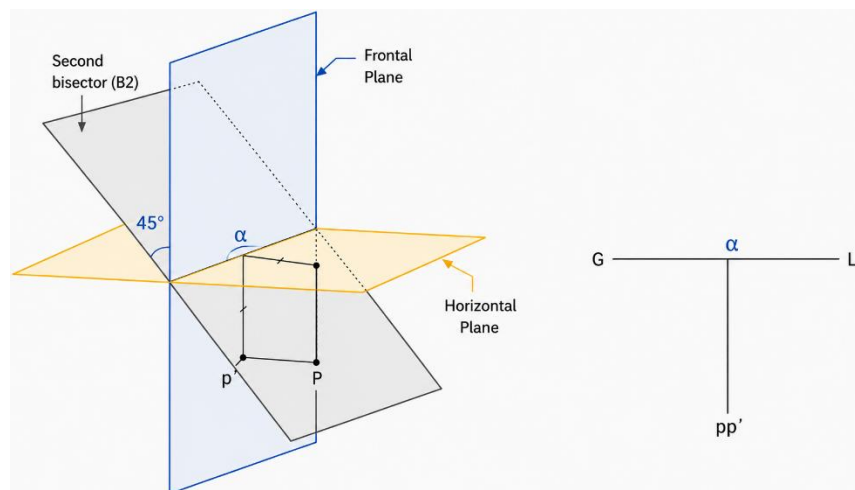


Figure II. 7: Point belonging to the 2nd and 4th dihedrals

1.7. Folding of three planes

To go from the 3d representation to the 2d representation, the frontal plane must be aligned with the horizontal plane on one side, and it must be aligned with the profile plane on the other side, by means of a geometric transformation (rotation).

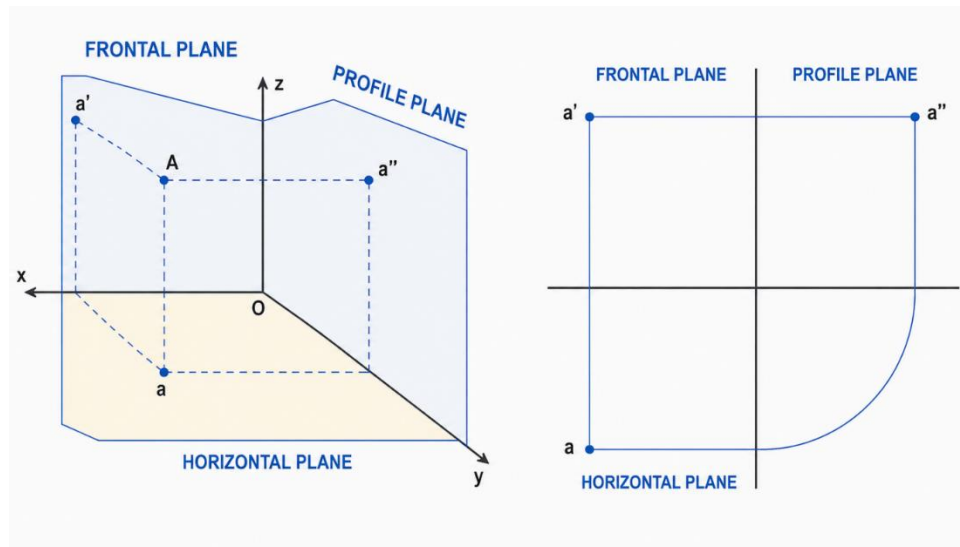


Figure II. 8: Folding of three planes

2. The line

2.1. Definition

Geometry teaches us that a line is entirely defined by two distinct points. Therefore, to determine a line in a diagram, it is sufficient to know the horizontal and frontal projections of two of its points. Thus, a line is characterized by its horizontal projection and its front projection.

2.2. Arbitrary line

A line is termed "arbitrary" when its position in space is neither parallel nor perpendicular to the two projection planes (PH) and (PF). In other words, this line does not exhibit characteristics of being parallel or perpendicular to the projection planes (PH) and (PF).

Consider two distinct points (A) and (B) in space.

- (a) and (b) are their horizontal projections
- (a') and (b') are their front projections.
- The line (ab) is the horizontal projection of the line (AB).
- The line (a'b') is the frontal projection of the line (AB).

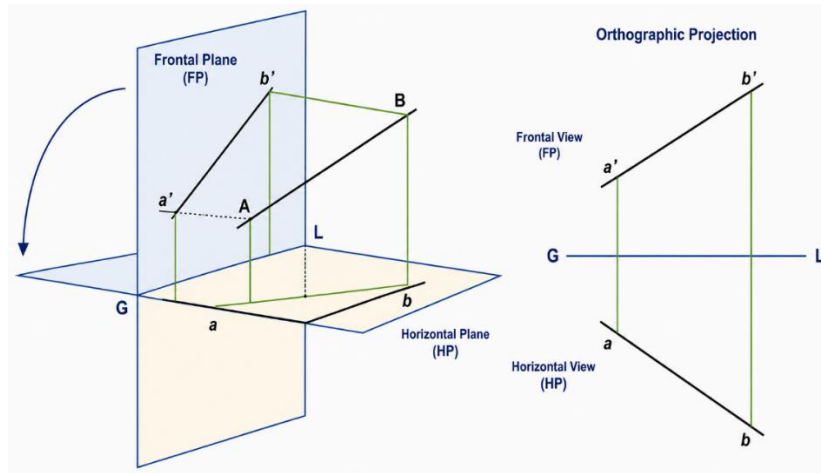


Figure II. 9: Projection of an Arbitrary line

2.3. Trace of the line

The trace of a line refers to the points where the line intersects the projection planes. The front trace (TF) of a line corresponds to the point where it intersects the frontal plane. This point on the line has zero distance. The horizontal trace (TH) is the point on the line with zero side.

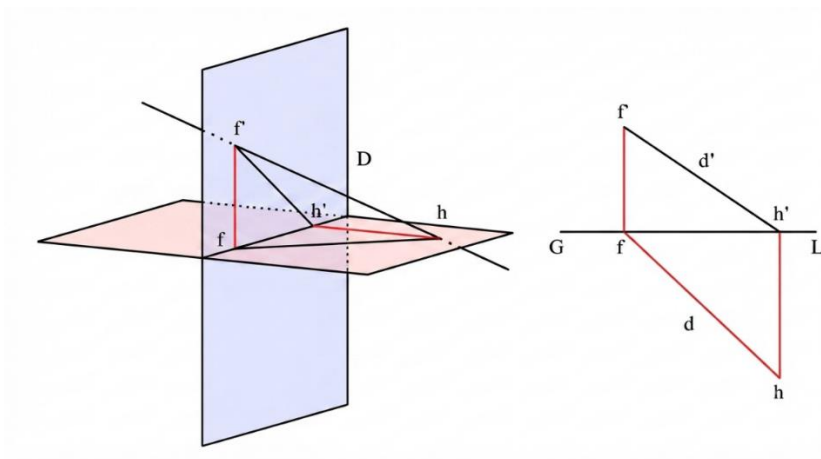


Figure II. 10: Trace of the line

2.4. Particular line

A line can occupy various positions in relation to the projection planes. It is termed as special or significant when it is parallel or perpendicular to one of the projection planes.

When a line is specific, it can only have a single trace concerning all projection planes. This implies that a specific line can intersect each projection plane at only one point, whether in terms of distance or side.

2.4.1. Vertical line

Vertical line $\perp$ (HP).

Vertical line $\parallel$ (FP) and (PP).

Its horizontal projection reduces to a point. Its frontal projection is perpendicular to the ground line (LT). Because it is perpendicular to the horizontal plane (HP), it is therefore parallel to the

frontal plane (FP). Consequently, it is called a "particular frontal line". Its front projection is represented in actual shape, at actual scale, and with actual dimensions. It leaves only one trace, which is its horizontal trace.

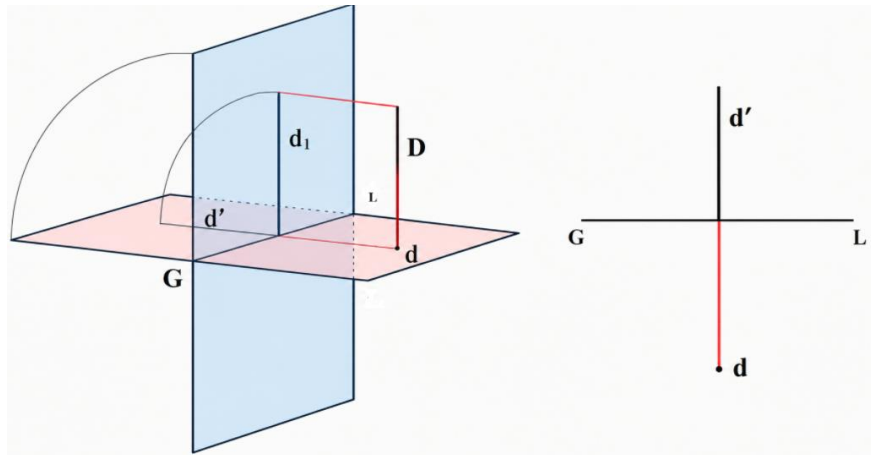


Figure II. 11: Projection of a Vertical line

2.4.2. Line perpendicular to (FP)

Line $\perp$ (FP).

Its frontal projection reduces to a point. Its horizontal projection is perpendicular to the ground line (LT). Being perpendicular to the frontal plane (FP), it is consequently parallel to the horizontal plane (HP). Hence, it is termed a "particular horizontal line". Its horizontal projection is represented in actual shape, at the actual scale, and with actual dimensions. It has only one trace, which is its frontal trace.

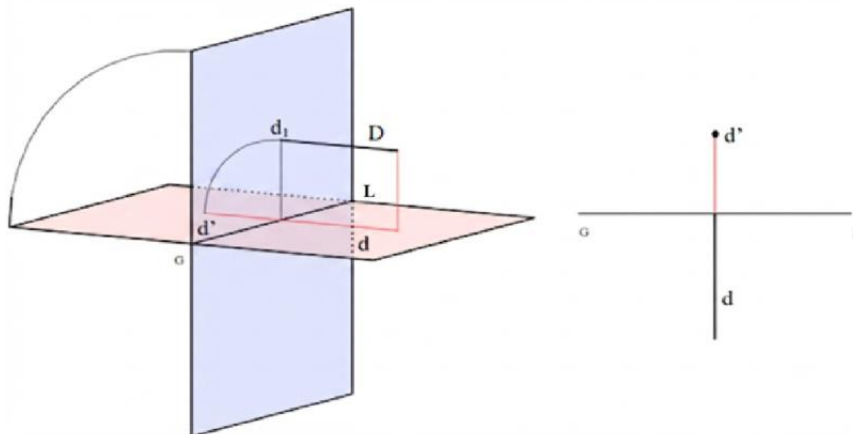


Figure II. 12: Projection of a line perpendicular to (FP)

2.4.3. Horizontal line

Horizontal line $\parallel$ (HP).

Each point on this line shares the same sides. Consequently, the frontal projections of all points on this line are equidistant from the ground line (LT). Therefore, the frontal projection is also parallel to the (LT). The horizontal line leaves only one trace, which is its frontal trace. In space,

the line itself and its horizontal projection are parallel. Hence, the horizontal projection is depicted in true form, to the true scale, and with true dimensions.

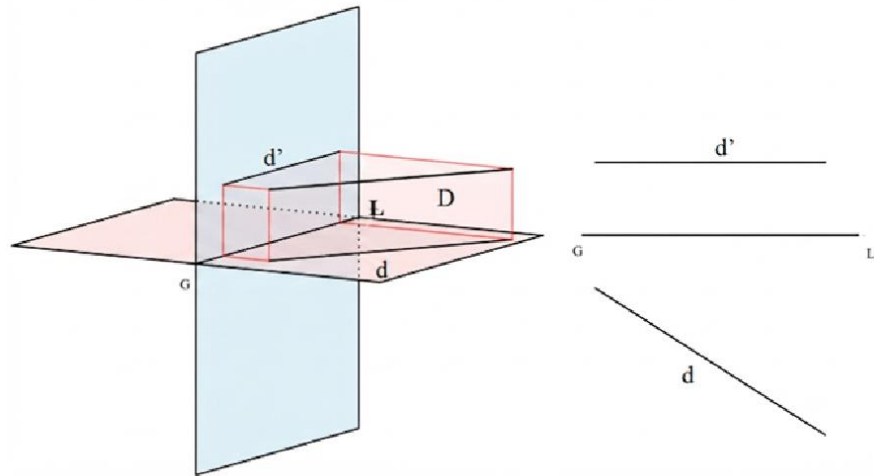


Figure II. 13: Projection of an horizontal line

2.4.4. Frontal line

Frontal line // (FP).

All points on this line are equidistant from this plane. Consequently, the horizontal projections of all points on the line are at an equal distance from the ground line (LT). Therefore, the horizontal projection is also parallel to the (LT). The frontal line leaves only one trace, which is its horizontal trace. In space, the line itself and its frontal projection are parallel. Therefore, the frontal projection is represented in true form, to the true scale, and with true dimensions.

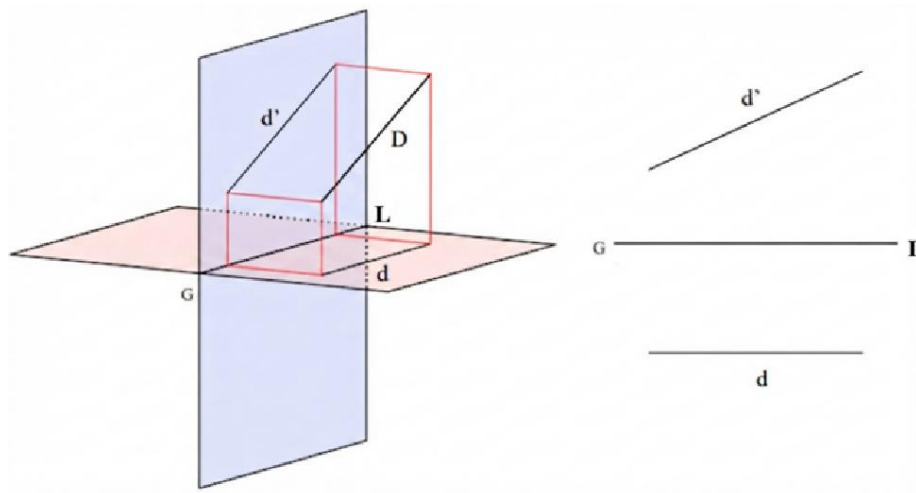


Figure II. 14: Projection of a frontal line

2.4.5. Horizontal-frontal line

Horizontal-frontal line // (HP) and (FP).

Each point on this line has the same side and distance. Its projections are parallel to the ground line. They are represented in true form, to the true scale, and with true dimensions. This line has no trace, either horizontal or frontal.

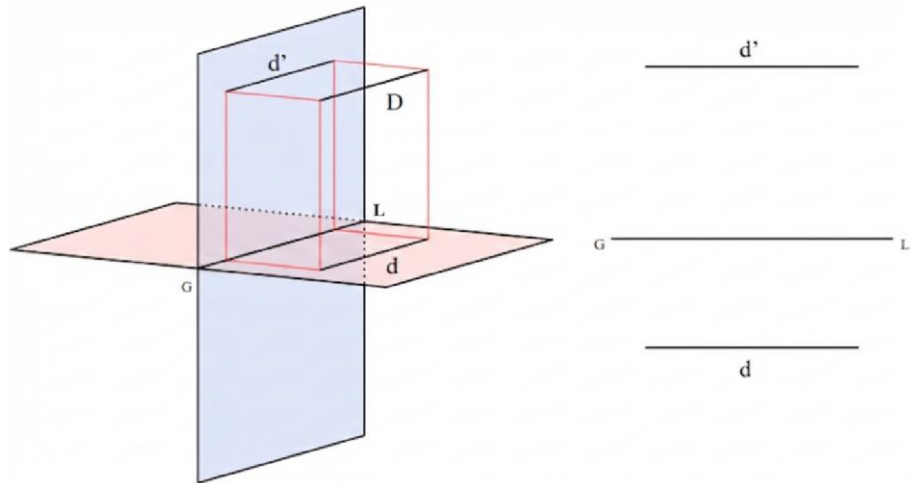


Figure II. 15: Projection of an horizontal-frontal line

2.4.6. Line belonging to a bisector plane

All points on such a line are at an equal distance from the projection planes.

- If this line lies in the first bisector, its horizontal and frontal projections are symmetrical with respect to the ground line (LT).

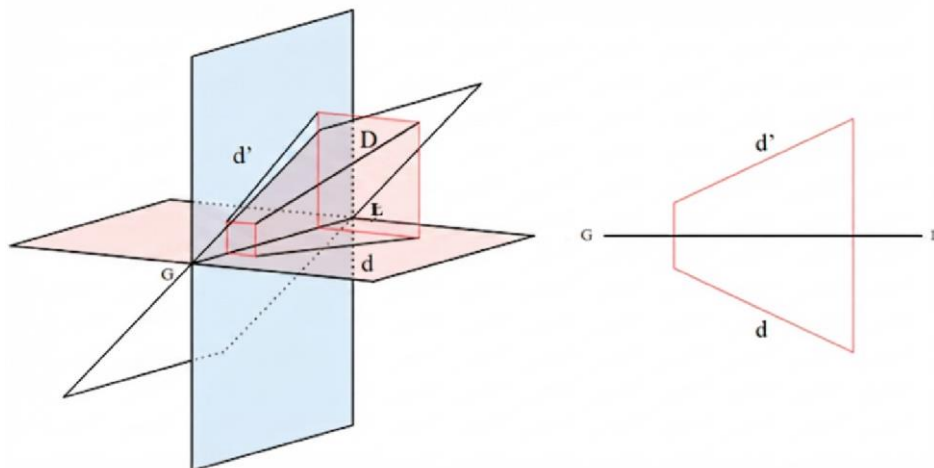


Figure II. 16: Projection of a line belonging to the 1st and 3rd bisector plane

- If the line belongs to the second bisector, its two projections overlap.

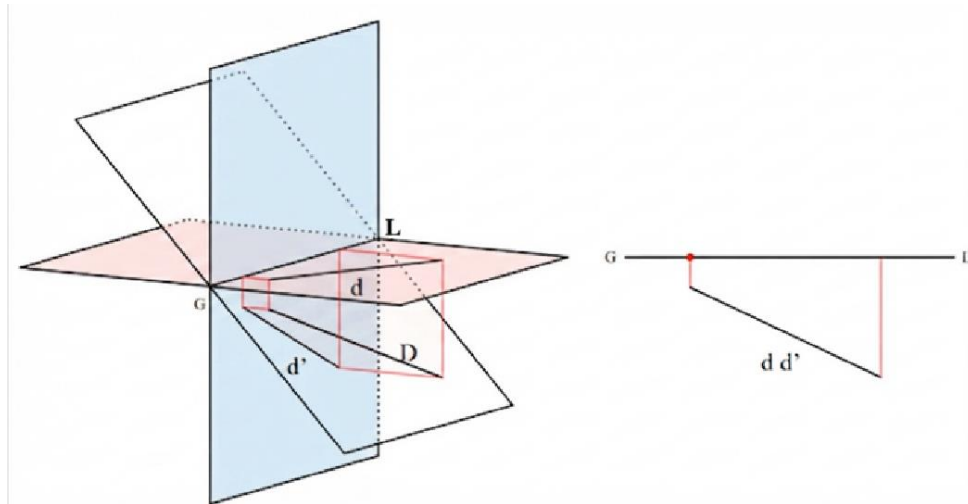


Figure II. 17: Projection of a line belonging to the 2nd and 4th bisector plane

2.4.7. Profil line

Profil line // (PP), and profil line $\perp$ (LT)

The two projections of such a line are indeed perpendicular to the ground line and aligned along the same projection line.

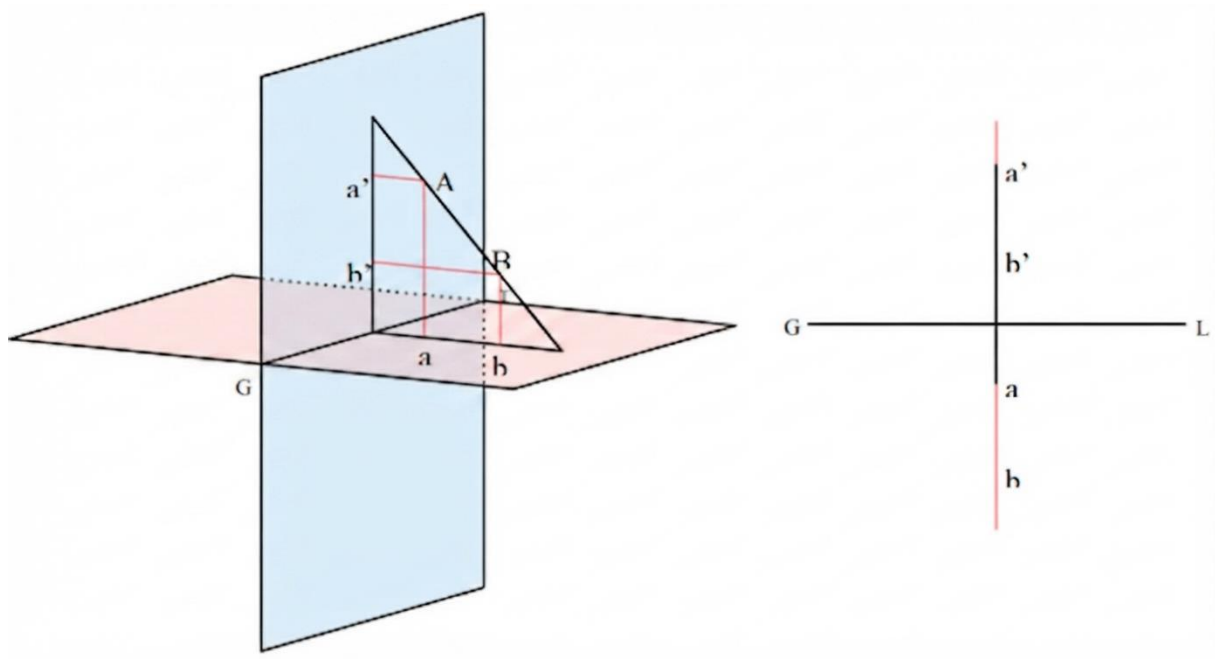


Figure II. 18: Projection of a profil line

NB: The vertical and line perpendicular to (FP) are two specific profile lines.

2.5. Representation of the point in a line

2.5.1. Mark a point on the line

Consider a line (D) whose horizontal and frontal projections are known, and let (m) be the horizontal projection of a point (M) on this line. The frontal projection (m') of this point necessarily belongs to the frontal projection (d') of the line (D) and is located on a projection

line from (m). This frontal projection (m') is therefore constructed graphically by raising the projection line from (m) and thus intercepting the frontal projection (d') of (D). Point (M) is then determined

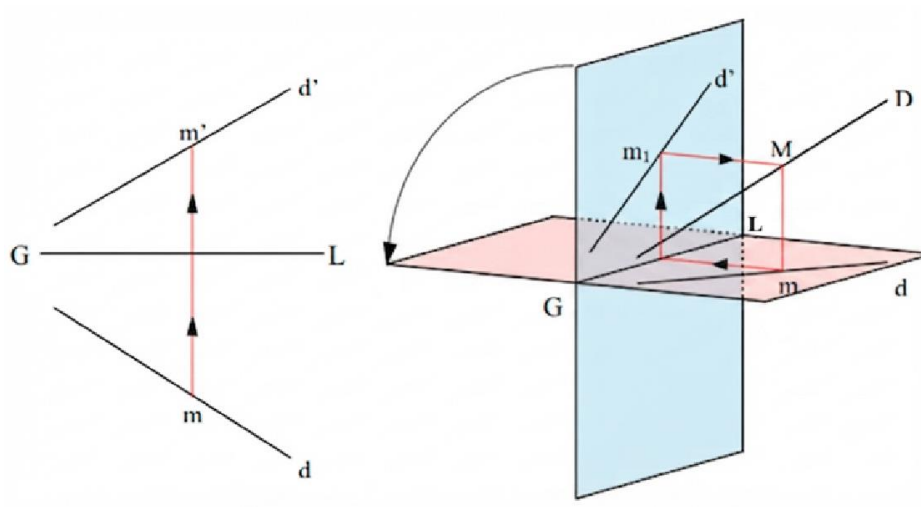


Figure II. 19: Mark a point on the line

2.5.2. Mark a point with a given side on the line

Let be the line (D) whose horizontal and frontal projections are known and let (c) be the side of the point sought. All the points with a side equal to (c) are projected frontally onto a line parallel to the ground line (LT), at a distance (c) from it.

This line intercepts the frontal projection (d') of the line (D) at point (m'), from which it is then sufficient to draw a projection line towards the horizontal projection (d) of the line (D). The point (M) belonging to (D) and with a side equal to (c) is thus determined.

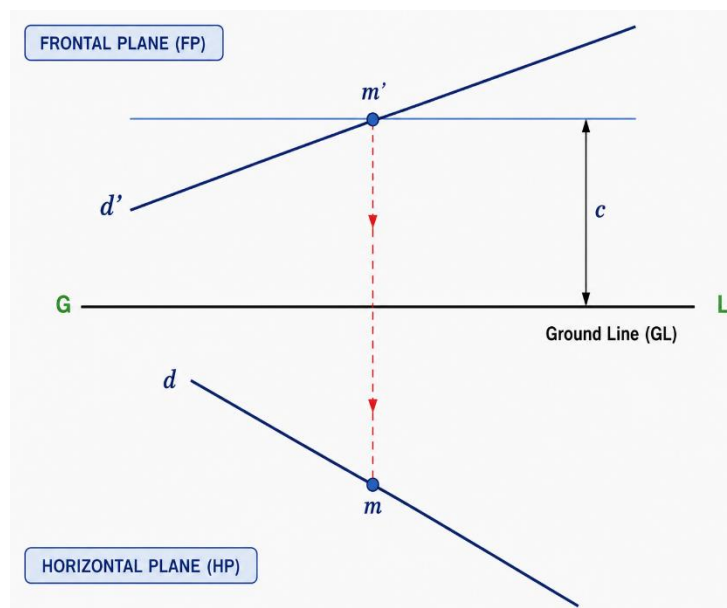


Figure II. 20: Mark a point with a given side on the line

The search for a point of given distance on a given line is treated in a similar way

2.6. Searching for the true length

2.6.1. Rotation method

For any line to be in true length, it must be parallel to the projection plane.

a- Exemple 1 : Transform the line AB by rotation into a frontal line.

Using point A, let's create a rotation such that point B is the same distance away as point A.

In horizontal projection: ($XA = XB$) so (μa) = (μbr)

In frontal projection: The sides of A and B remain unchanged, so (ZB) = (ZBr). The line ($a' b'r$) represents the true length of AB.

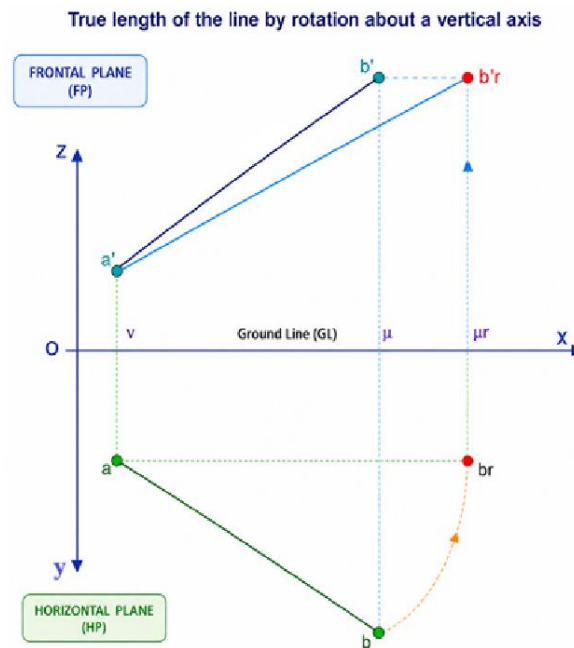


Figure II. 21: Transform the line AB by rotation into a frontal line

b- Exemple 2 : Transform the line AB by rotation into an horizontal line.

Using point B, let's create a rotation such that point A has the same side as point B.

In frontal projection: ($ZA = ZB$) so ($\mu a'r$) = (μb).

In horizontal projection: The distances of A and B remain unchanged, so (XA) = (XAr).
The line ($ar b$) represents the true length of AB.

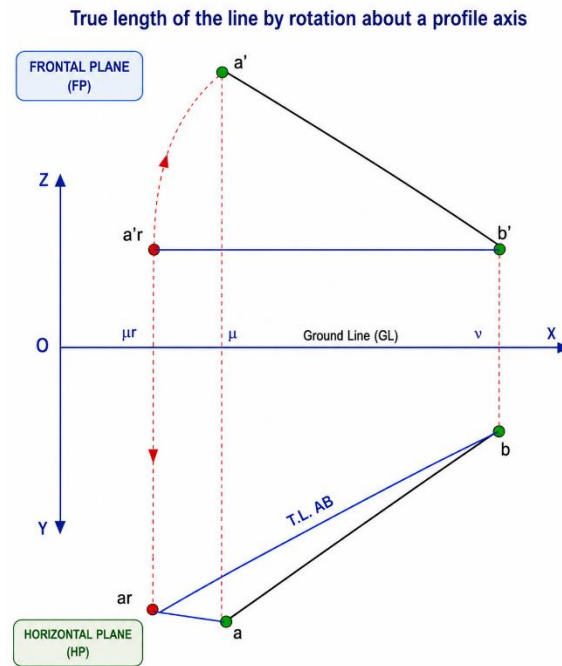


Figure II. 22: Transform the line AB by rotation into an horizontal line

2.6.2. The Pythagoras method

a- Exemple 1 : Sides and horizontal projection

The true length of a line is the hypotenuse of a right-angled triangle whose 2 right-angled sides are respectively equal to:

(ZB-ZA) is the difference of sides between A and B.

(a br) = (ab) is the horizontal projection of the line.

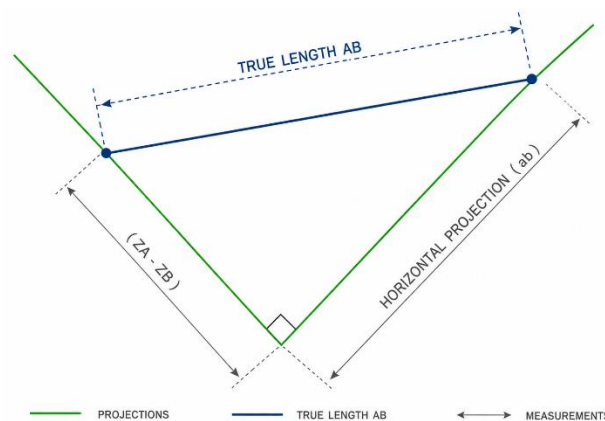


Figure II. 23: Pythagoras method; Sides and horizontal projection

b- Exemple 2 : Distances and frontal projection

The true length of a line is the hypotenuse of a right-angled triangle whose 2 sides of the right angle are respectively equal to:

(XB-XA) is the difference in distance between A and B.

$(a' b' r') = (a' b')$ is the frontal projection of the line.

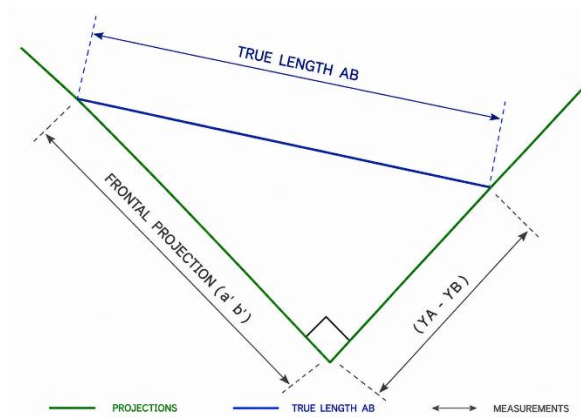


Figure II. 24: Pythagoras method; Distances and frontal projection

3. Plane

3.1. Definition

A plane can be defined in different ways:

Using 3 points that are not aligned.	Based on 1 point and a line that do not intersect.	Using 2 lines that intersect at a point.	Using 2 distinct parallel lines.
$P(A, B, C) / A \neq B \neq C$ Point P is defined by three non-collinear points A, B and C.	$P(A, d) / A \notin d$ Point P is defined by point A and line d, where A is not on d.	$P(m \times n = O)$ Point P is defined by the intersection of two lines m and n, which intersect at point O.	$P(m // n)$ Point P is defined by two parallel lines m and n, which are in the same plane.

Table II. 1: Plane Definition Methods

3.2. Arbitrary plane

It is any plane relative to the two projection planes (horizontal plane) and (frontal plane), and is neither parallel nor perpendicular to either (HP) or (FP).

In descriptive geometry, a plane is often characterised by the intersection of two intersecting lines. This method of characterisation is used in particular to determine the properties of a plane through its traces.

3.3. Trace of the plane

The traces of a plane are the lines formed by the intersection of that plane with the projection planes. The two lines that define the traces of a plane are the horizontal trace (P) and the frontal trace (Q) which intersect on the ground line at a point (α).

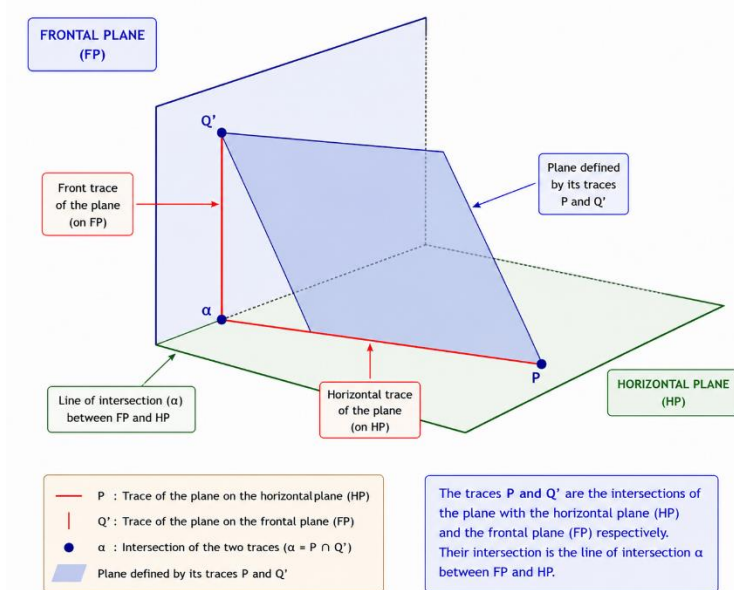


Figure II. 25: Trace of a plane

3.4. Particular planes

The planes considered remarkable are those that are parallel or perpendicular to the projection planes, as well as to the bisector planes.

3.4.1. Vertical plane

Vertical plane $\perp$ (HP).

(FT) (frontal trace) $\perp$ (LT)

(HT) = Horizontal projection.

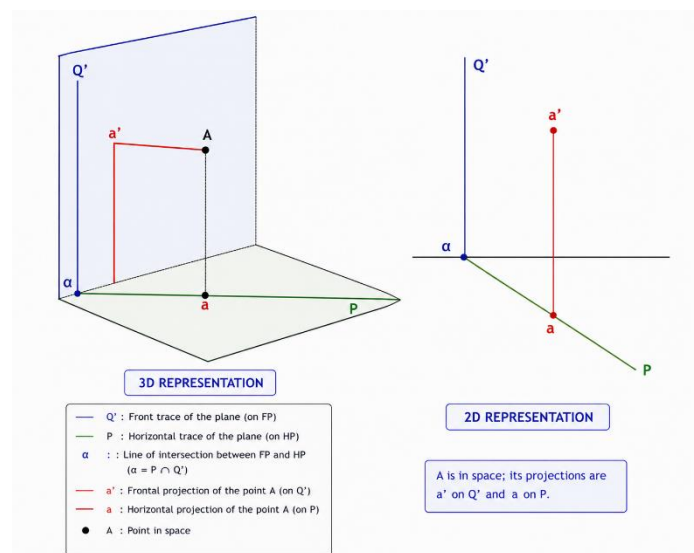


Figure II. 26: Representation of a vertical plane

NB: The frontal plane is a specific vertical plane.

3.4.2. Plane perpendicular to (FP)

Plane $\perp$ (FP)

(HT) $\perp$ (LT).

(FT) = frontal projection.

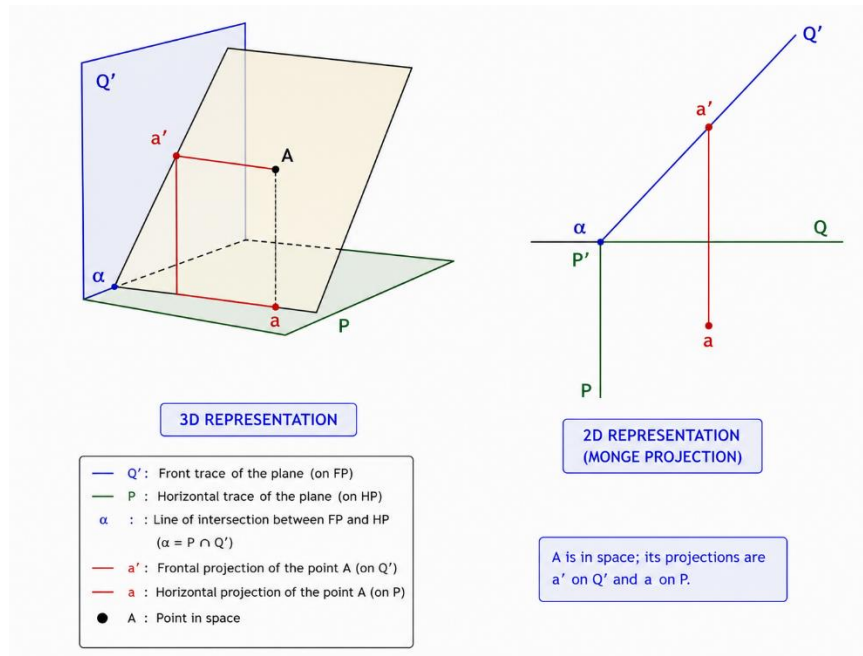


Figure II. 27: Representation of a plane perpendicular to (FP)

NB: The horizontal projection plane is a particular plane perpendicular to (FP).

3.4.3. Profil plane

Profil plane $\perp$ (FP), and profil plane $\perp$ (HP).

(HT) $\perp$ (LT), and (FT) $\perp$ (LT).

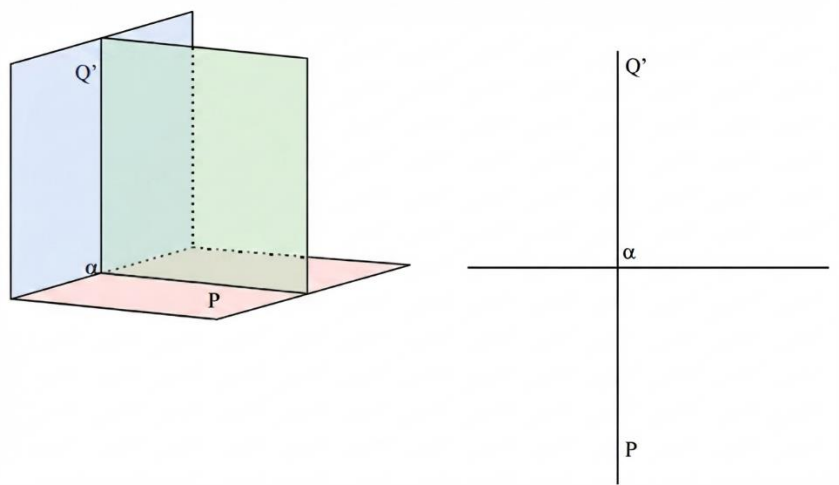


Figure II. 28: Representation of a profil plane

NB: The profile plane is a particular vertical plane.

3.4.4. Horizontal plane

Horizontal plane // (HP), and horizontal plane $\perp$ (FP)

(FT) // (LT).

This type of plane has only one trace, its front trace, It has not an horizontal trace.

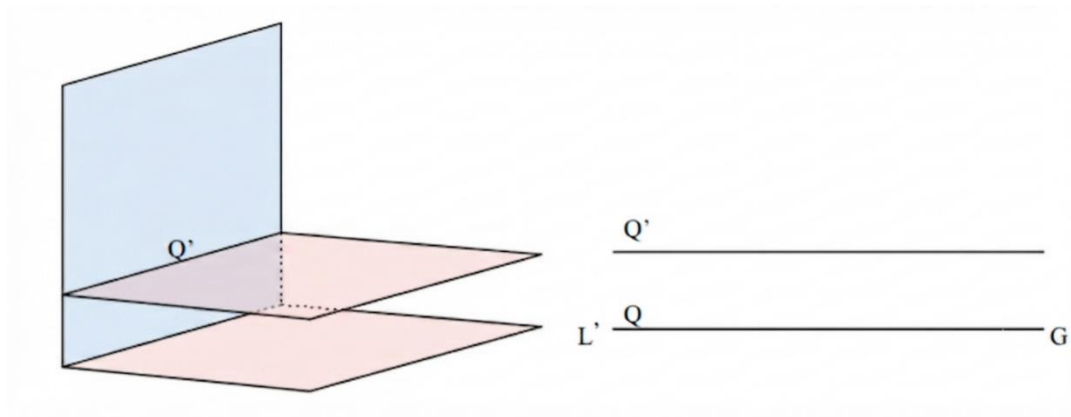


Figure II. 29: Representation of a horizontal plane

NB: Any horizontal plane is also a plane perpendicular to (FP).

3.4.5. Frontal plane

Frontal plane // (FP), and frontal plane $\perp$ (HP).

(HT) // (LT).

This plane has only one trace, which is its horizontal trace. However, it has no frontal trace.

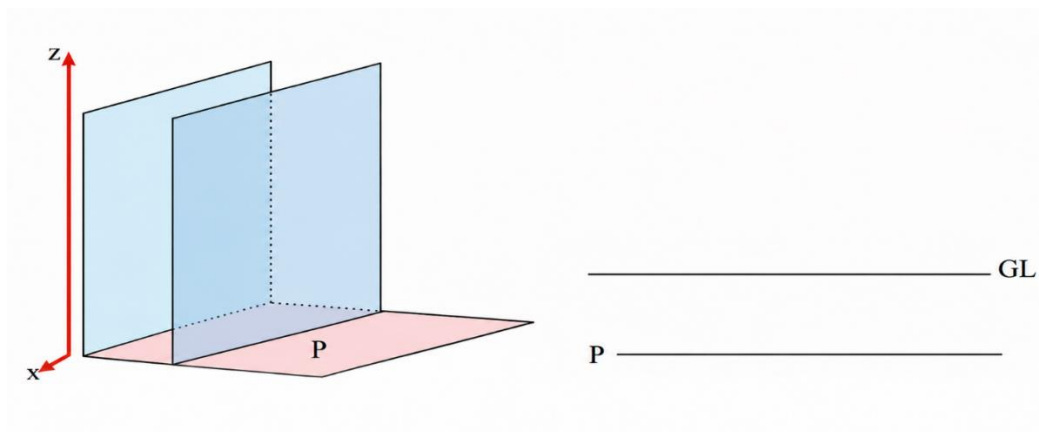


Figure II. 30: Representation of a frontal plane

3.4.6. Bisector plane

A plane passing through the ground line (LT) has its traces merged with the ground line. To define such a plane, it is necessary and sufficient to know one of its points (A). Any line joining this point and any point on the ground line (B) belongs to this plane.

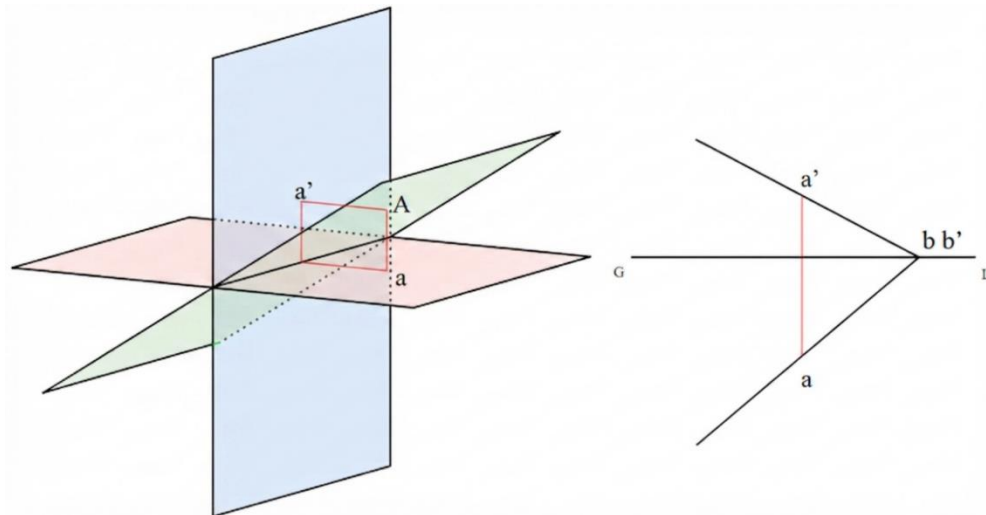


Figure II. 31: Representation of a bisector plane

a- 1st Bisector plane

A plane parallel to the 1st bisector has its horizontal and frontal traces merged and parallel to the ground line (LT).

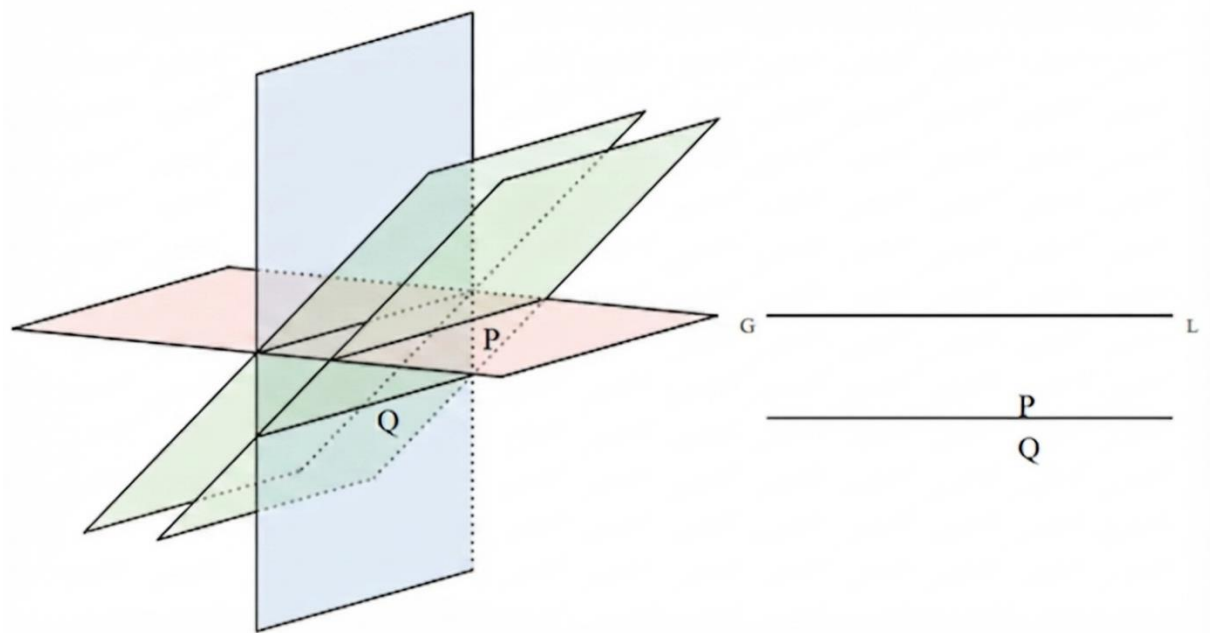


Figure II. 32: Representation of a plane parallel to the 1st bisector plane

b- 2nd Bisector plane

A plane parallel to the 2nd bisector has its horizontal and frontal traces parallel to the ground line (LT) and symmetrical to it.

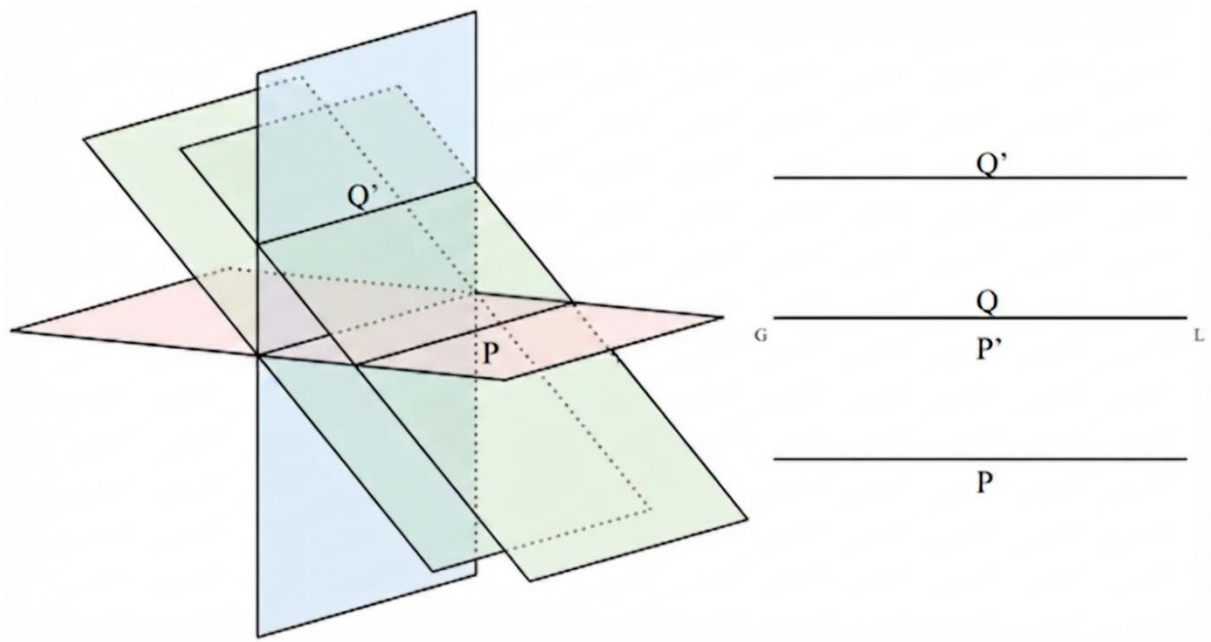


Figure II. 33: Representation of a plane parallel to the 2nd bisector plane

3.4.7. Plane parallel to the ground line (LT)

It is a plane parallel to the ground line (LT), and can be:

- Frontal plane: it has just an horizontal trace.
- Horizontal plane: it has just a frontal trace
- Bisector plane: it has two traces, horizontal and frontal, parallel to the (LT).

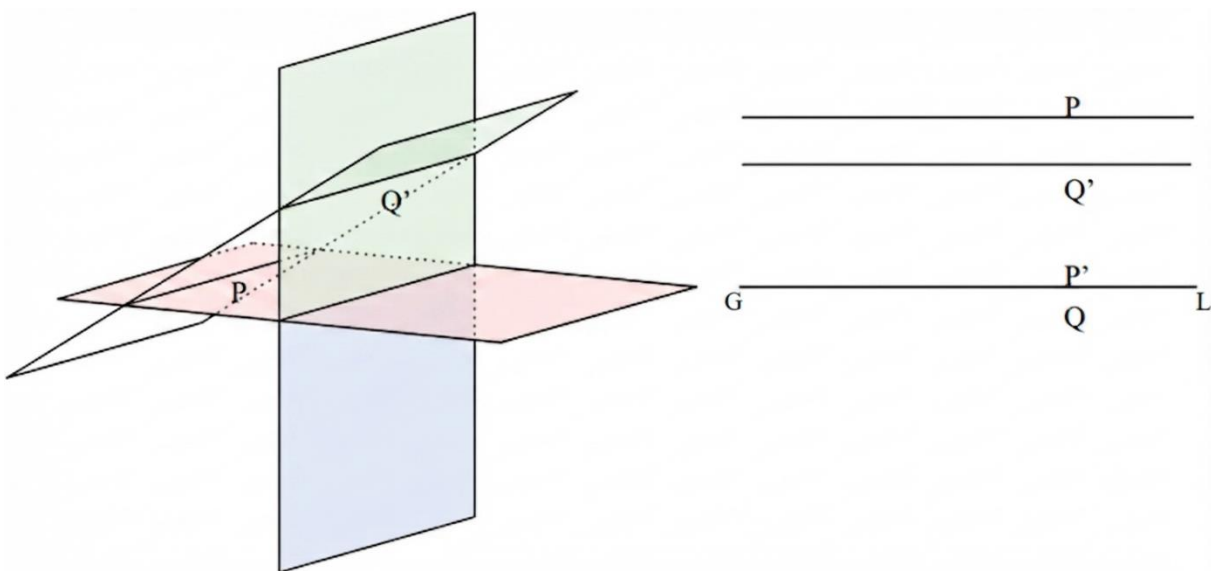


Figure II. 34: Representation of a plane parallel to the ground line (LT)

3.5. Principal lines of a plane

The principal lines of a plane are those lines in the plane which are parallel to the frontal plane or to the horizontal plane of projection, as well as those lines which are perpendicular to the frontal plane or to the horizontal plane of projection, i.e. the lines of greatest slope.

3.5.1. Horizontal of a plan

The lines in a plane that are parallel to the horizontal plane of projection are called the horizontals of a plane.

These lines are obtained by intersecting the plane with the horizontal planes.

The horizontal trace (P) of the plane is the horizontal line of zero side of this plane.

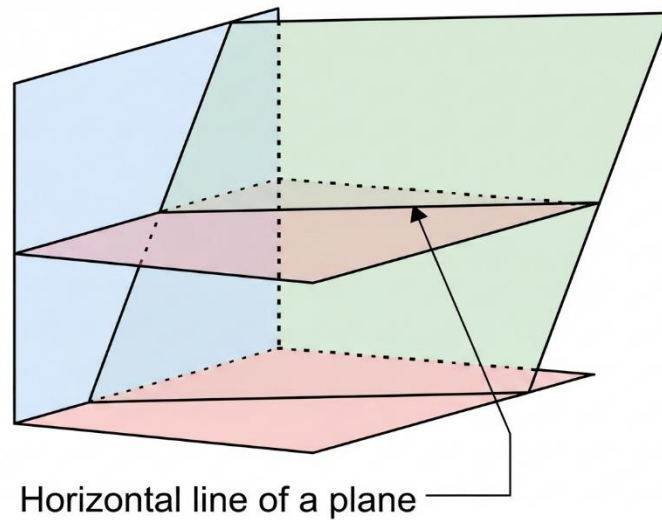


Figure II. 35: Horizontal of plane

3.5.2. Frontal of a plane

The lines in a plane that are parallel to the frontal plane of projection are called the frontal lines of a plane.

These lines are obtained by intersecting the plane with the frontal planes.

The frontal trace (Q') of the plane is the frontal line of zero distance from this plane.

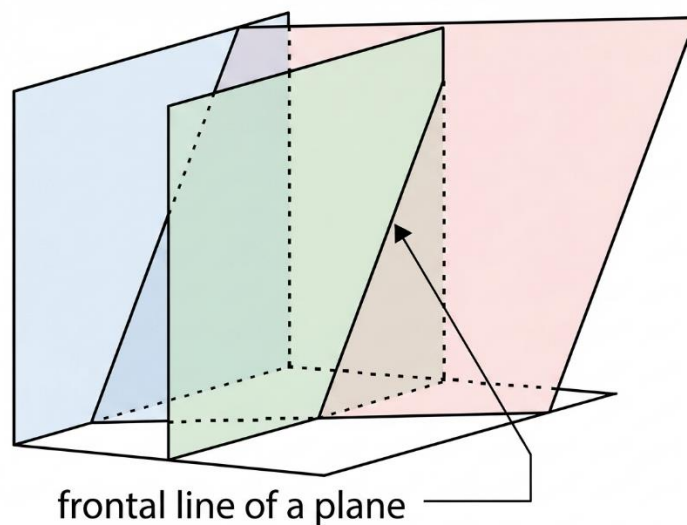


Figure II. 36: Frontal of a plane

3.5.3. Line of greatest slope of a plane

The lines of greatest slope of a plane with reference to a projection plane are the lines of this plane that form the greatest possible angle with this projection plane.

In descriptive geometry, a classic distinction is made between lines of greatest slope with the horizontal plane and lines of greatest slope with the frontal plane.

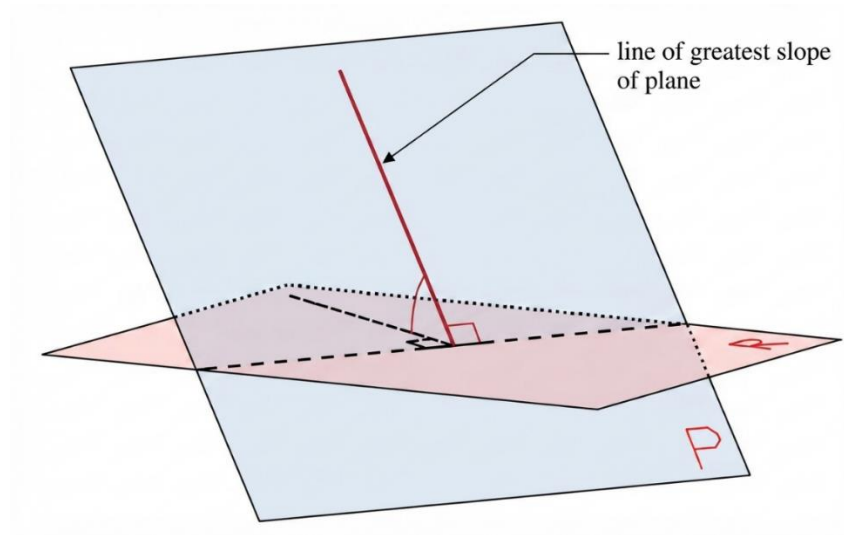


Figure II. 37: Line of greatest slope of a plane

Theorem: Let (P) and (R) be two planes intersecting along a line (D). The lines of greatest slope of the plane (P) relative to the reference plane (R) are the lines of (P) perpendicular to (D).

- **Traces of Line of greatest slope of a plane**

The horizontal trace of a plane represents the line of intersection of this plane with the horizontal projection plane.

Depending on the reference plane considered, the lines of greatest slope of the plane are therefore the lines perpendicular to the corresponding trace of the plane.

Furthermore, as the horizontal or frontal lines of the plane are parallel to the horizontal or frontal traces respectively, the lines of greatest slope of a plane are also perpendicular to the horizontal or frontal lines of this same plane.

A line of greatest slope is sufficient to fully characterise a plane. Knowing the line of greatest slope (G) with, for example, the horizontal plane, it is possible to construct its horizontal trace, i.e. point (A). We know that the horizontal trace of the plane, the line (P), passes through point (A) and is perpendicular to (G). Once constructed, this horizontal trace gives us the point (a) where this line intersects the ground line (LT). This point (a) also belongs to the front trace (Q') of the plane. All that remains is to construct the front trace of (G), i.e. point (B), to obtain a second point on this front trace of the plane.

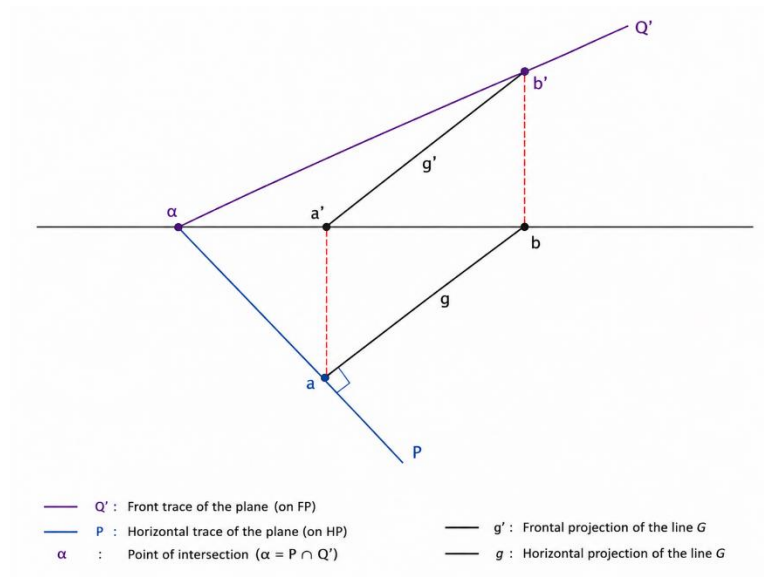


Figure II. 38: Traces of line of greatest slope of a plane

3.6. Construction to planes

3.6.1. Mark a point in a plane

Here, one of the projections of the point is known, and we are looking for its other projection, knowing that the point belongs to the plane. The problem can be broken down into two stages:

- Firstly, we need to construct a line passing through this point and belonging to the plane.
- The point is then recalled on the projections of the line.

Case of a plane defined by its traces (PaQ'):

This time, the frontal projection (d') of the plane line is known and we are looking for its horizontal projection (d). Let (A) be the horizontal trace of (D). This point belongs to the horizontal plane of projection as well as to the plane defined by (PaQ'). It therefore also belongs to the horizontal trace (P) of the plane. Similarly, the frontal trace (B) of (D) belongs to the frontal plane of projection as well as to the plane (PaQ'), and therefore to the frontal trace (Q') of this plane. These traces of the line (D) are known and therefore allow us to construct its horizontal projection.

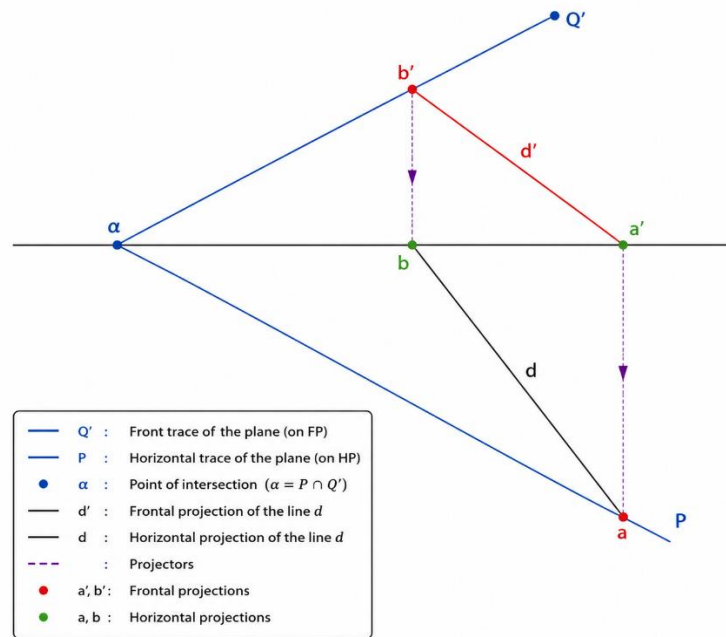


Figure II. 39: Mark a point in a plane

Conclusion:

The study of points, lines, and planes forms the cornerstone of descriptive geometry, as these elements define the structure of all spatial representations. Mastery of their properties and graphical representations enables a precise understanding of geometric relationships in three-dimensional space.

Throughout this chapter, the point has been examined as the fundamental element of geometry, with its various representations across different projection planes and spatial configurations. The line has been analyzed in its general and special forms, highlighting its role as a connector of points and its geometric behavior in space. Methods for determining true length and locating points on a line have reinforced the practical aspects of line representation.

The plane, as a more complex geometric entity, has been studied through its traces, orientations, and principal lines, allowing a deeper understanding of spatial orientation and structure. Construction techniques involving points within planes have further strengthened the ability to manipulate and interpret geometric configurations.

In conclusion, the concepts presented in this chapter provide the essential foundation for all subsequent studies in descriptive geometry. They develop spatial reasoning, precision in graphical construction, and analytical skills necessary for advanced applications in architecture, engineering, and technical design.

Chapter III Intersection of lines, planes, and volumes

Introduction

The study of intersections between lines, planes, and volumes is a central topic in descriptive geometry, as it allows the analysis of spatial relationships and the determination of common elements between geometric entities. These intersection principles are essential for understanding how different forms interact within three-dimensional space and for solving complex geometric construction problems in architecture and engineering.

This chapter introduces the fundamental concept of intersection, beginning with the simplest case of two concurrent lines, and progressively extending to more complex situations involving planes and solids. The different methods for determining the intersection of two planes are presented, whether they are defined by intersecting lines, by traces, or by mixed configurations.

The chapter also examines the intersection of a line with a plane, distinguishing between particular and arbitrary planes, and highlighting systematic construction methods for each case. These operations form the basis for spatial reasoning and graphical problem-solving in descriptive geometry.

In addition, the notions of parallelism and orthogonality are introduced as fundamental geometric relationships. The conditions for parallel lines, parallel planes, and combinations of lines and planes are analyzed in detail, followed by the study of perpendicularity between lines, planes, and mixed configurations. Through these concepts, students develop a structured approach to understanding spatial interactions and geometric constraints.

1. Intersections

1.1. Definition

- Intersection of Two Lines: The intersection of two lines corresponds to the point where these lines cross in space, thereby defining a common point.
- Intersection of a Line and a Plane: When a line intersects a plane in descriptive geometry, their intersection results in either a point (if the line intersects the plane at a single point).
- Intersection of Two Planes: In descriptive geometry, the intersection of two planes forms a line. The planes can either intersect to create a line.

1.2. Intersection of Two Lines (Concurrent Lines)

To ensure that two lines are concurrent (intersecting), it is necessary and sufficient that they share a common point. The like-named projections of these two lines intersect on a common projection line.

Let (D) and (L) be two lines in space: $(D) \cap (L) = M$:

- $(d) \cap (l) = m$ (horizontal projection of point M).
- $(d') \cap (l') = m'$ (frontal projection of point M).
- It is necessary for m and m' to lay on the same projection line.

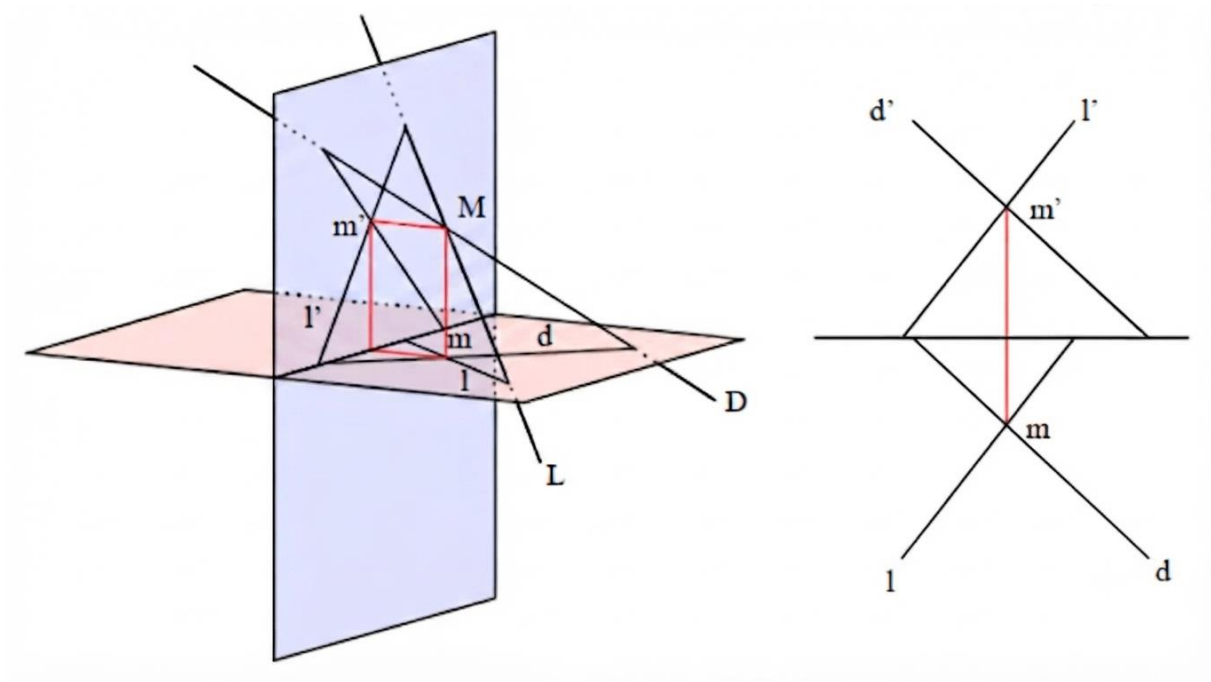


Figure III. 1: Intersection of Two Lines

The two lines (D) and (Δ) are neither concurrent nor parallel; they are referred to as non-coplanar.

❖ **Example of Two Non-Coplanar Lines:**

Consider (S) and (T) as two lines in space:

- A and D are two points on line (T).
- B and C are two points on line (S).

The lines (S) and (T) are non-coplanar.

- Distance of C = Distance of D
- side of C $\neq$ side of D.

Thus, points C and D are different (no intersection points between lines S and T).

- Distance of A $\neq$ Distance of B .
- side of A = side of B.

Thus, points A and B are different (no intersection points between lines S and T).

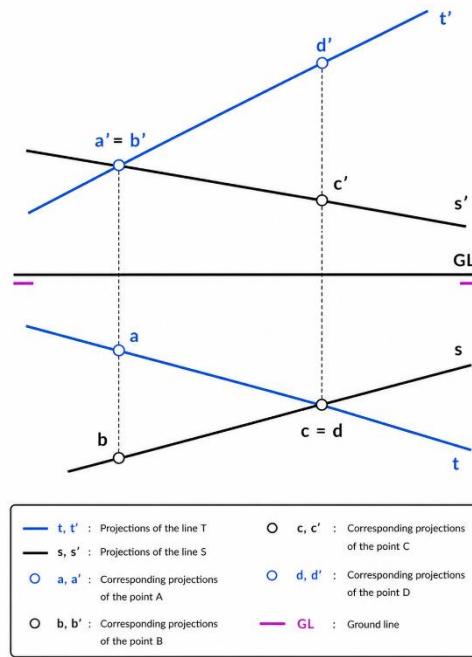


Figure III. 2: Non-Coplanar Lines

1.3. Intersection of two planes

The intersection of two planes results in either a line or is null. If two planes intersect, it occurs along a line, and it is sufficient to determine two points on this line to characterize it.

1.3.1. Two planes defined by two intersecting lines

So, consider a plane $(P) = (D) \cap (E) = A$. Another plane $(V) = (F) \cap (G) = B$.

- $(H1)$ and $(H2)$ are two auxiliary horizontal planes. $(H1) \cap (P) = (U)$.
- $(H1) \cap (V) = (Y)$.
- $(H2) \cap (P) = (T)$.
- $(H2) \cap (V) = (X)$.
- Thus, $(I) = (P) \cap (V)$.

(I) is determined by the intersection point of the lines (T) and (X) and the intersection point of the lines (U) and (Y) .

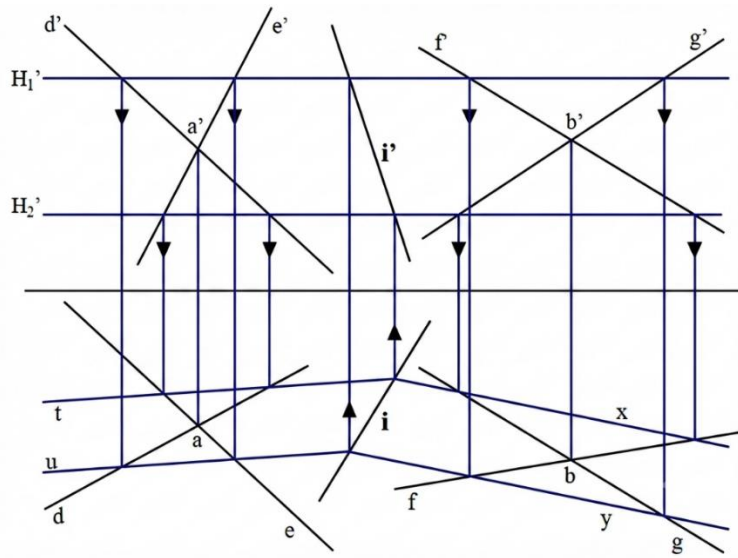


Figure III. 3: Intersection of two planes defined by two intersecting lines

1.3.2. Two planes defined by their traces

Consider two planes $(P\alpha Q')$ et $(T\beta R')$ defined by their traces:

$$(P\alpha Q') \cap (T\beta R') = (D).$$

$(D) \in (P\alpha Q')$:

- $THD \in (P\alpha) \Rightarrow thd \in (\alpha p)$ and $thd' \in (LT)$.
- $TFD \in (\alpha'Q')$. $\Rightarrow tfd \in (LT)$ and $tfd' \in (\alpha'q')$.

$(D) \in (R\beta R')$:

- $THD \in (T\beta) \Rightarrow thd \in (\beta t)$ and $thd' \in (\beta't')$.
- $TFD \in (\beta'R') \Rightarrow tfd \in (\beta r)$ and $tfd' \in (\beta'r')$.

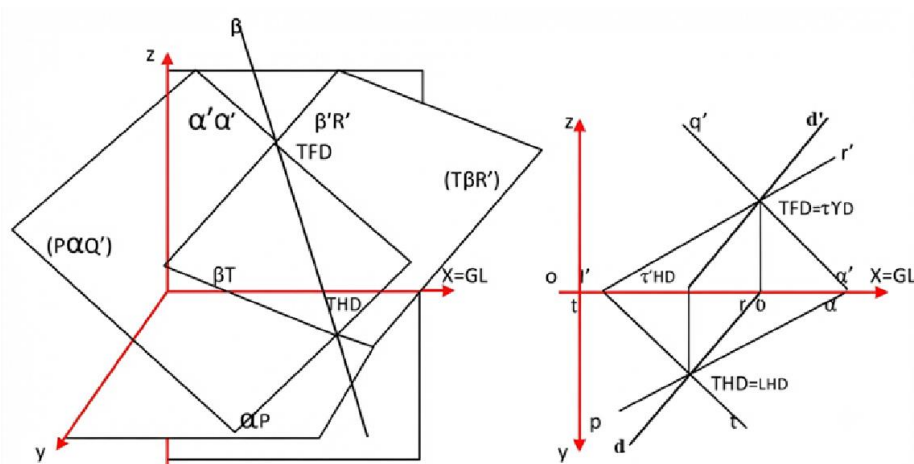


Figure III. 4: Intersection of two planes defined by their traces

1.3.3. Two planes defined, one by its traces and the other by two intersecting lines

Consider two planes:

$(P \cap Q')$ is represented by their traces.

$(T \cap R')$ is represented by two lines (D) and (Δ) .

$(P \cap Q') \cap (P \cap Q') = (L)$.

(D) and $(\Delta) \cap (T \cap R') = A$ and B .

The points A and B belong to the plane (L) . Therefore, to represent (L) , it is sufficient to represent the points A and B .

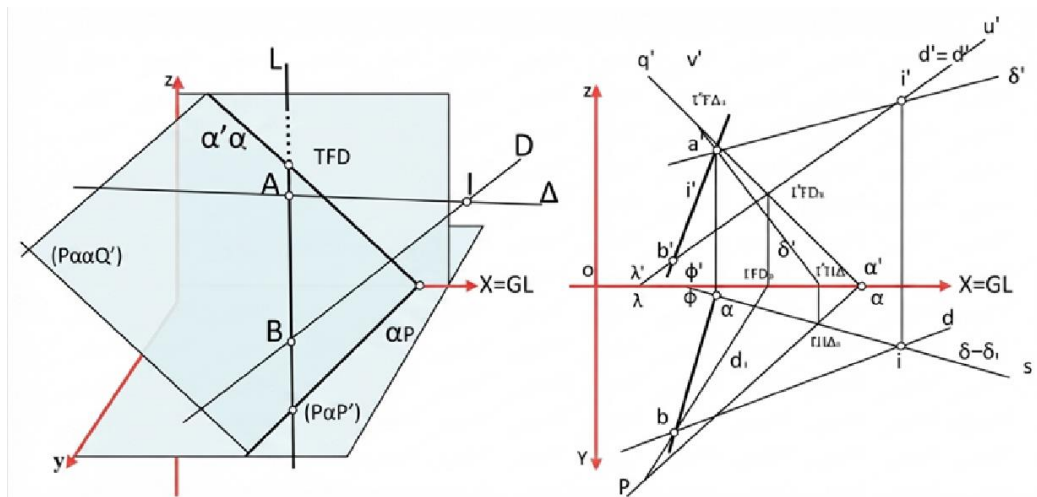


Figure III. 5: Intersection of two planes defined, one by its traces and the other by two intersecting lines

1.4. Intersection of a line and a plane

1.4.1. Intersection of a line and a particular plane

a. Horizontal plane:

The construction of the intersection of a line and an horizontal plane is immediate in frontal projection, as all points of an horizontal plane project frontally onto its frontal trace. It only remains to recall the frontal projection of the intersection point onto the horizontal projection of the line.

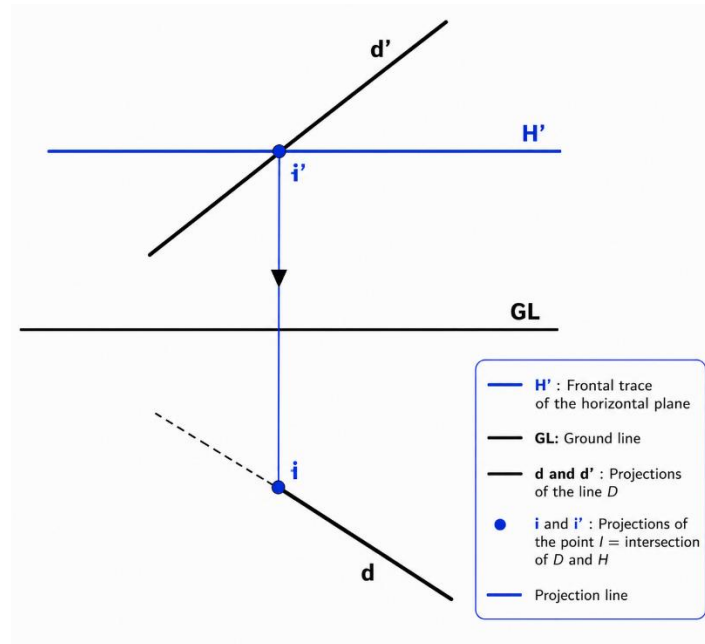


Figure III. 6: Intersection of a line and an horizontal plane

b. Frontal plane:

Here, the intersection is immediate in horizontal projection since all points of the plane (F) project onto its horizontal projection.

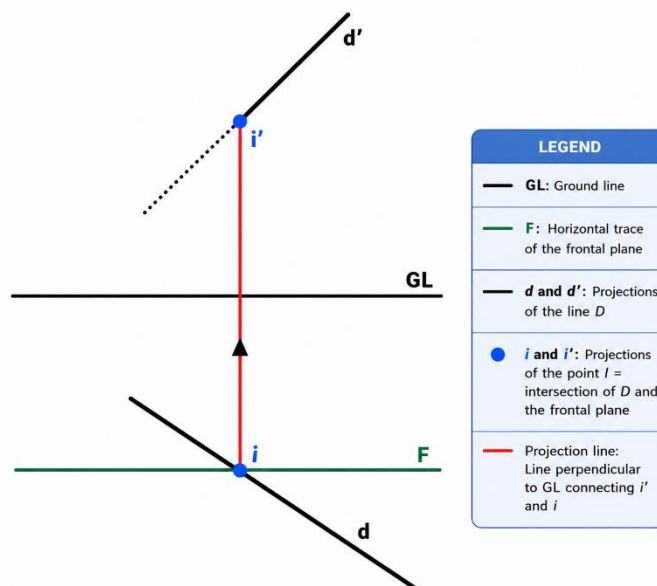


Figure III. 7: Intersection of a line and a frontal plane

c. Vertical plane:

All points of the vertical plane (PaQ') project onto its horizontal projection. Once again, the construction of the intersection (I) with the line (D) is immediate in horizontal projection.

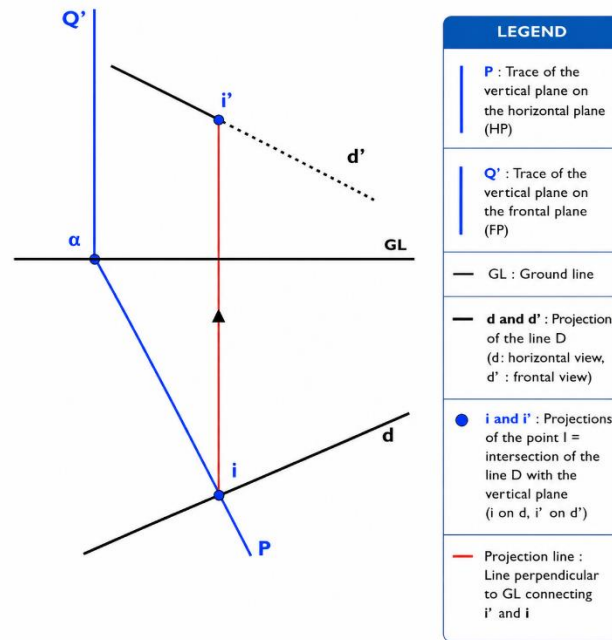


Figure III. 8: Intersection of a line and a vertical plane

d. Plane perpendicular to (FP):

The intersection between a plane (PaQ') and a line (D) is immediate in frontal projection.

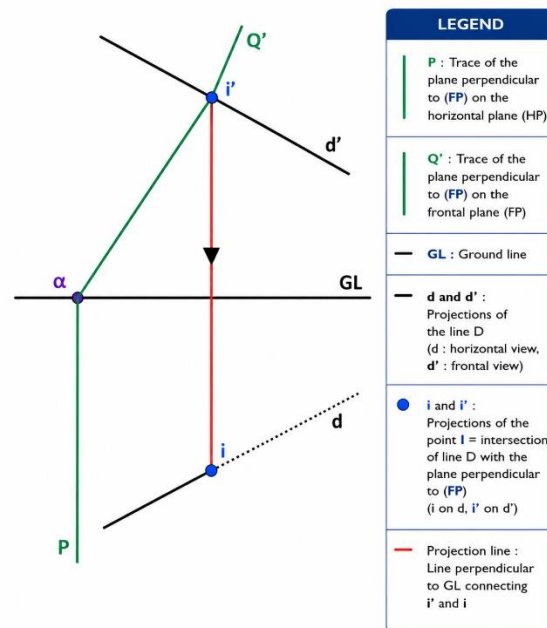


Figure III. 9: Intersection of a line and a plane perpendicular to (FP)

1.4.2. Intersection of a line and an arbitrary plane

To locate the point of intersection between a line and a plane, geometry recommends the following operations:

- Introduce an auxiliary plane through the line (the auxiliary planes being the vertical plane and the plane perpendicular to FP).

- Find the intersection line of the two planes (the given plane and the chosen auxiliary plane).
- The intersection of the given line with the intersection line of the two planes is the sought-after point of intersection.

Consider a line (D) and a plane (P α Q'), two planar figures in space:

The chosen auxiliary plane passing through the line (D) is the vertical plane (T β R').

- (P α Q') $\cap$ (T β R') = (Δ).
- (D) $\cap$ (Δ) = A.

Therefore, A is the sought-after point of intersection between the line (D) and the plane (P α Q').

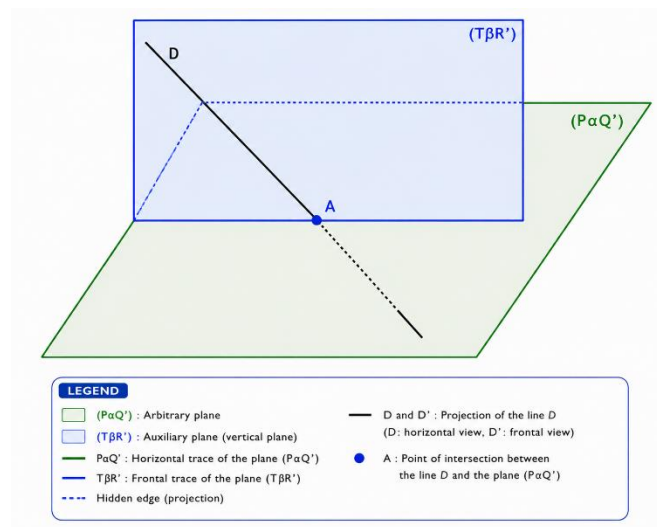


Figure III. 10: Intersection of a line and an arbitrary plane

a. Plane defined by two intersecting lines:

Let (R) be a plane defined by two intersecting lines (D) and (Δ), and (L) be a line given by its two projections (l) and (l').

- (D) $\cap$ (Δ) = A.
- (R) $\cap$ (L) = I (the sought-after point of intersection).
- Introduce a vertical plane (P α Q') through the line (L).
- (D) $\cap$ (P α Q') = C
- (Δ) $\cap$ (P α Q') = B
- (BC) $\cap$ (L) = (R) $\cap$ (L) = I $\Rightarrow$ i is the horizontal projection of point I, i' is the frontal projection of point I (the sought-after point).

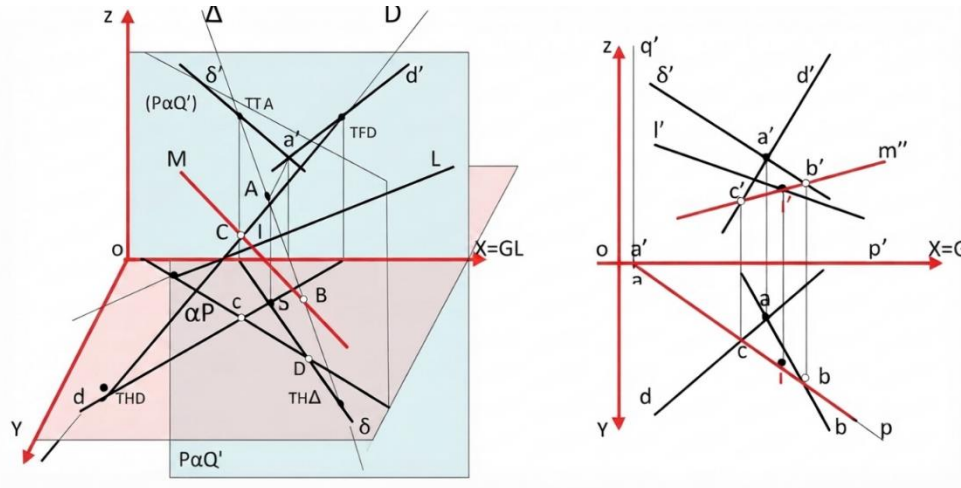


Figure III. 11: Intersection of a line and an arbitrary plane defined by two intersecting line

b. Plane defined by its traces:

Let $(P\alpha Q')$ be a plane defined by its traces, and (D) be a line.

- $(P\alpha Q') \cap (D) = A$ (the sought-after point of intersection).
- Introduce a vertical plane $(T\beta R')$ through the line (D) .
- $(P\alpha Q') \cap (T\beta R') = (\Delta)$.
- $(p\alpha) \cap (t\beta) = th\Delta$, recall $th\Delta$ on the (LT) , obtaining $th'\Delta$.
- $(\alpha'q') \cap (\beta'r') = tf\Delta$, recall $tf\Delta$ on the (LT) , obtaining $tf'\Delta$.
- Connect $th\Delta$ and $tf\Delta \Rightarrow (\delta)$ (horizontal projection of (Δ)).
- Connect $th'\Delta$ and $tf'\Delta \Rightarrow (\delta')$ (frontal projection of (Δ)).
- $(\Delta) \cap (D) = A$ (the sought-after point of intersection).

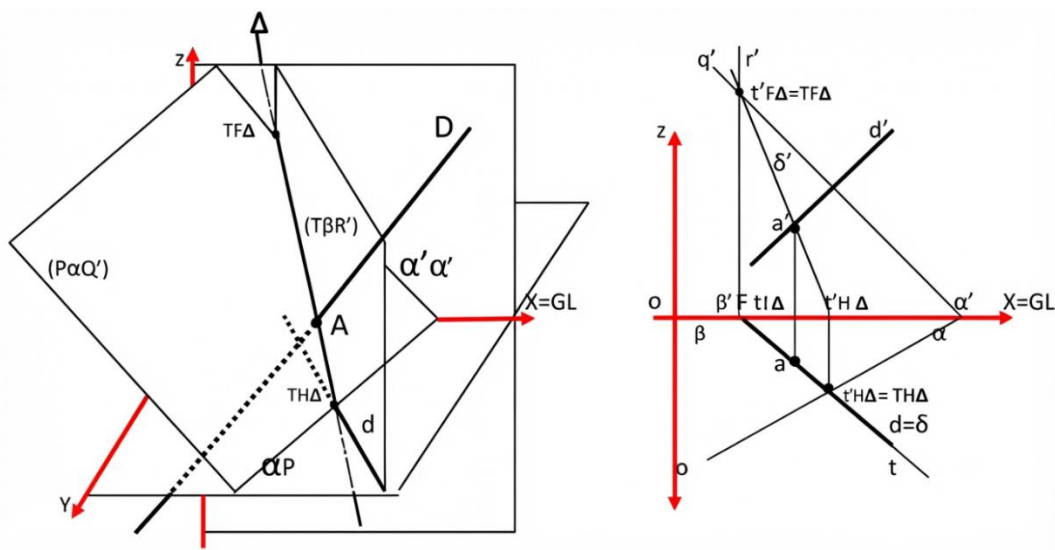


Figure III. 12: Intersection of a line and an arbitrary plane defined by its traces

2. Parallel

2.1. Parallel of two lines

Two parallel lines are two coplanar non-concurrent lines.

With the exception of profile lines, two parallel lines have their corresponding projections also parallel.

Projections of the same name can be coincident.

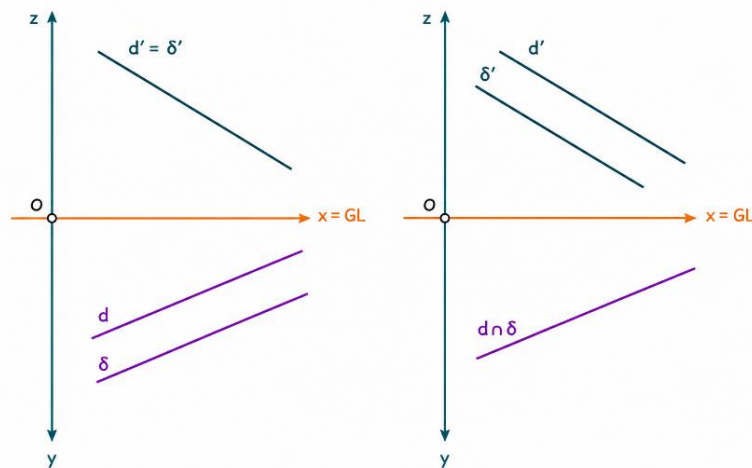


Figure III. 13: Parallel of two lines

2.2. Parallel of two planes

Two planes are parallel if one contains two intersecting lines that are parallel to the other.

2.2.1. Planes defined by their traces

Except for two planes parallel to the ground line, two parallel planes have their traces of the same name parallel.

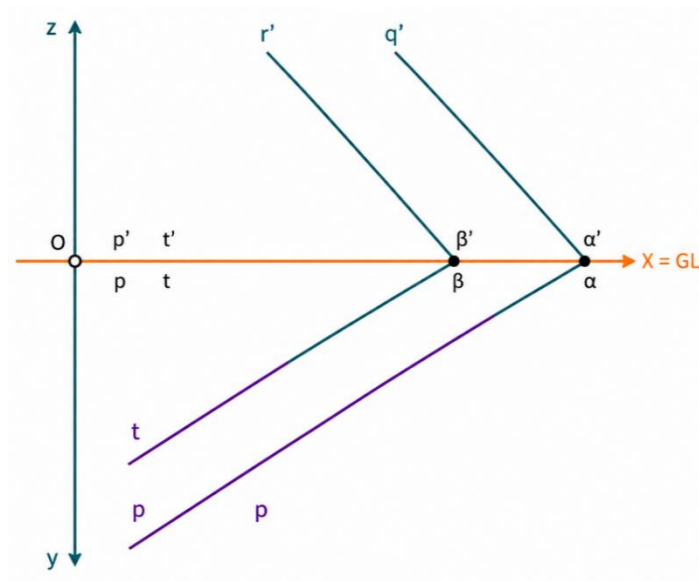


Figure III. 14: Parallel of two planes defined by their traces

2.2.2. Plane defined by two intersecting lines

Let $(P\alpha Q')$ and $(T\beta R')$ be two planes:

$$(P\alpha Q') = (D) \cap (\Delta).$$

$$(T\beta R') = (D1) \cap (\Delta1).$$

- $(D) // (D1) \Rightarrow (d) // (d1)$ et $(d') // (d1')$.
- $(\Delta) // (\Delta1) \Rightarrow (\delta) // (\delta1)$ et $(\delta') // (\delta'1)$.

Therefore; $(P\alpha Q') // (T\beta R')$.

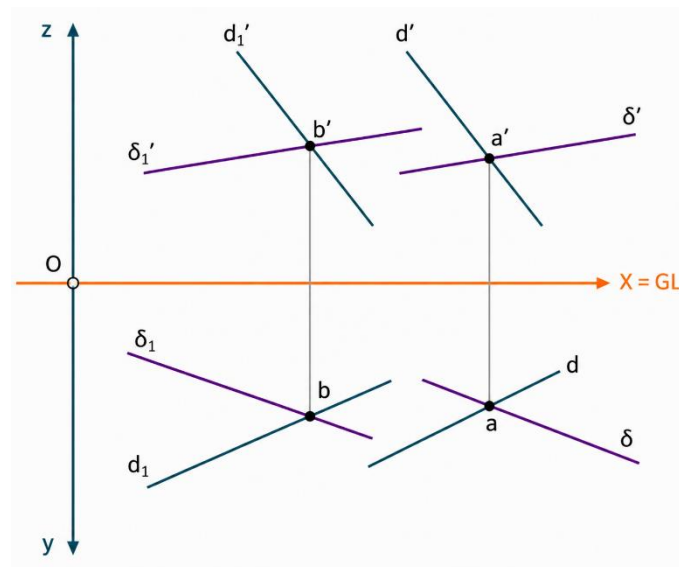


Figure III. 15: Parallel of two planes defined by two intersecting lines

2.3. Parallel of a line and a plane

A line is parallel to a given plane if there exists a line in the plane parallel to the given line.

2.3.1. Plane defined by its traces

$(P\alpha Q')$ is a plane represented by its traces.

$$(\Delta) \in (P\alpha Q').$$

For (D) to be parallel to $(P\alpha Q')$, it is necessary that:

$$A \in (D).$$

From a (horizontal projection of point A), draw (d) , knowing that $(d) // (\delta)$.

From a (horizontal projection of point A), draw a (frontal projection of point A), $\Rightarrow (d') // (\delta')$.

So; $(D) // (P\alpha Q')$.

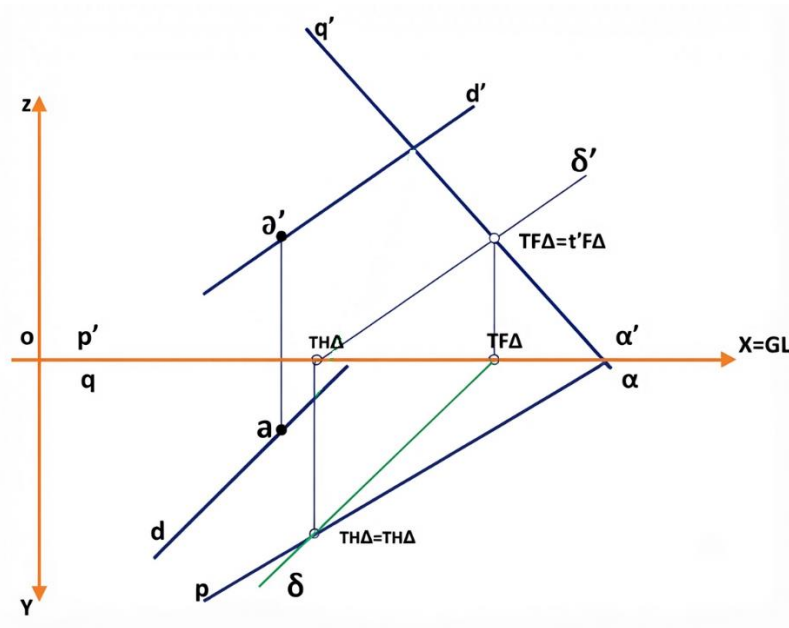


Figure III. 16: Parallel of a line and a plane defined by its traces

2.3.2. Plane defined by two intersecting lines

The plane is represented by two intersecting lines (Δ) and (L).

It involves drawing through point A the line (D) parallel to one of the lines (Δ) or (L).

- Draw $a \Rightarrow (d) \parallel (\delta)$.
- Draw $a' \Rightarrow (d') \parallel (\delta')$.

Or:

- Draw $a \Rightarrow (d) \parallel (l)$.
- Draw $a' \Rightarrow (d') \parallel (l')$.

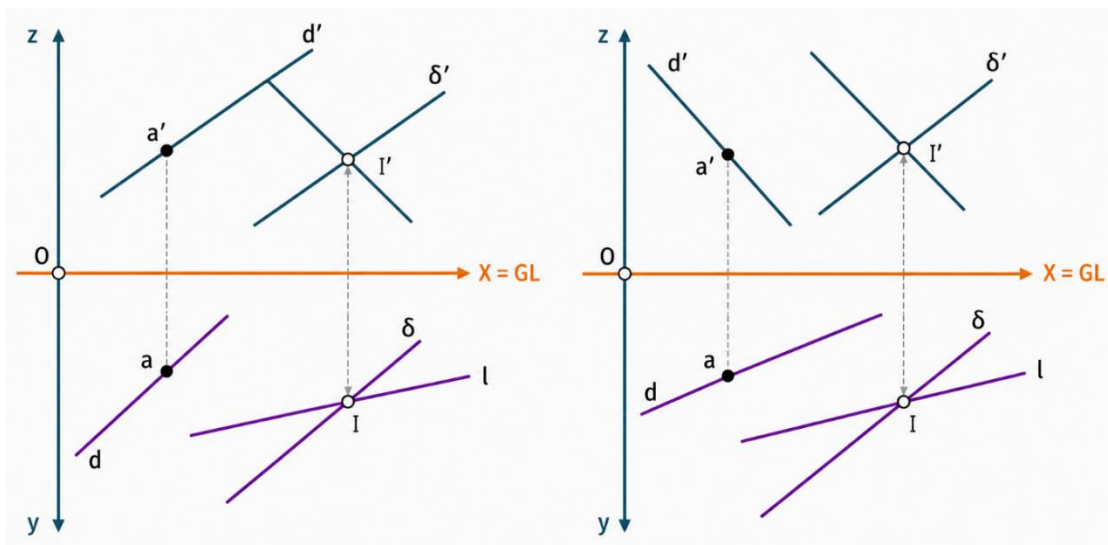


Figure III. 17: Parallel of a line and a plane defined by two intersecting lines

3. Orthogonality

3.1. Orthogonality of two lines

Two perpendicular lines are coplanar.

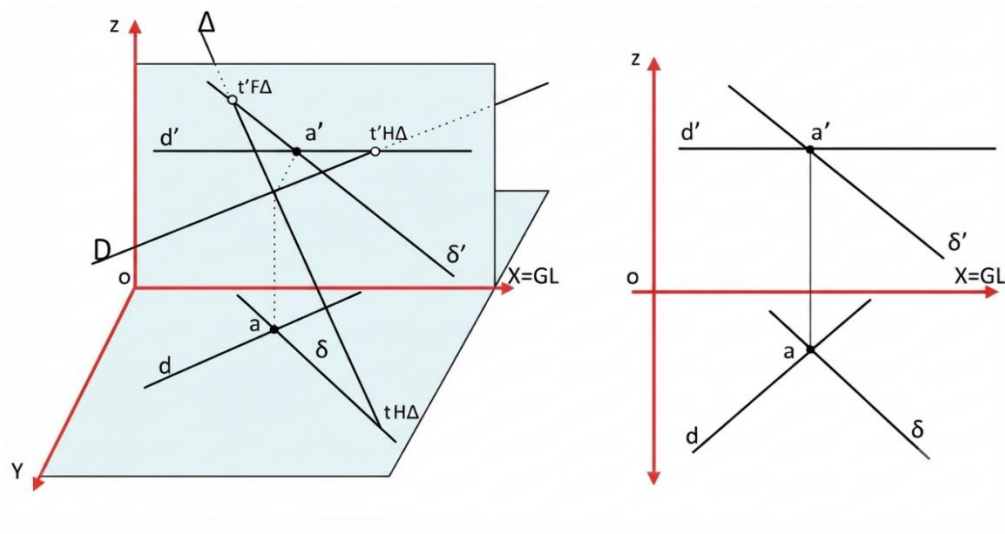


Figure III. 18: Orthogonality of two lines

3.2. Orthogonality of two planes

Two planes are perpendicular when one contains a line perpendicular to the other.

- Let $(P\alpha Q')$ be a plane defined by its traces.
- Let $(T\beta R')$ be another plane defined by its traces.
- Let A be a point in $(T\beta R')$.

For $(T\beta R')$ to be perpendicular to $(P\alpha Q')$, it is necessary to:

Draw a line (D) perpendicular to $(P\alpha Q')$ through point A.

From point A, draw (d) perpendicular to $(P\alpha)$.

From point A', draw (d') perpendicular to $(\alpha'Q')$.

Represent the two traces of the line: (th, th') and (tf, tf') .

$(T\beta R')$ is a plane, where $(t\beta)$ passes through (th) , and $(\beta'R')$ passes through (tf') .

$\Rightarrow (T\beta R') \perp (P\alpha Q')$.

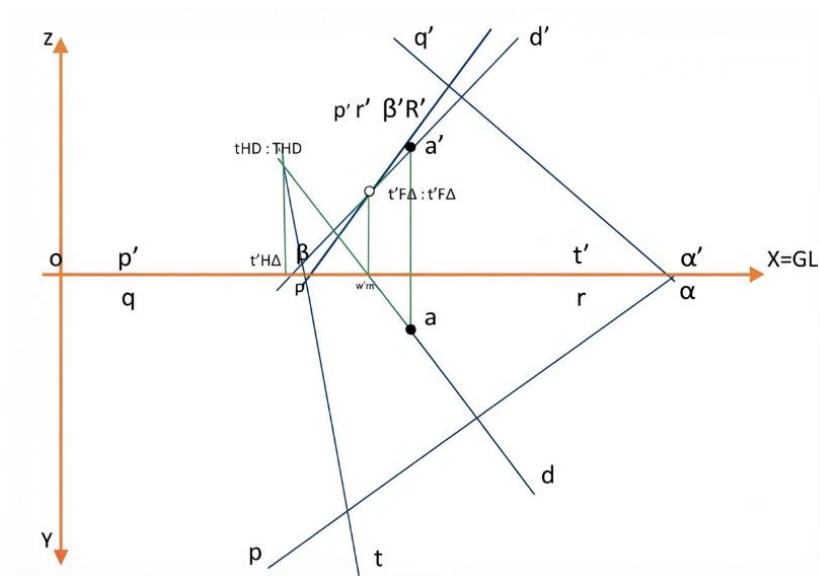


Figure III. 19: Orthogonality of two planes

3.3. Orthogonality of a line and a plane

A line is orthogonal to a plane if it is orthogonal to two intersecting lines in the plane.

If a line is orthogonal to a plane, it is orthogonal to all lines in that plane.

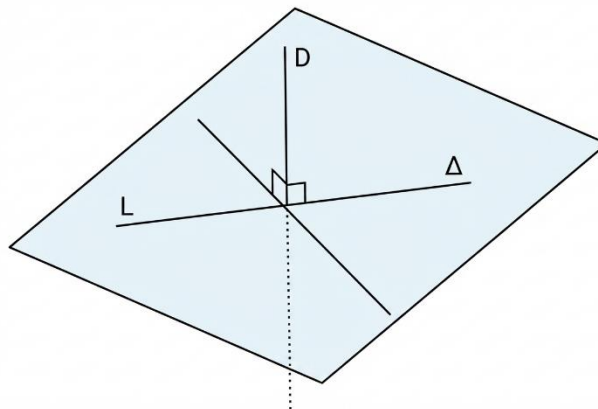


Figure III. 20: Orthogonality of a line and a plane

3.3.1. Plane defined by its traces

For a line to be perpendicular to a plane, it is necessary and generally sufficient:

- The horizontal projection of the line must be perpendicular to the horizontal projection of any horizontal line in the plane. (Reminder: the horizontal trace of a plane is one of the horizontals of that plane).
- The front projection of the line must be perpendicular to the front projection of any front line of this plane. (Reminder: the front trace of a plane is one of the fronts of that plane).

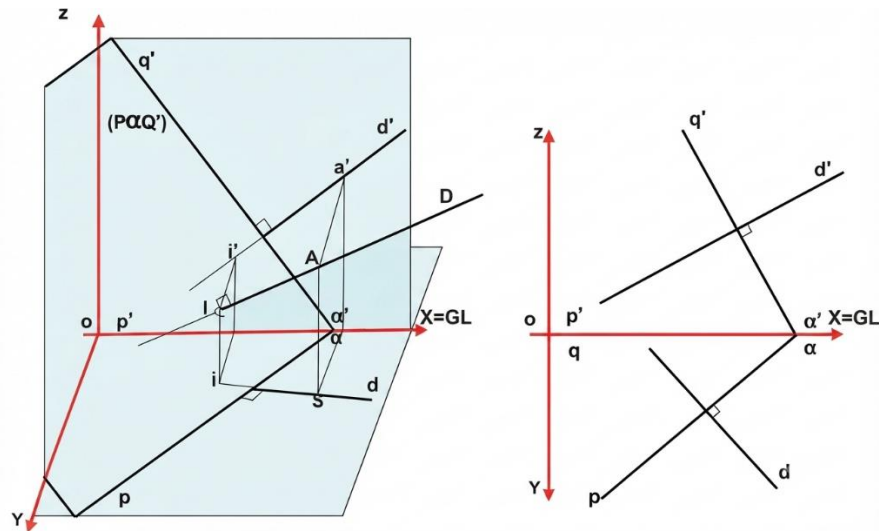


Figure III. 21: Orthogonality of a line and a plane defined by its traces

3.3.2. Plane defined by two intersecting lines

For a plane defined by two intersecting lines $(D) \cap (\Delta)$:

To be perpendicular to a line (L) , it is necessary that:

- Its horizontal projection is perpendicular to the horizontal projection of the horizontal line of the plane, $\Rightarrow (l) \perp (h)$.
- Its front projection is perpendicular to the front projection of the front line of the plane, $\Rightarrow (l') \perp (f')$.

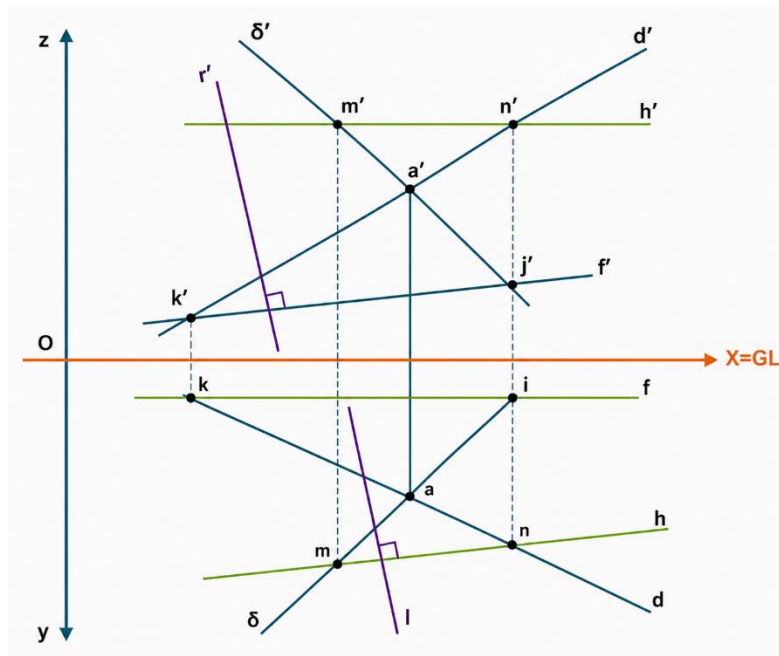


Figure III. 22: Orthogonality of a line and a plane defined by two intersecting lines

Conclusion

In conclusion, the analysis of intersection, parallelism, and orthogonality between two lines, two planes, and a line and a plane is crucial in descriptive geometry.

- For two lines, they can either intersect, forming a point of intersection, be parallel with no point of intersection, or be coincident, occupying the same space.
- Two planes can intersect, forming a line of intersection, be parallel with no point of intersection, or be coincident, occupying the same space.
- As for a line and a plane, they can intersect at a unique point, be parallel with no intersection, or the line can lie in the plane, forming an infinity of intersection points.

These concepts are essential for understanding the geometric and spatial relationships between different entities in a three-dimensional space.

Chapter IV Folding and affinity

Introduction

The techniques of folding, change of projection planes, and rotation constitute essential tools in descriptive geometry for transforming and simplifying spatial representations. These methods allow the transition from one projection system to another, making it possible to reveal true dimensions and facilitate the geometric analysis of points, lines, and planes.

This chapter introduces the concept of plane change, which consists of replacing one projection plane by another in order to obtain clearer or more convenient representations of geometric elements. The application of this method to points and lines is discussed for both horizontal and frontal planes, highlighting its role in improving spatial readability.

The chapter then explores the method of rotation, which involves turning geometric elements around a defined axis to bring them into a more favorable position for measurement or analysis. Rotations around vertical axes and axes perpendicular to the frontal plane are examined for both points and lines.

Finally, the concept of folding is presented as a fundamental technique used to bring projection planes into a single plane of representation. The folding of points and lines is studied in different configurations, depending on their orientation, including arbitrary lines, horizontal lines, and lines perpendicular to a given plane. Through these methods, students gain powerful tools for simplifying complex spatial problems and obtaining accurate geometric constructions.

1. Plane change

1.1. Definition

In the method of changing planes, the geometric figure remains fixed in space, while the position of the projection planes can be changed. The planes are modified while remaining perpendicular to each other; only one plane can be changed.

1.2. Change of horizontal plane

The change of horizontal plane in descriptive geometry refers to the selection of a new horizontal projection plane to represent three-dimensional objects, while the frontal projection plane remains unchanged.

The frontal projection of the figure remains unchanged and the distances between the points are preserved in the new outline.

As for the horizontal projection, the ground line and point dimensions change.

1.2.1. Change of horizontal plane for a point

Point (A) remains unchanged. A new horizontal projection plane (H1) is given, perpendicular to the frontal projection plane (F). The frontal projection of point (A) is unchanged and its distance is maintained.

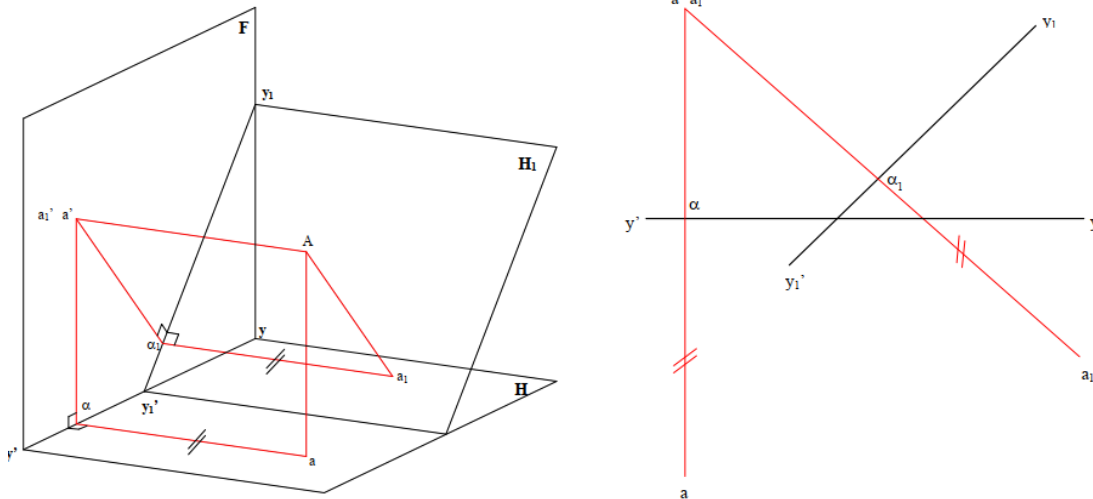


Figure IV. 1: Change of horizontal plane for a point

The representation ($a\ a'$) of point (A) is changed to ($a_1\ a'_1$). The frontal projection of point (A) is unchanged: (a') is merged with (a'_1) and the distance from point (A) is maintained: (aa) = (a_1a_1).

1.2.2. Change of horizontal plane for a line

Consider a line (D) of any position in space, represented by its two projections (d) and (d'). By changing the horizontal plane, make the line (D) an horizontal line.

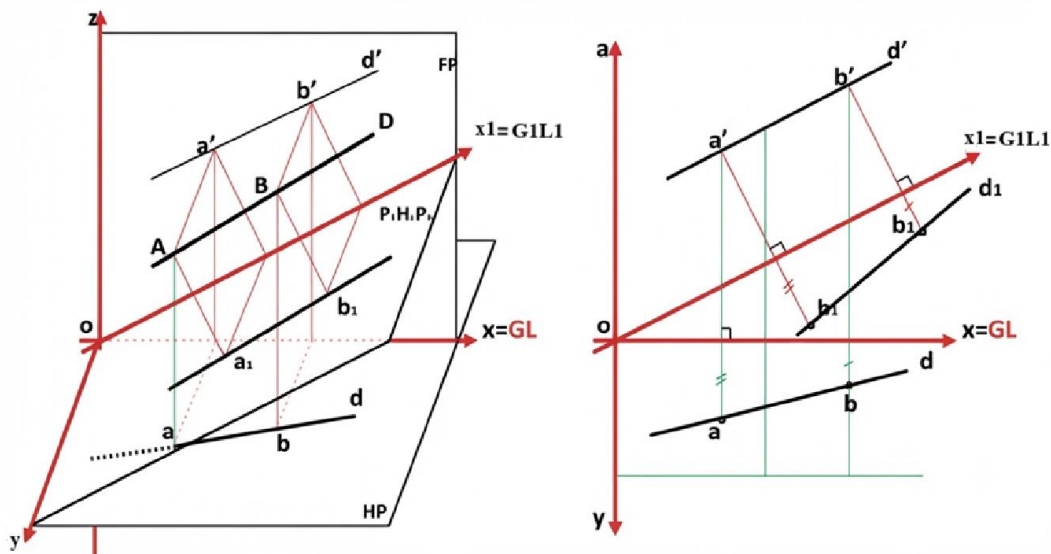


Figure IV. 2: Change of horizontal plane for a line

Knowing that the front projection (d') of the line (D) remains unchanged, and the question of making this line horizontal: Show the new ground line ($L1T1$) // at (d'). Find the transform A_1 and B_1 of points A and B respectively, i.e. a_1 and b_1 . At a_1 and b_1 we obtain (d_1), the horizontal projection of the line (D_1) (the transform of the line (D)).

NB: (d_1) is on true form and true length

1.3. Change of frontal plane

Changing the frontal plane in descriptive geometry refers to the selection of a new frontal projection plane to represent three-dimensional objects. The horizontal projection of the figure remains unchanged and the dimensions of the points are retained in the new outline. As for the frontal projection, the ground line and point distances change.

1.3.1. Change of frontal plane for a point

Point (A) remains unchanged. A new frontal projection plane (F_1) is given, perpendicular to the horizontal projection plane (H). The horizontal projection of point (A) is unchanged and its coordinate is retained.

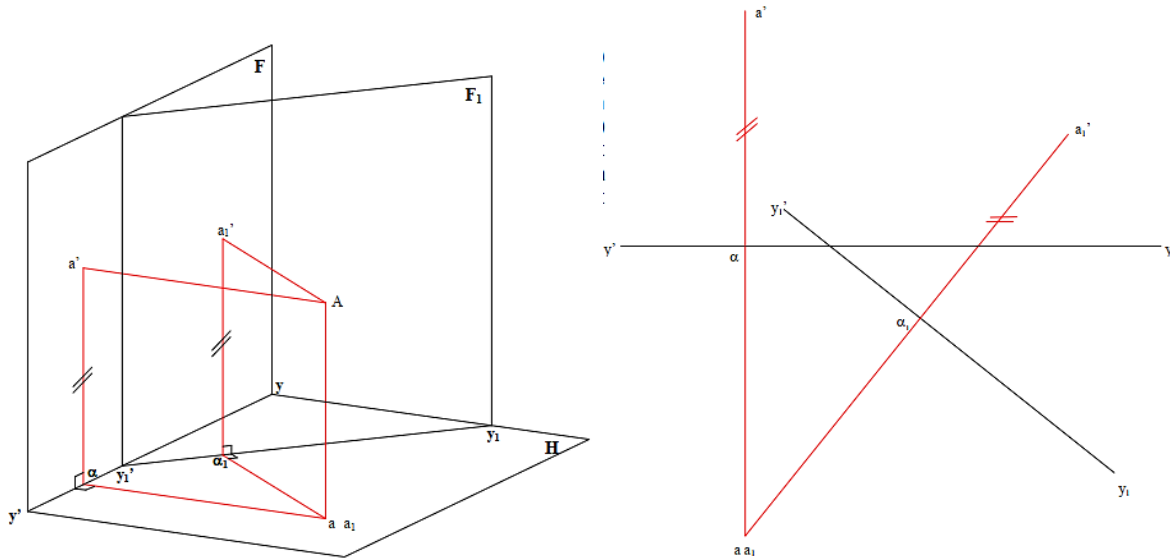


Figure IV. 3: Change of frontal plane for a point

The representation ($a\ a'$) of point (A) is changed to ($a_1\ a'_1$). The horizontal projection of point (A) is unchanged: (a) is merged with (a_1) and the dimension of point (A) is retained: (aa') = ($a_1a'_1$).

1.3.2. Change of frontal plane for a line

Consider a line (D) of any position in space, represented by its two projections (d) and (d'). By changing the frontal plane, make the line (D) a frontal line.

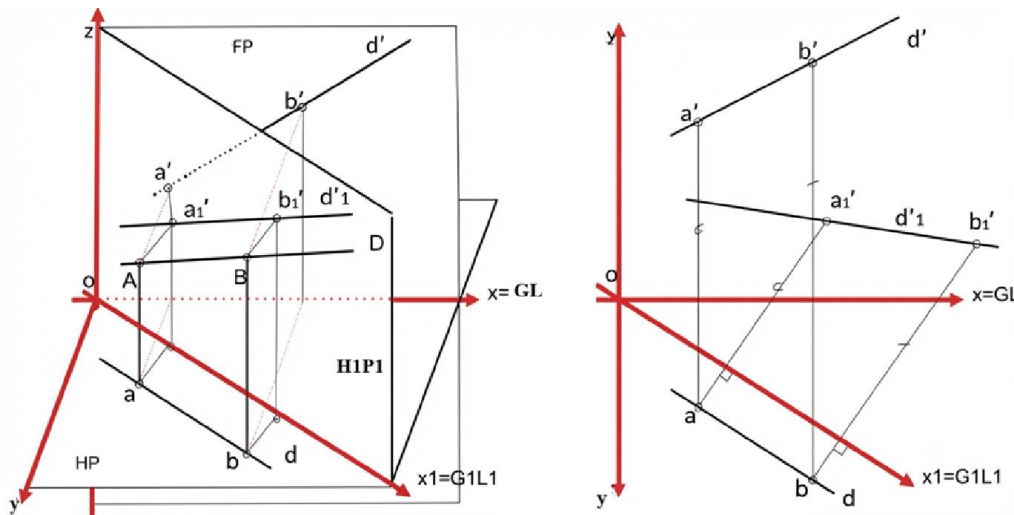


Figure IV. 4: Change of frontal plane for a line

Knowing that the horizontal projection (d) of the line (D) remains unchanged, and the question of making this line frontal: Show the new ground line ($L1T1$) // at (d). Find the transform $A1$ and $B1$ of points A and B respectively, i.e. $a'1$ and $b'1$. From $a'1$ and $b'1$ we obtain ($d'1$), the front projection of the line ($D1$) (the transform of the line (D)).

NB: ($d'1$) is on true form, and true length.

2. Rotation

2.1. Definition

In the rotation method, the projection planes remain fixed; it is the figure to be represented that rotates around a fixed line called the axis of rotation, or simply the axis. These axes must be vertical or axes perpendicular to (FP).

2.2. Rotation around a vertical axis

2.2.1. For a point

A point rotating around a vertical axis describes a circular arc parallel to the horizontal plane and perpendicular to the frontal plane. This arc of a circle is projected horizontally as a true form, and frontally as a segment.

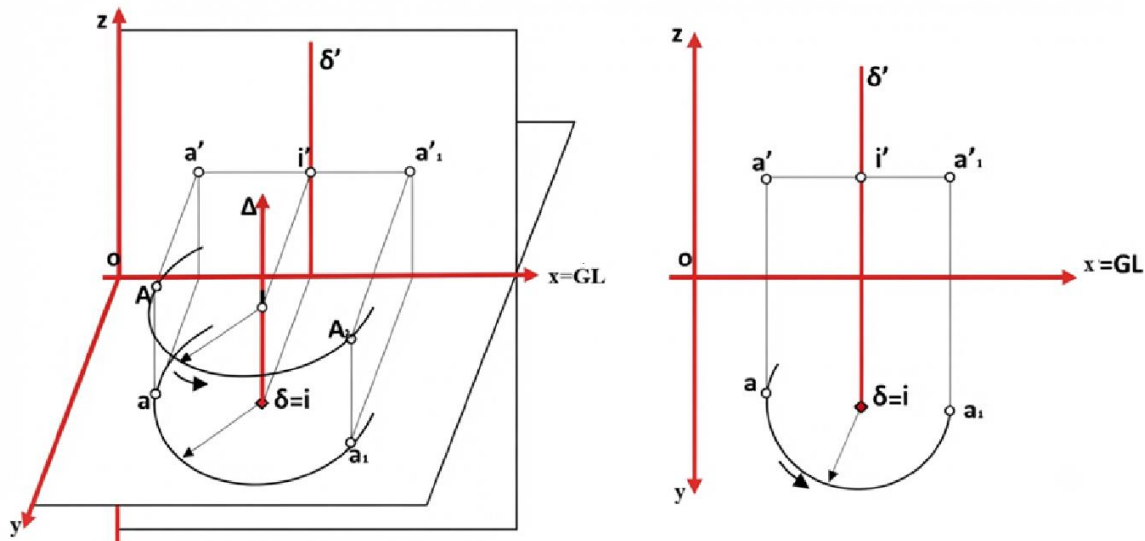


Figure IV. 5: Rotation around a vertical axis for a point

Point I is called the invariant point.

Point A1 is the new position of point A, geometrically called the transform of point A.

2.2.2. For a line

Given any line (D) (arbitrary line) in space and a line (Δ), representing the axis of rotation. Rotate the line (D) so that it faces frontally.

NB: The only particular positions that a line can assume as it rotates around a vertical axis are the frontal position and the profile position.

If the line (D) is brought back to a frontal position $\Rightarrow$ the horizontal projection (d1) of the line (D1) (the transform of (D)) will be parallel to the ground line (LT) and will pass through i (the horizontal projection of the invariant point I).

- To represent (d'1) we need to rotate any point on the line (D).
- We have chosen point A and looked for a'1 (the frontal projection of point A1, which is the transform of A).
- Join a'1 to i' and obtain (d'1) the front projection of the line (D1) (the transform of (D)).

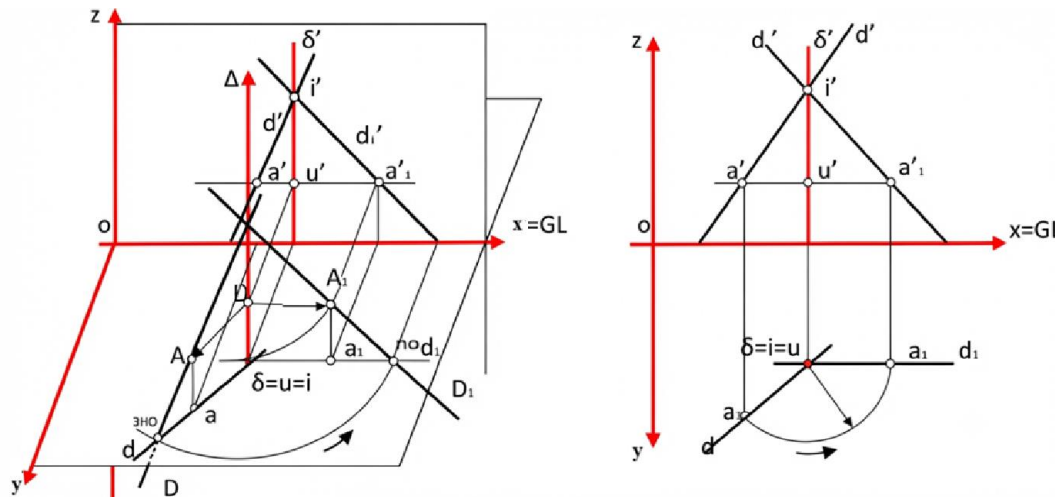


Figure IV. 6: Rotation around a vertical axis for a line

2.3. Rotation around an axis perpendicular to (FP)

2.3.1. For a point

A point rotating around a right axis describes a circular arc parallel to the frontal plane and perpendicular to the horizontal plane. This arc of a circle is projected frontally into a true form, and horizontally into a segment.

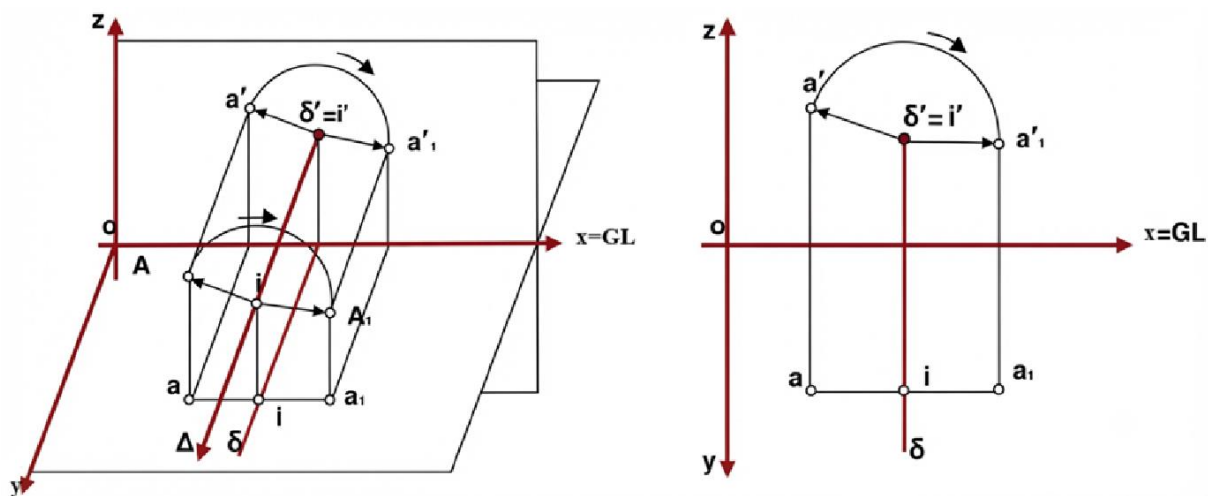


Figure IV. 7: Rotation around an axis perpendicular to (FP) for a point

Point I is called the invariant point.

Point A1 is the new position of point A, geometrically called the transform of point A.

2.3.2. For a line

Consider any line (D) (arbitrary line) in space and a line (Δ), representing the axis of rotation. Rotate the line (D) to make it horizontal.

NB: The only particular positions that a line can assume as it rotates around an axis perpendicular to (FP) are the horizontal position and the profile position.

If the line (D) is returned to a horizontal position $\Rightarrow$ the front projection (d'1) of the line (D1) (the transform of (D)) will be parallel to the ground line (LT) and will pass through (i') (the front projection of the invariant points I).

- To represent (d1), we need to rotate any point on the line (D).
- We have chosen point A and looked for (a1) (the horizontal projection of point A1, which is the transform of A).
- Join (a1) to i and we obtain (d1), the horizontal projection of the line (D1) (the transform of (D)).

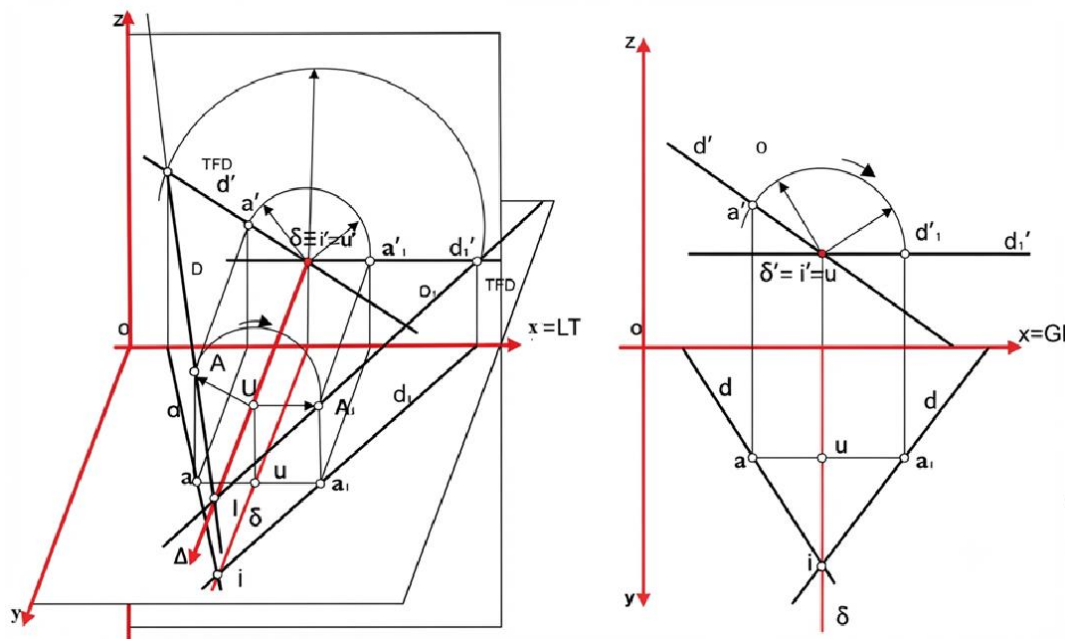


Figure IV. 8: Rotation around an axis perpendicular to (FP) for a line

3. Folding

3.1. Definition

The “folding” is a technique used in descriptive geometry to represent three-dimensional objects on a two-dimensional plane. It essentially involves projecting points, lines, and surfaces from a three-dimensional space onto a projection plane, typically using orthogonal projections. Folding facilitates the graphical representation of complex objects by providing a clearer and simplified view on a plane.

Folding consists of rotating a figure (whose points are all coplanar) around one of the lines in the plane that carries it, so as to make the figure parallel to one of the projection planes. The line around which the figure rotates is called the hinge. If the figure is folded onto a horizontal plane, the hinge is necessarily an horizontal line, and if it is folded onto a frontal plane, it is necessarily a frontal line. The hinge is invariant when folded.

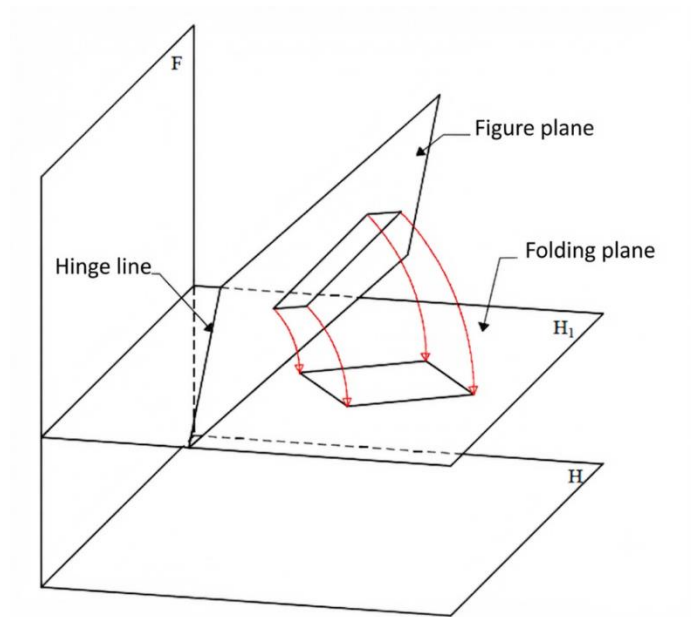


Figure IV. 9: Folding elements

3.2. Folding of a point

The horizontal trace of the plane ($P\alpha Q'$), which contains point A, is denoted as αP . The line (αP) serves as the hinge for folding. To represent the transformed point A1 by folding of the point A onto the horizontal plane (PHP):

- Draw a perpendicular (ai) from a (the horizontal projection of point A) to a line (h) parallel to ($p\alpha$).
- Place A on (h) with (aA1) equal to the Side of A.
- Draw an arc with center i and radius (iA1). The intersection of this arc with the perpendicular (ai) gives A1, the transformed point A by folding.

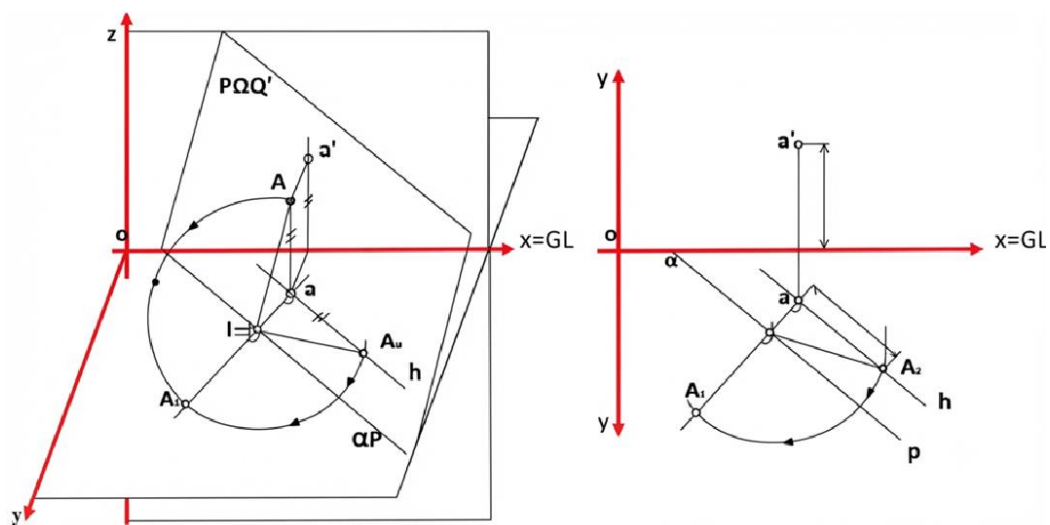


Figure IV. 10: Folding of a point

3.3. Folding of a line

To represent the transformation of a given line by folding, it is necessary to perform folding for two points on the line. The first point can be any chosen point, and the second point is a reference point, often the invariant point (the trace of the line).

3.3.1. (D) is an arbitrary line

The horizontal trace (thD) of the line D is the invariant point, meaning $thD1 = thD$.

To represent the transformed line (D1) by folding:

- It is sufficient to perform folding for a single point on the line (D).
- For example, take point A. Connect A to thD, and you obtain (D1).

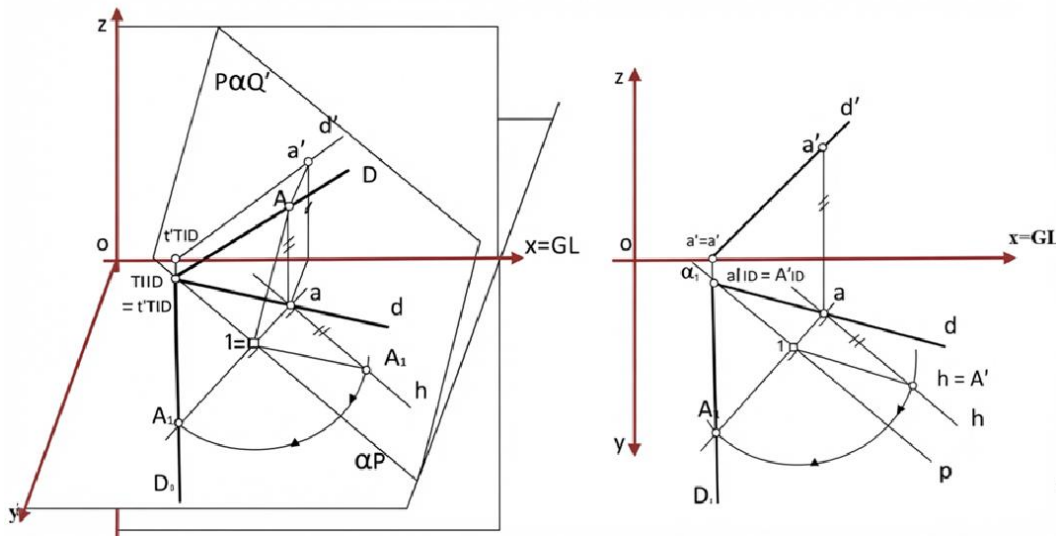


Figure IV. 11: Folding of a line when it is arbitrary

3.3.2. (D) is an horizontal line

The line (D) is horizontal, it is parallel to $(\alpha P) \Rightarrow (D1)$, the transformed line by folding, will be parallel to (αP) .

To represent (D1):

- It suffices to perform folding for a single point on the line (D), taking point A as an example, and find A1, the transformed point by folding.
- Finally, draw (D1) from A1, representing the transformed line by folding, and parallel to (αP) .

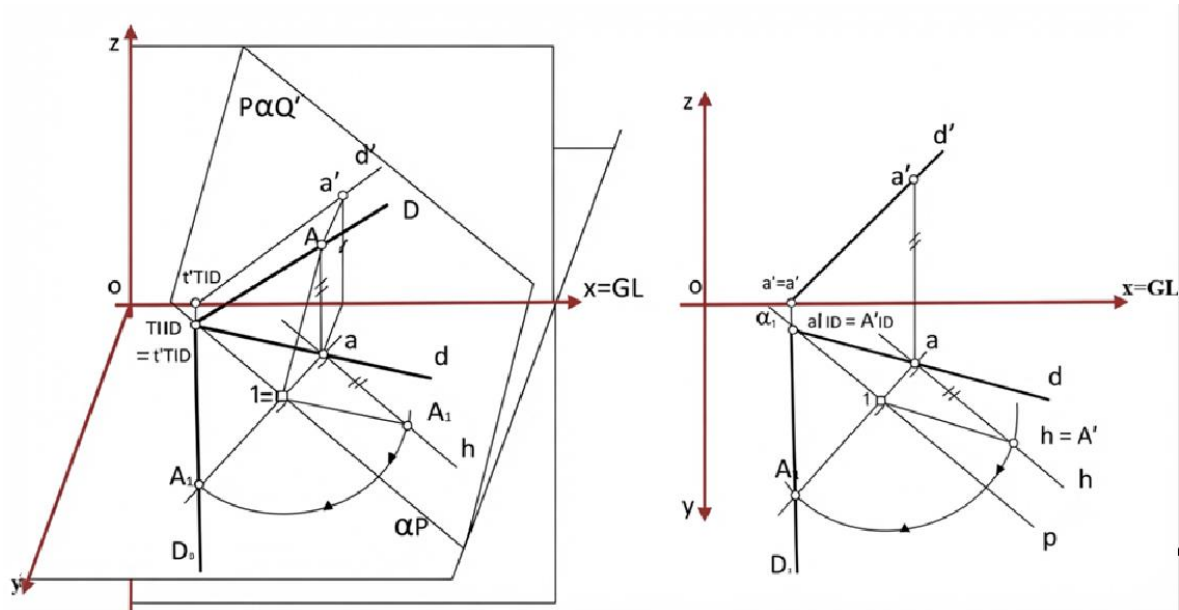


Figure IV. 12: Folding of a line when it is horizontal

3.3.3. (D) is perpendicular to (αP)

The line (D) is perpendicular to (αP), (D1) is the transformed line by folding, remaining perpendicular to (αP).

To represent (D1):

It suffices to draw it through thD1 and thD, (thD is the invariant point), and perpendicular to (αP).

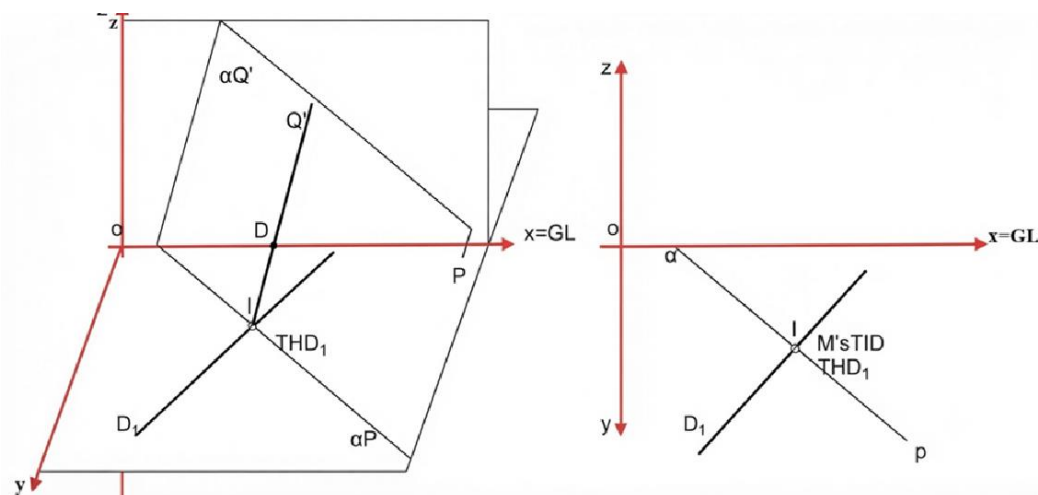


Figure IV. 13: Folding of a line when it is perpendicular to (αP)

3.3.4. Elevations

Let (D), (D1), (d) and (PαQ') be given. Show the two projections of point A knowing A1. Similarly, show the front projection (d') of the line (D).

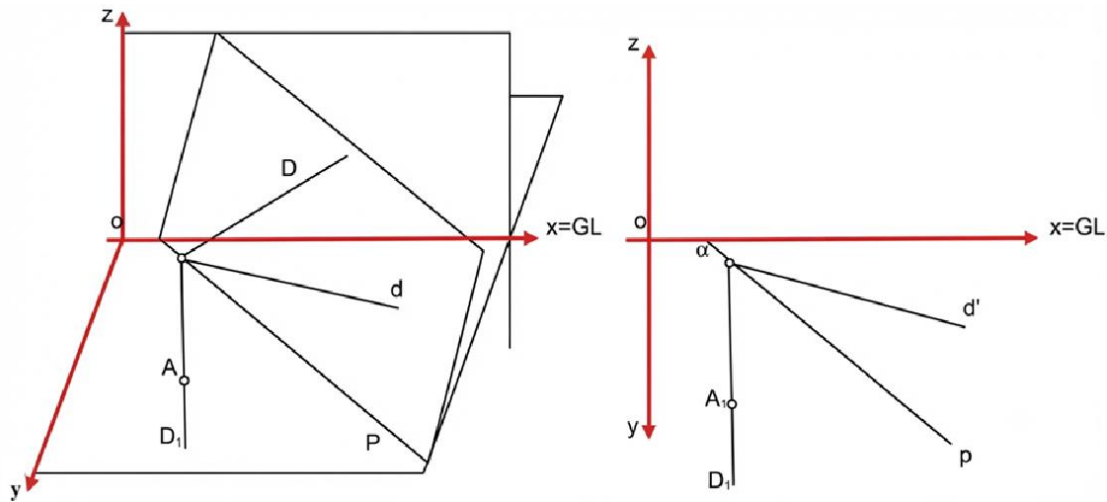


Figure IV. 14: Elevation

Draw by A1 a line perpendicular to (αP) , at its intersection with (d) , we obtain (a) (the horizontal projection of point A).

- From (a) draw a line parallel to (αP) , this will be (h) .
- Represent an arc of a circle with centre (i) and radius $= (iA1)$, and its intersection with (h) gives us $A1$, so $(aA1)$ represents the side of point A.
- For the representation of the frontal projection (d') of the line (D) , (a') must be joined to $th'D$ ($th'D$ is obtained by recalling thD to LT).

Conclusion :

The methods of plane change, rotation, and folding are fundamental operations in descriptive geometry that enable the transformation and simplification of spatial representations. They provide essential tools for revealing true dimensions and improving the clarity of geometric constructions.

Throughout this chapter, the change of projection planes has demonstrated its usefulness in repositioning points and lines to more convenient reference systems. The study of rotation has shown how geometric elements can be reoriented around specific axes to facilitate measurement and analysis in space.

The folding technique has proven to be a particularly important step in unifying projection planes into a single representation system, allowing for a clearer interpretation of spatial relationships. The different cases studied for points and lines illustrate the versatility of this method in handling various geometric configurations.

In conclusion, mastery of folding and affinity techniques is essential for effective spatial reasoning and accurate graphical representation. These methods play a key role in solving complex geometric problems and constitute a solid foundation for advanced studies in descriptive geometry, architecture, and engineering design.

Chapter V Representation of circles, cones, cylinders and spheres

Introduction:

This chapter introduces the graphical representation of the principal geometric surfaces commonly encountered in descriptive geometry: the circle, cylinder, cone, and sphere. These fundamental shapes serve as the basis for the study of more complex solids used in architecture, engineering, industrial design, and technical drawing.

The chapter begins with the representation of the circle, emphasizing its definition and graphical construction in different projection planes. It then examines the cylinder, cone, and sphere, focusing on their geometric characteristics, orthographic representations, and surface developments. The development of these solids is particularly important for practical applications such as sheet metal fabrication, architectural modeling, packaging design, and manufacturing processes.

By mastering the representation and development of these elementary solids, students will strengthen their spatial visualization skills and acquire essential techniques for solving descriptive geometry problems involving curved surfaces and three-dimensional objects.

1. Circle:

1.1. Presentation:

In descriptive geometry, a circle is a closed plane curve consisting of all points located at a constant distance from a fixed point called the center. This constant distance is known as the radius.

From a spatial perspective, a circle is defined by:

- its center;
- its radius;
- the plane in which it lies.

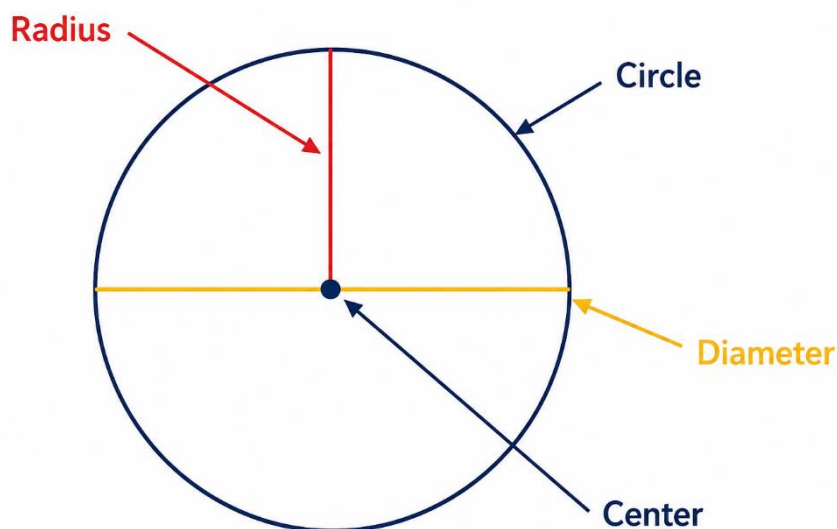


Figure V. 1: Representation of a circle

1.2. Graphical Representation

where:

- (a,b) are the coordinates of the center;
- r is the radius of the circle.

For a course in Descriptive Geometry of Space, it is also important to emphasize that the circle is widely used in the representation and study of cylinders, cones, and spheres, as well as in the determination of plane sections and surface intersections.

2. Cylinder:

2.1. Presentation:

The cylinder is a three-dimensional geometric figure defined as a surface made up of all the points located at a constant distance from a central axis, and whose cross-section is a closed curve, generally a circle. The height of the cylinder is the distance between the centers of the two circles, and is also the axis of revolution around which the shape is created.

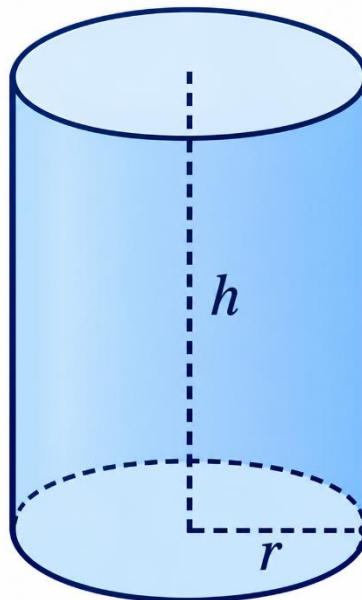


Figure V. 2: Representation of a cylinder

2.2. Development

The pattern of a cylinder of revolution is a rectangle which, when cut and folded, can be assembled to form a cylinder. For a cylinder of revolution, the pattern is particularly simple. It consists of:

- A rectangle: where the length is equal to the circumference of the base of the cylinder, and the height is equal to the height of the cylinder.
- Two circles: which define the bases of the cylinder:

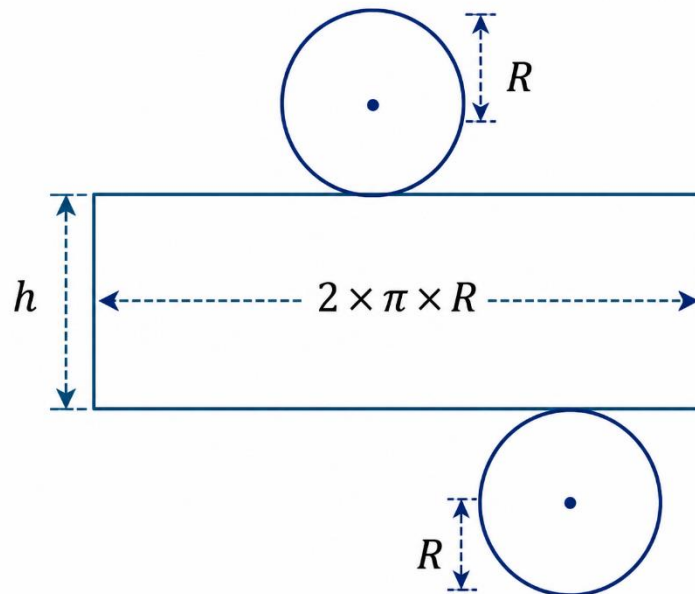


Figure V. 3: Development of a cylinder

3. Cone:

3.1. Presentation:

A cone of revolution is a three-dimensional shape created by rotating a triangle around one of its sides, called the axis of revolution. When the triangle rotates around this axis, it creates a conical shape. The main characteristics of a cone of revolution include its axis, which is also the height of the cone, and the radius of its base, which is the distance between the apex of the cone and the center point of its circular base.

Cone

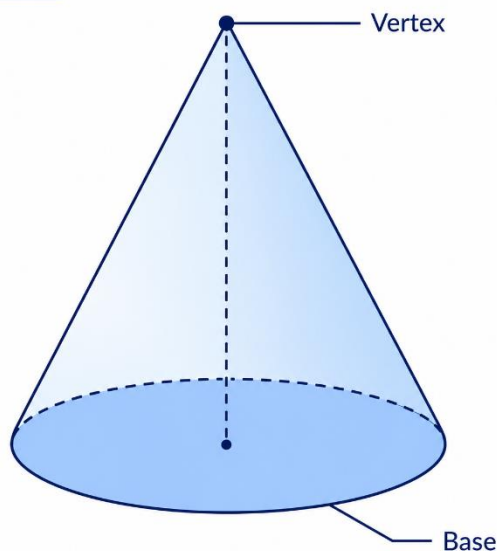


Figure V. 4: Presentation of a cone

3.2. Development

Steps to obtain a cone pattern:

- Base of the cone: Draw a circle to represent the base of the cone, where the size of the circle determines the radius of the base.
- Circular surface: This is the arc of a circle. Its length is equal to the perimeter of the base circle.
- Cone generator: This is a line from the outside of the circle to the center, representing the axis of revolution around which the cone is formed.

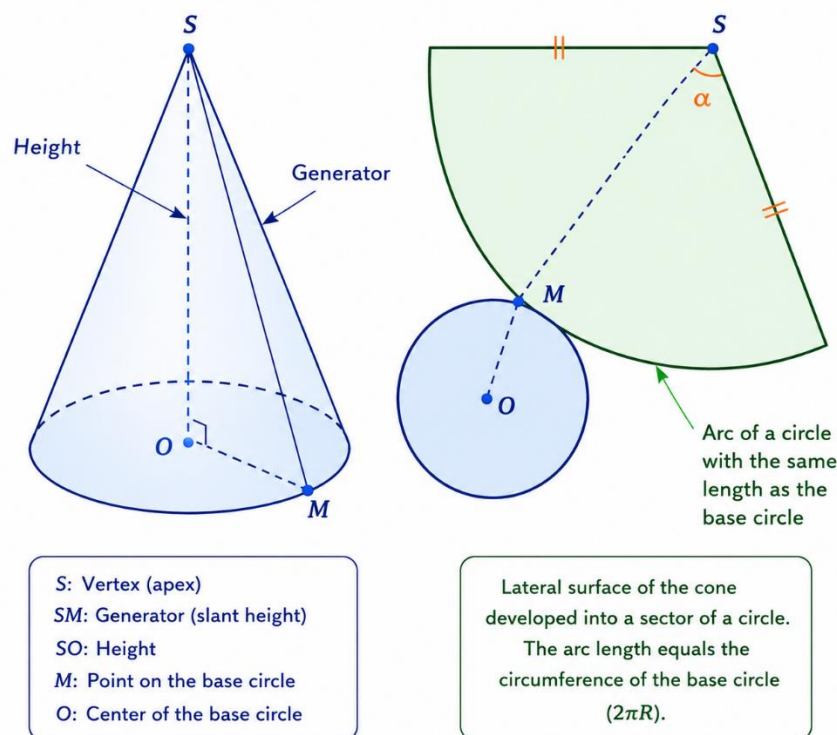


Figure V. 5: Development of a cylinder

4. Sphere:

4.1. Presentation

In geometry, the sphere is defined as the set of points in space equidistant from a given central point. Important characteristics of the sphere include its radius, which is the distance between the centre of the sphere and any point on its surface. The value of this distance from the centre is called the radius. A sphere S of centre O and radius R is the set of points M such that $OM=R$.

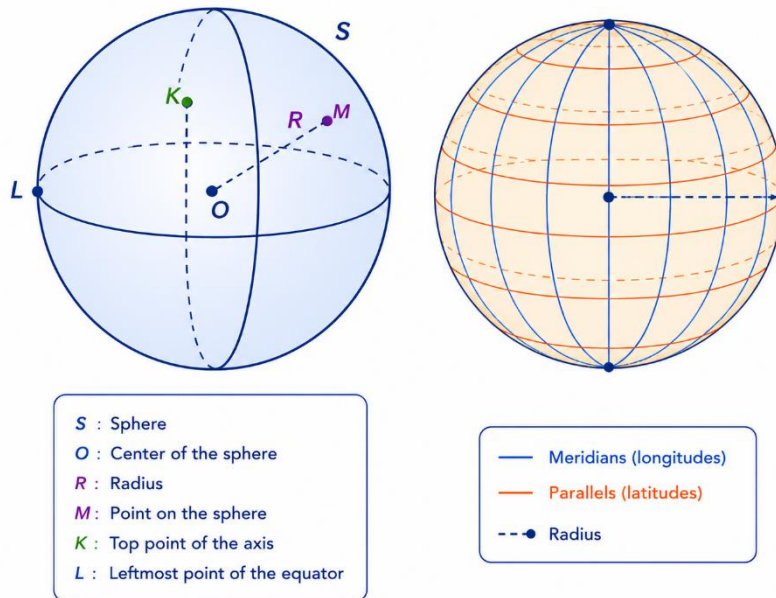


Figure V. 6: Presentation of a sphere

4.2. Development

The sphere is one of the non-developable surfaces, so it is only possible to study approximate development. The procedures used in practice are: Development by zones. Development by spindles.

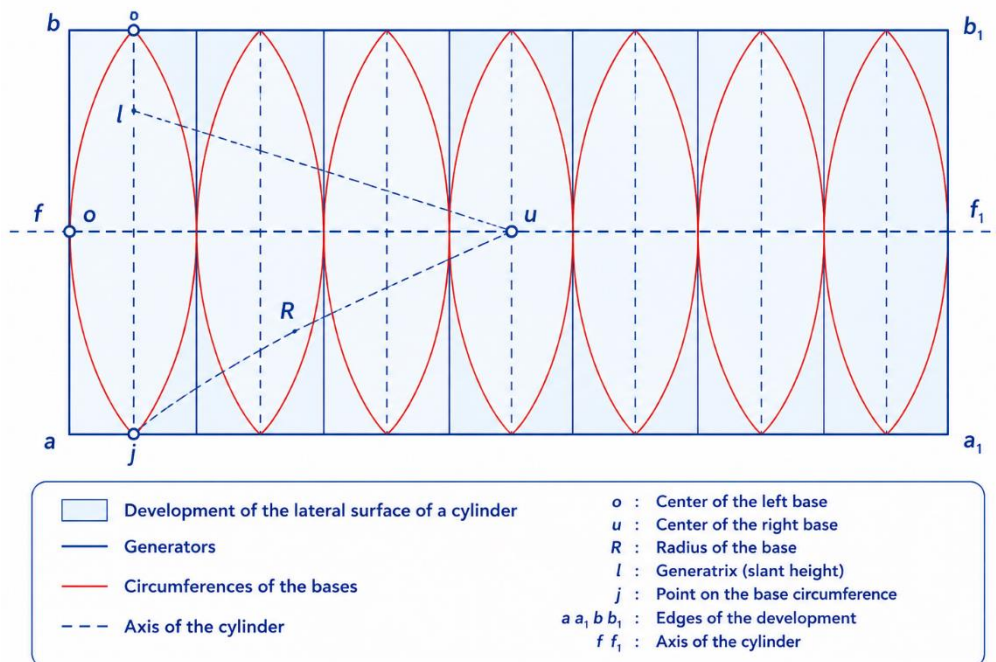


Figure V. 7: Development of a sphere

All the arcs shown have the same radius and have their centres on the axis of symmetry of the large rectangle (ff_1). To locate the centres of these arcs, for example, we joined h and I , then

from the middle of this segment (hi), we led a perpendicular, and at the intersection with the axis (ff1), we obtained the centre u of the arc (hij), and also the radius $R=(uj)=(ui)=(uuh)$.

The development is all the more approximate if the number of spindles chosen is larger. And the larger the sphere, the closer the development.

Conclusion:

The study of circles, cylinders, cones, and spheres constitutes a fundamental step in descriptive geometry, as these geometric forms are frequently encountered in architectural, engineering, and industrial applications. Understanding their graphical representation enables students to accurately visualize and interpret three-dimensional objects through two-dimensional projections.

Furthermore, the development of cylinders and cones provides essential techniques for transforming curved surfaces into plane figures, facilitating their construction and fabrication. Although the sphere cannot be developed exactly onto a plane, its representation remains indispensable for understanding spherical geometry and its applications.

The concepts presented in this chapter establish a solid foundation for the analysis of more advanced geometric forms, intersections of surfaces, and complex spatial constructions that will be explored in subsequent chapters.

Chapter VI Methods of perspective representation

Introduction:

Perspective representation is a fundamental tool in descriptive geometry that allows the transformation of three-dimensional objects into realistic two-dimensional images, closely resembling human visual perception. Unlike axonometric or orthographic projections, perspective drawing introduces depth and spatial realism by simulating the way objects appear smaller as their distance from the observer increases.

This chapter explores the theoretical and practical foundations of perspective representation in architectural and engineering drawing. It begins by defining the basic concepts and identifying the key elements involved in constructing a perspective, including planes, lines, and points, which form the structural basis of any graphical projection.

The chapter then examines different viewpoints, such as elevated views, human eye-level perspectives, and ground-level views, highlighting how the observer's position influences spatial perception. It further introduces central conical perspective, emphasizing its principles, definition, and the parameters that affect its construction. Oblique conic perspective, including two-point perspective, is also discussed as an alternative method for representing space with greater flexibility.

Additional sections address interior perspective representation and the geometric rules governing the depiction of elements such as parallel lines, inclined lines, circles, arcs, and staircases. Through these topics, students will gain a comprehensive understanding of how perspective techniques are applied to accurately and effectively represent architectural spaces.

1. Definition:

Conical perspective is a means of representing by drawing (on a plane) an object or building as it appears as seen from a particular point of view.

- The conical perspective of an object is the conical projection of this object onto a projection plane (the "table").
- With a conic projection all the projections converge towards the same point O called the center of projection.
- The projectors form a cone, hence the name "conical projection".

A distinction is made between oblique conical perspective and central conical perspective. It is the position of the object in relation to the painting that determines the nature of the perspective.

- Oblique conical perspective is constructed with two vanishing points.
- Central conic perspective is constructed with a single vanishing point.

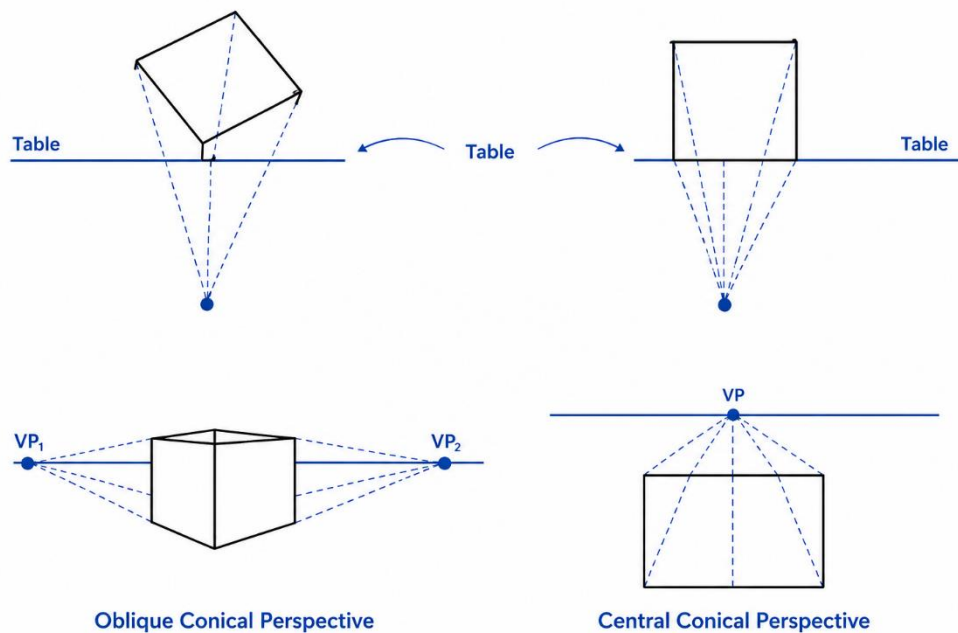


Figure VI. 1: Central and oblique conical perspective

2. Perspective stakeholders

2.1. Planes:

- Object: This is the element to be represented (point, line, surface, volume).
- Ground plane: is the horizontal plane on which the object to be drawn is placed in perspective and from which the vertical measurements are taken. This is the level on which the table is based and it is sometimes assumed that the viewer is standing on it.
- Horizon plane: is the plane that crosses the viewer's eye, and is the plane that determines the viewer's height in relation to the ground plane. It is perpendicular to the picture plane and parallel to the ground plane.
- Image plane (Table) T: This is the plane perpendicular to the ground plane on which the image of the figure is drawn in perspective.
- Viewing plane: is a plane perpendicular to the table and to the ground plane and on the horizon plane and passes through the viewer's eye (centre of view).

2.2. Lines:

- Horizon line (LH): is the line resulting from the intersection of the horizon plane with the table. The horizon line determines the height of the viewer's eye.
- Land line (LT): is the line resulting from the intersection of the table with the ground plane.
- Line of sight (line of sight): This is the line resulting from the intersection of the view plane with the horizon plane. It passes through the viewer's eye and is perpendicular to the table.

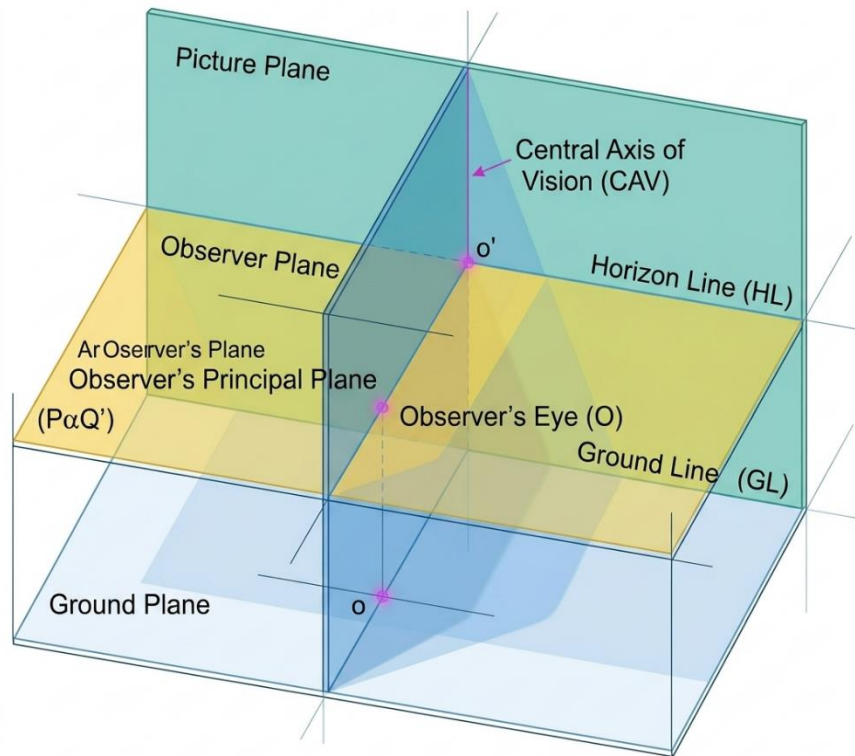


Figure VII. 2: Perspective stakeholders

2.3. Points:

- Principal Point (PP): This is the point of view from which we choose to view the object.
- Centre of view: This is the point where the line of sight intersects the picture and is considered to be the main vanishing point of the vertical lines on the picture plane.
- Cone of vision: This is a cone whose head is the viewer's eye and whose base is on the table.
- Main distance: distance between the eye and the table.
- Distance: distance between the eye and the object.

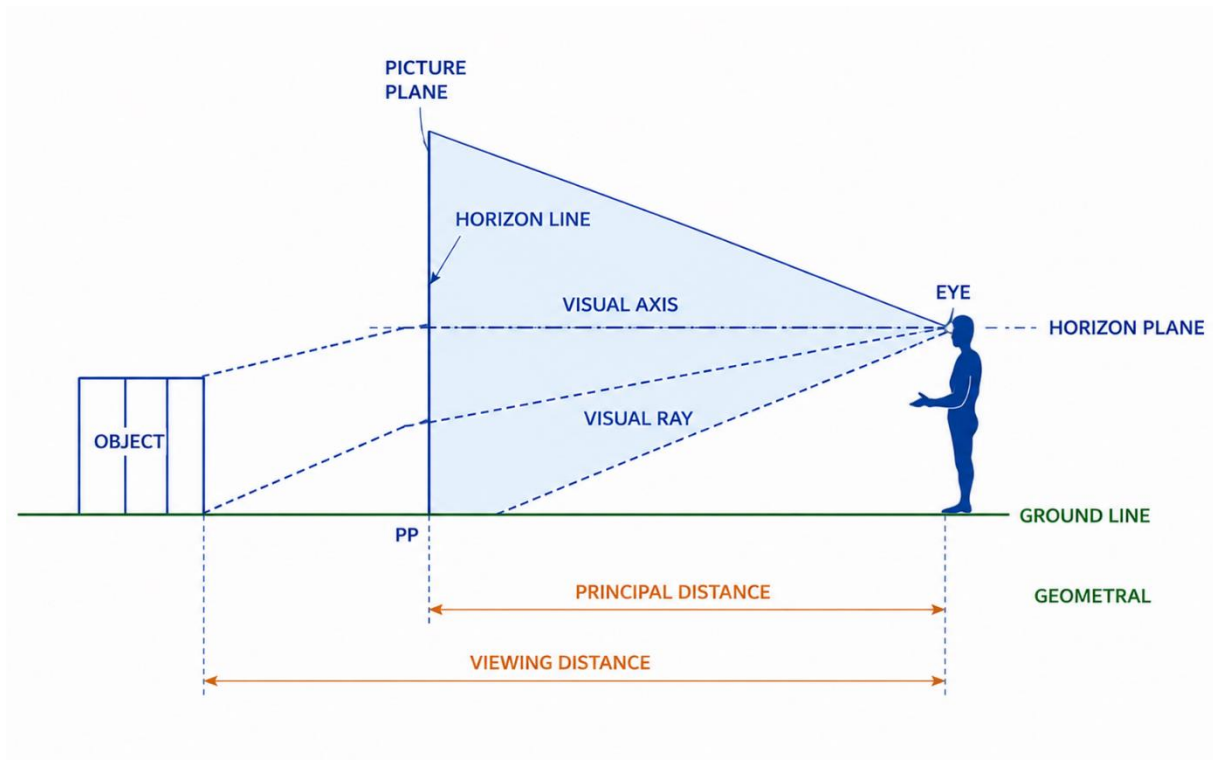


Figure VI. 3: Perspective elements

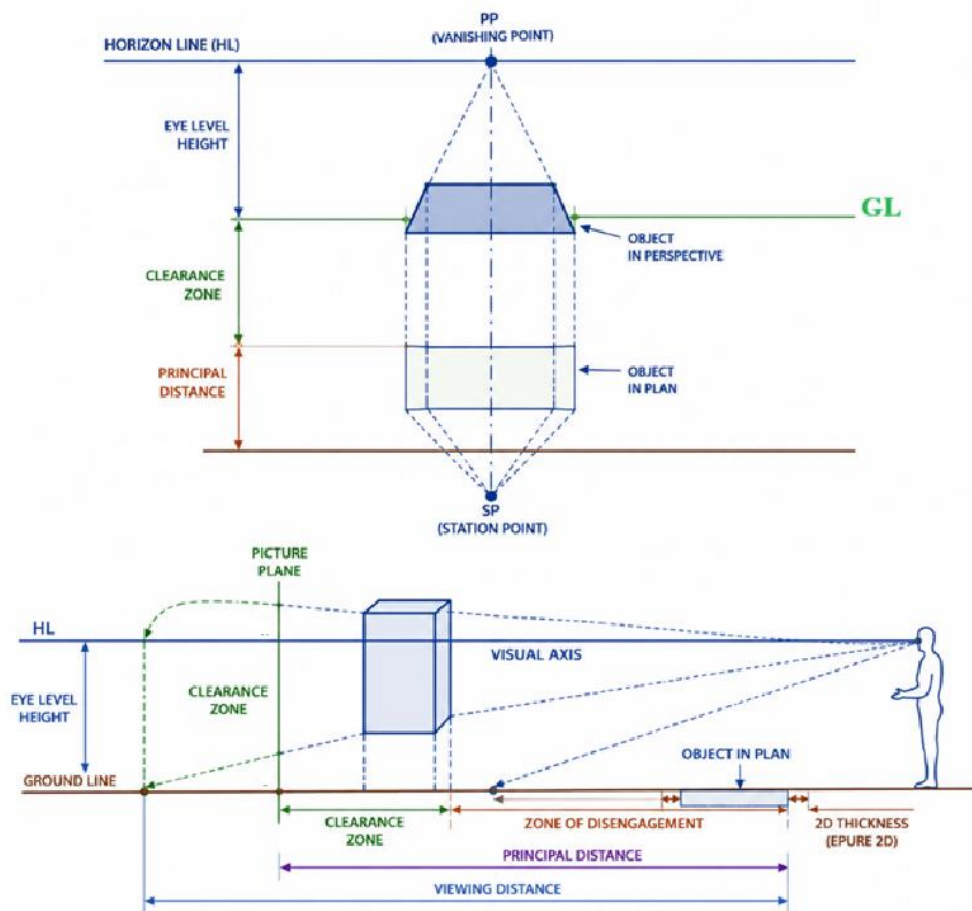


Figure VI. 4: Projection and section of perspective elements

3. Different views

3.1. Elevated view

If you are looking from a high vantage point (plane, hill, etc.), the horizon line will be higher up.

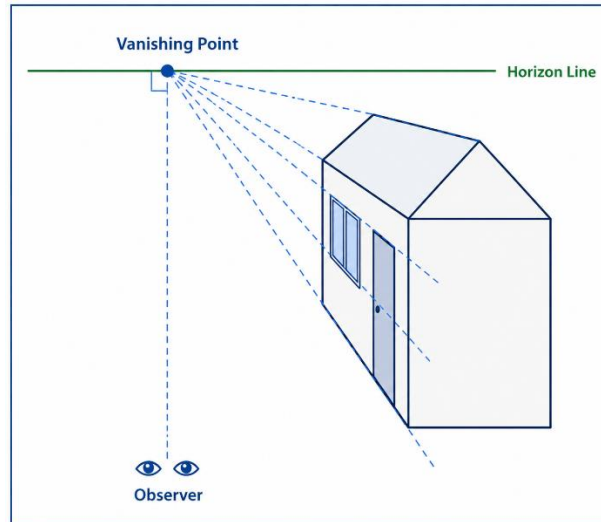


Figure VI. 5: Elevated view

3.2. View from a human perspective:

When viewed from human height, the horizon line is slightly lower than the centre of the page.

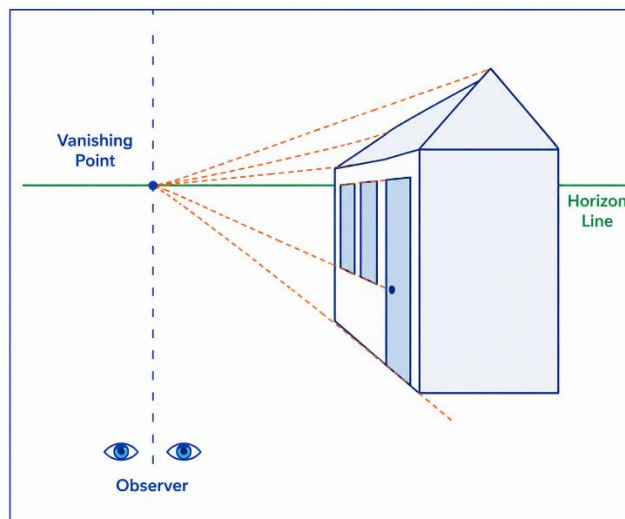


Figure VI. 6: View from a human perspective

3.3. Ground level view:

And if you're looking from a lower vantage point (crouched, in a cellar, etc.), the horizon line will be lower.

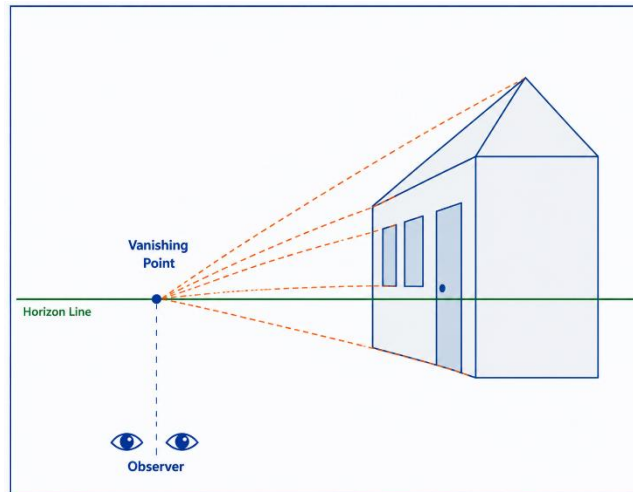


Figure VI. 7: Ground level view

4. Central conical perspective

4.1. Definition

Central conic perspective (linear perspective with one vanishing point) is a method of graphic representation that uses a single point of view, called the vanishing point or perspective point, located on the horizon line. Objects are drawn so that the vanishing lines converge at this point, creating an illusion of depth and realistic perspective. This technique is commonly used in art and architecture to represent three-dimensional scenes on a two-dimensional surface.

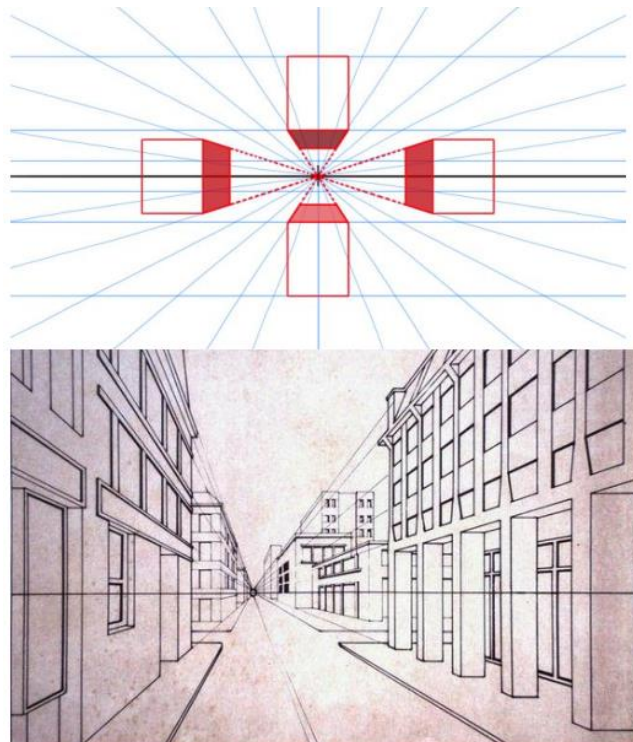


Figure VI. 8: Central conical perspective

4.2. Parameters affecting perspective

- Location of the focal point (and vanishing point) on the horizon.

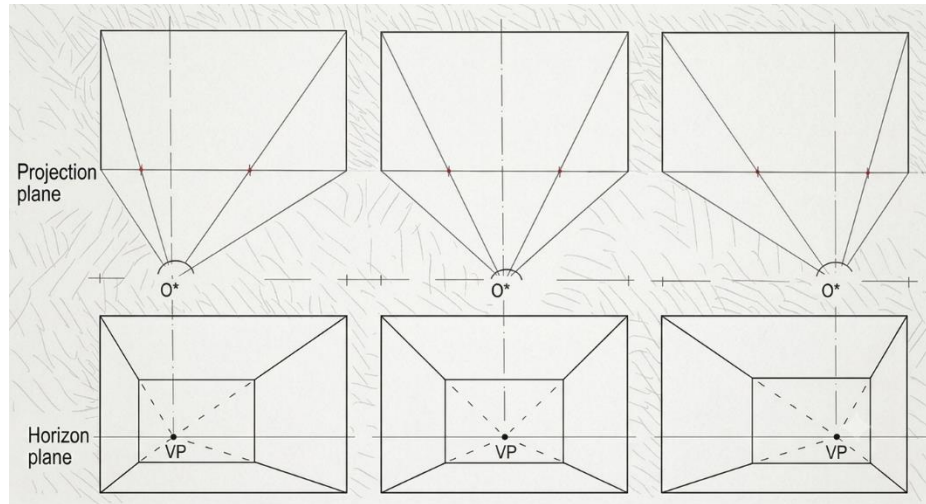


Figure VI. 9: Location of the vanishing point

- Height of horizon line.

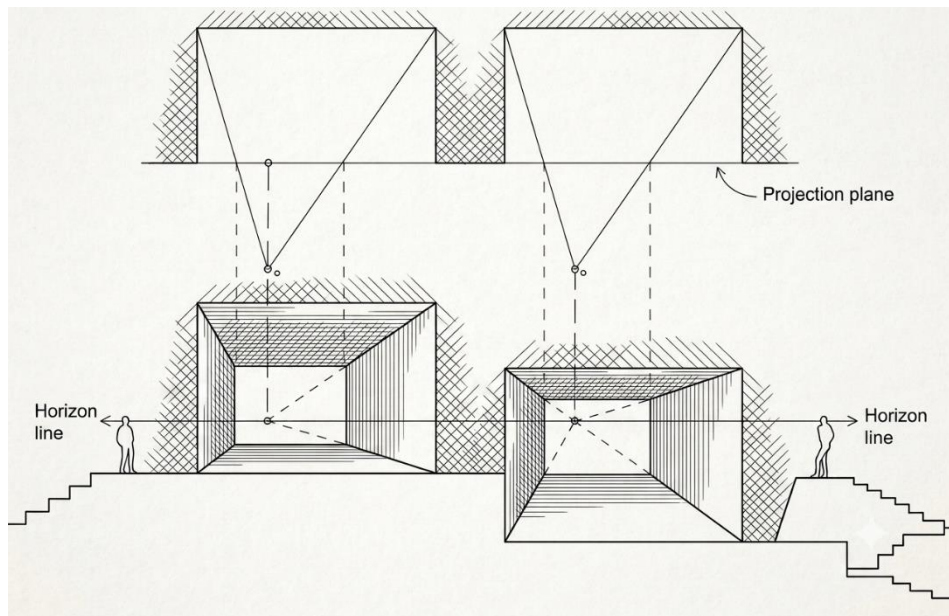


Figure VI. 10: Height of the horizon line

- Distance of the focal point from the table.

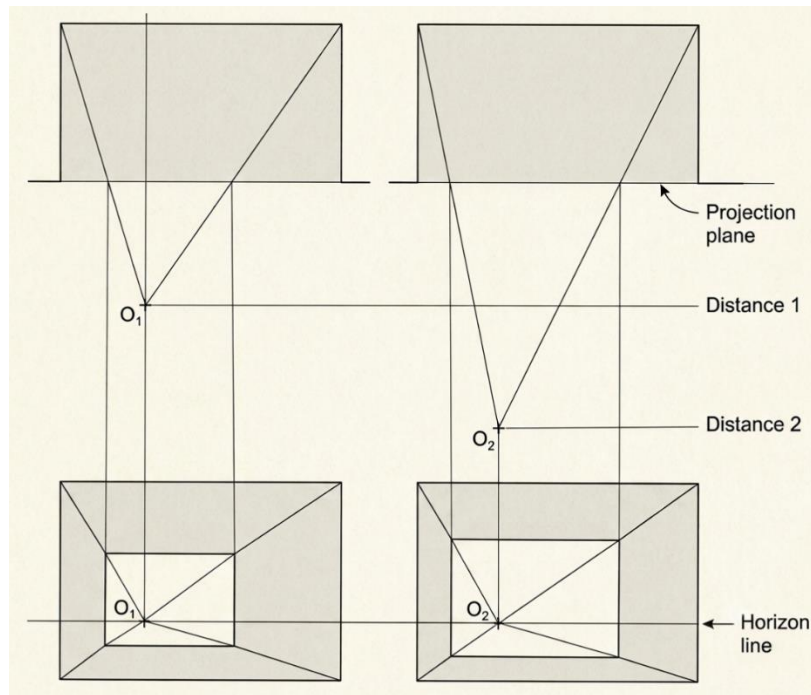


Figure VI. 11: Distance of the vanishing point from the table

5. Oblique conic perspective

5.1. Definition

Unlike central conic perspective, which uses a single vanishing point, oblique conic perspective uses several vanishing points located on the horizon line. These vanishing points make it possible to represent the vanishing lines of objects in different directions, creating an illusion of depth and realistic perspective. Oblique conic perspective is often used in architecture and engineering to create technical drawings and construction planes.

5.2. Conical perspective with two vanishing points

The essence of this method of spatial drawing is the use of two vanishing points, with which bodies and surfaces can be represented in an oblique view. For a single surface, two vanishing lines are required from the two vanishing points. Their intersections then define the corner points of the surface. The vertical edges of a body can be moved vertically upwards. No spatial distortion is shown here with a corner perspective.

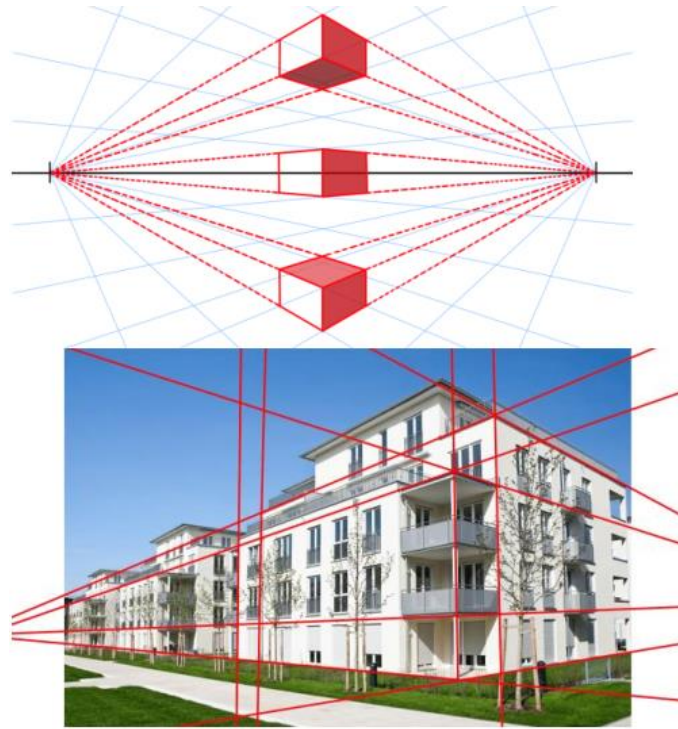


Figure VI. 12: Conical perspective with two vanishing points

5.3. Conical perspective with three vanishing points (aerial)

The third level of perspective representation is the three-point perspective: The upward distortion is also shown in the corner perspective. This perspective representation is often used for architectural drawings Example:

- To draw skyscrapers.
- In the drawing below, you can see how the vertical edges of a cube are distorted in perspective via the third vanishing point (FP-3).

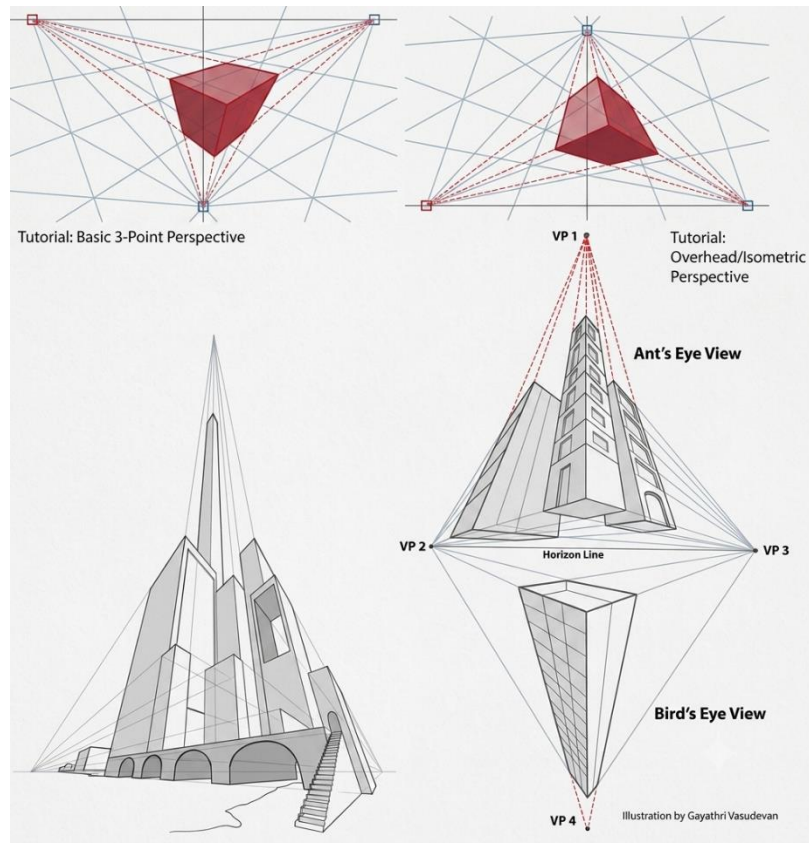


Figure VI. 13: Conical perspective with three vanishing points (aerial)

6. Interior perspective

Interior perspective is a graphic representation technique that aims to draw the interior of a three-dimensional space on a flat surface. It focuses on internal aspects such as the rooms in a house, furniture, decorations, etc. This technique creates realistic drawings that give the impression of looking through a window onto the interior of the space. Used in architecture, interior design, illustration and other fields where an accurate representation of the interior is required.

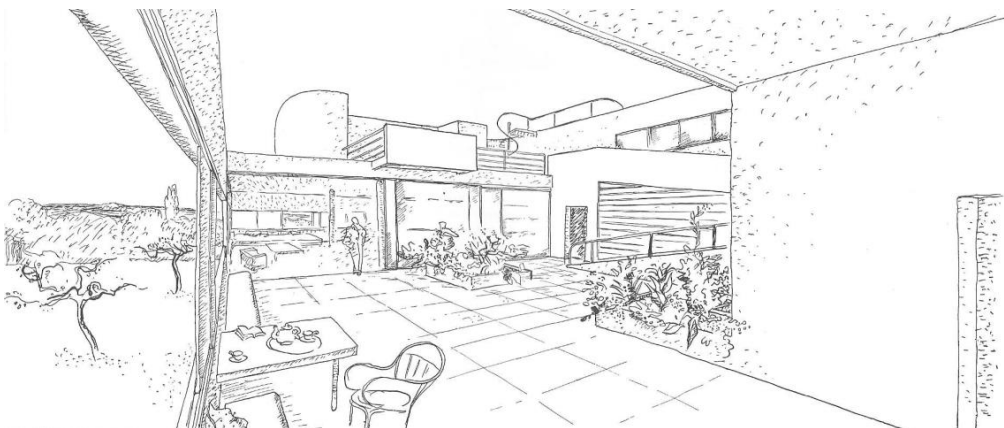


Figure VI. 14: Interior perspective

7. Perspective geometry

7.1. Parallel lines of constant distance

They retain their orientation, but their length and the distance between them decreases as a function of their distance from the observer.

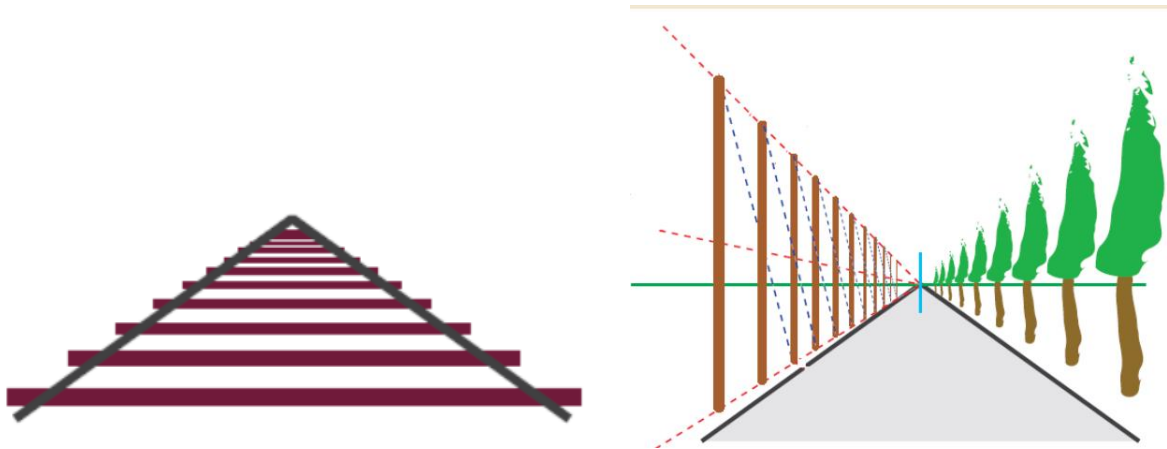


Figure VI. 15: Parallel lines of constant distance

7.2. Inclined lines

7.2.1. Inclined lines parallel to the table

They retain their orientation, but their length decreases as a function of their distance from the observer.

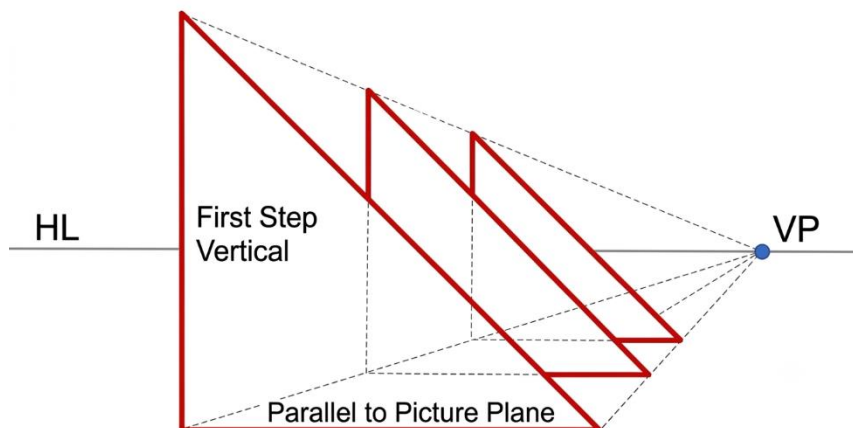


Figure VI. 16: Inclined lines parallel to the table

7.2.2. Inclined lines that are perpendicular or oblique to the table

leads to a vanishing point above or below the horizon line (LH). If you have to draw several parallel sloping lines, as in the case of a sloping roof, a ramp or a staircase, it is a good idea to know where the sloping lines seem to converge.

If the image rises as it moves away, its vanishing point will be above the horizon, whereas if it falls as it moves away, it will appear to converge below the horizon.

If the image rises as it moves away, its vanishing point will be above the horizon, whereas if it falls as it moves away, it will appear to converge below the horizon line.

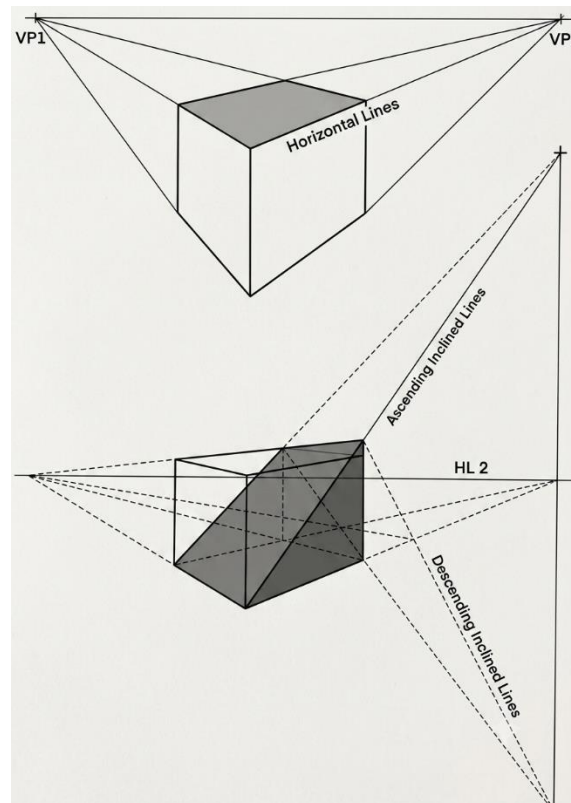


Figure VI. 17: Inclined lines that are perpendicular or oblique to the table

7.3. Representation of circles and arcs:

To draw a circle in perspective:

First draw a perspective view of a square circumscribing the circle. Then draw the diagonals of the square and, using additional lines parallel to the sides of the square or tangential to the circumference of the circle, indicate the straight lines where the circle crosses the diagonals.

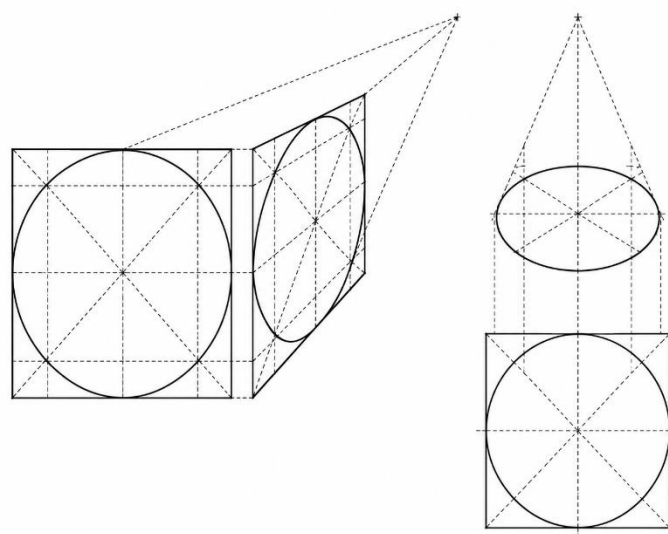


Figure VI. 18: Representation of a circle in perspective

7.4. The stairs in perspective :

To draw a staircase in conical perspective with two leakage points:

- 1- Choose the angle from which you wish to show the staircase.
- 3- Place the horizon line (the height of the viewpoint), and the two vanishing points.

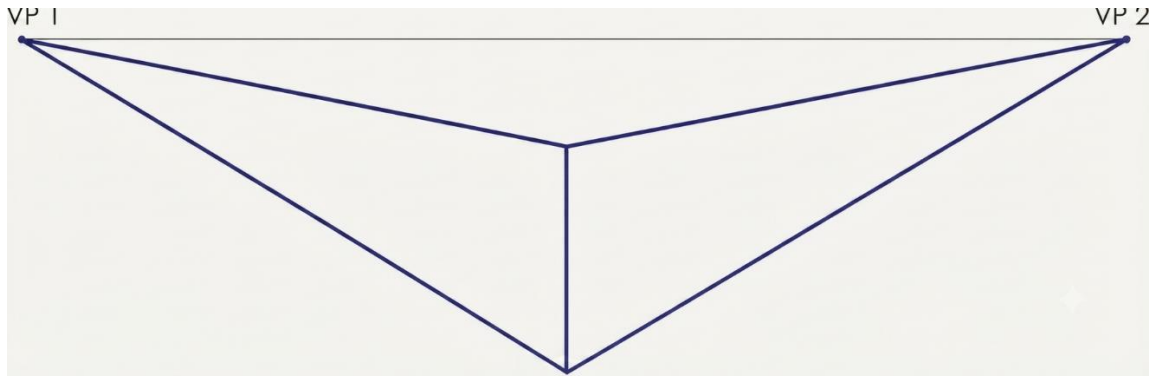


Figure VI. 19: Place of the horizon line and the vanishing points

- 3- Draw the volume containing the flight of stairs to be represented.
- 4- Divide the height of the flight of stairs (heights of the risers) by the number of steps.
- 5- Draw the two diagonal lines linking the first stair nosing to the landing.

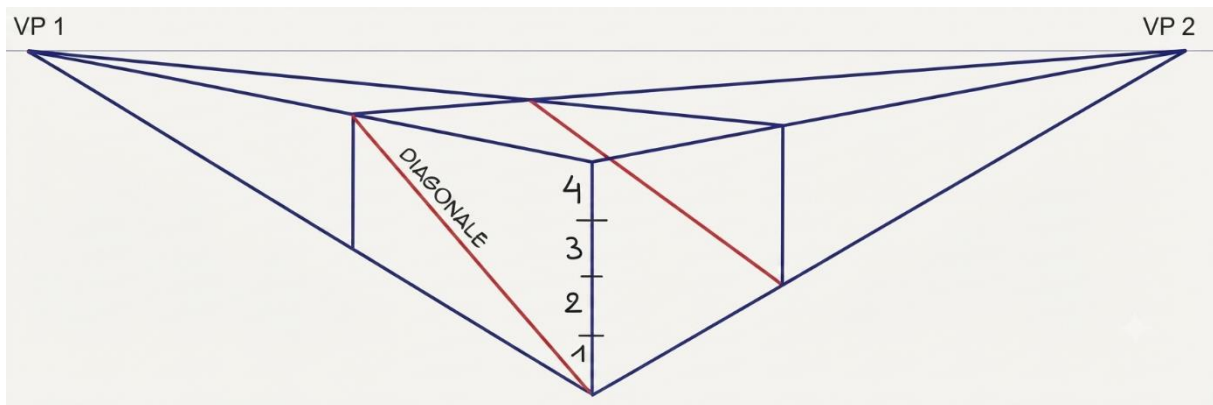


Figure VI. 20: Divide the height of the flight of stairs

- 6- Trace the slopes of each step and find the depth of the steps where they intersect the diagonal.
- 7- The height of the steps is the intersection of the verticals with the slopes.

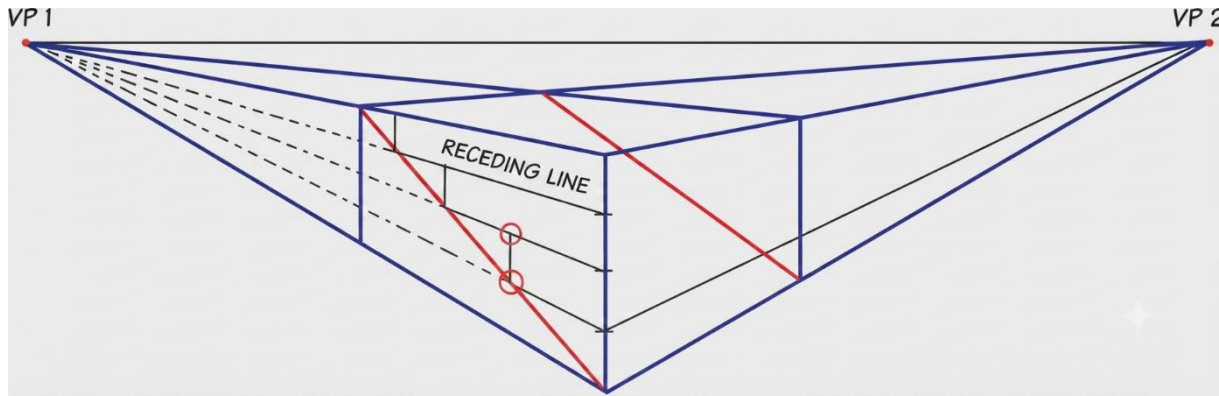


Figure V. 8: Trace the slopes of each steps

8- Finally, trace the leads to PF1 and PF2, then trace the vertical of the risers.

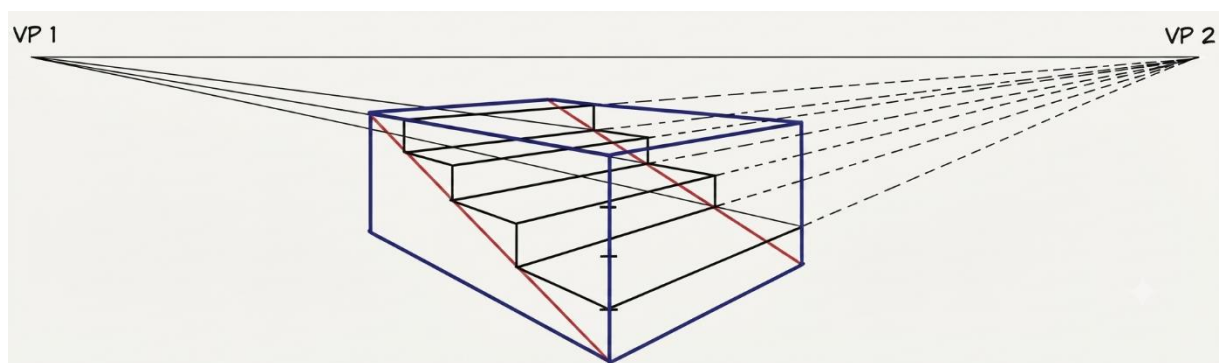


Figure VI. 21: Trace the leads to PF1 and PF2

Conclusion:

Perspective representation plays a central role in the visualization and communication of architectural and spatial designs, offering a realistic interpretation of three-dimensional environments on a two-dimensional plane. By simulating human visual perception, it enables a more intuitive understanding of scale, depth, and spatial relationships.

Throughout this chapter, the essential principles of perspective construction have been studied, starting from fundamental geometric elements to more advanced concepts such as central conical perspective and oblique perspective systems. The analysis of different viewpoints has shown how the observer's position significantly influences the perception and representation of space.

The study of geometric rules governing the representation of lines, planes, circles, and architectural elements such as stairs has provided practical tools for constructing accurate and coherent perspective drawings. These techniques are essential for translating complex spatial configurations into clear visual representations.

In conclusion, mastering perspective representation enhances both technical precision and visual communication skills. It constitutes an indispensable competence for architects, engineers, and designers, enabling them to effectively communicate spatial ideas and support the design process through realistic and expressive graphical representation.

Chapter VII Methods of axonometric representation

Introduction:

Axonometric representation is one of the most effective methods for visualizing three-dimensional objects on a two-dimensional surface while preserving their geometric proportions and spatial relationships. Unlike perspective drawings, where parallel lines may converge toward vanishing points, axonometric projections maintain the parallelism of lines, allowing dimensions and forms to be represented with greater accuracy and clarity.

In architecture, engineering, and design, axonometric drawings play a crucial role in communicating spatial concepts, construction details, and complex geometric configurations. They provide a comprehensive view of an object by simultaneously displaying multiple faces, making them particularly useful for technical documentation, project presentation, and design analysis. Various forms of architectural axonometry, such as exploded views, detail axonometries, and ceiling axonometries, offer additional possibilities for illustrating the organization and composition of buildings and structures.

This chapter introduces the fundamental principles of axonometric representation and its applications in architectural drawing. It presents the main types of axonometric projections, including military, cavalier, isometric, dimetric, and trimetric axonometries, highlighting their characteristics and fields of application. Particular emphasis is placed on the practical use of the 30°/60° military axonometry, a widely used method for representing architectural spaces and objects. Through these concepts and techniques, students will develop the skills required to accurately construct and interpret axonometric drawings in professional and academic contexts.

1. Definition

Axonometry presents both top and corner views using oblique and parallel projection within an orthogonal spatial coordinate system. This method enables the complexity of a whole to be understood more quickly. In the context of architecture, it can be used to represent a building with its roofs, to present certain interior aspects by means of cross-sections, and to improve overall understanding by incorporating transparent walls in perspective.

Axonometries are simple and quick to create, because they can be drawn to a precise scale based on the existing plane, elevation or section, depending on the importance attached to the element to be represented.

2. How axonometry is used in architecture?

Depending on the information you want to show, there are a number of options:

2.1. Exploded axonometry:

All the parallel perspectives break down the overall volume into several layers.

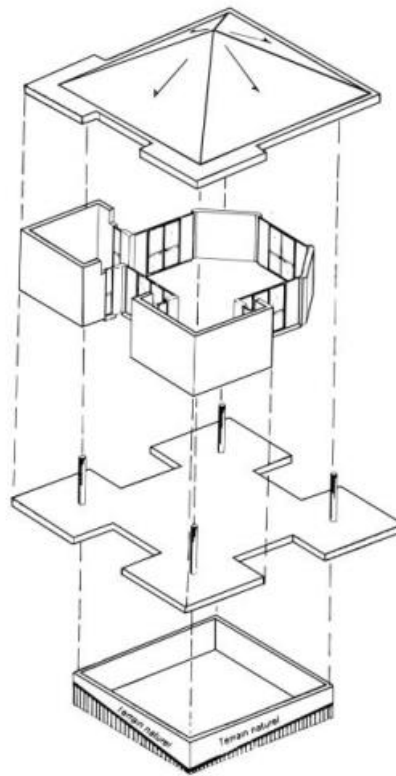


Figure VII. 1: Exploded axonometry

2.2. Axonometry details:

Parallel perspectives are intended to show an element that is usually technical.

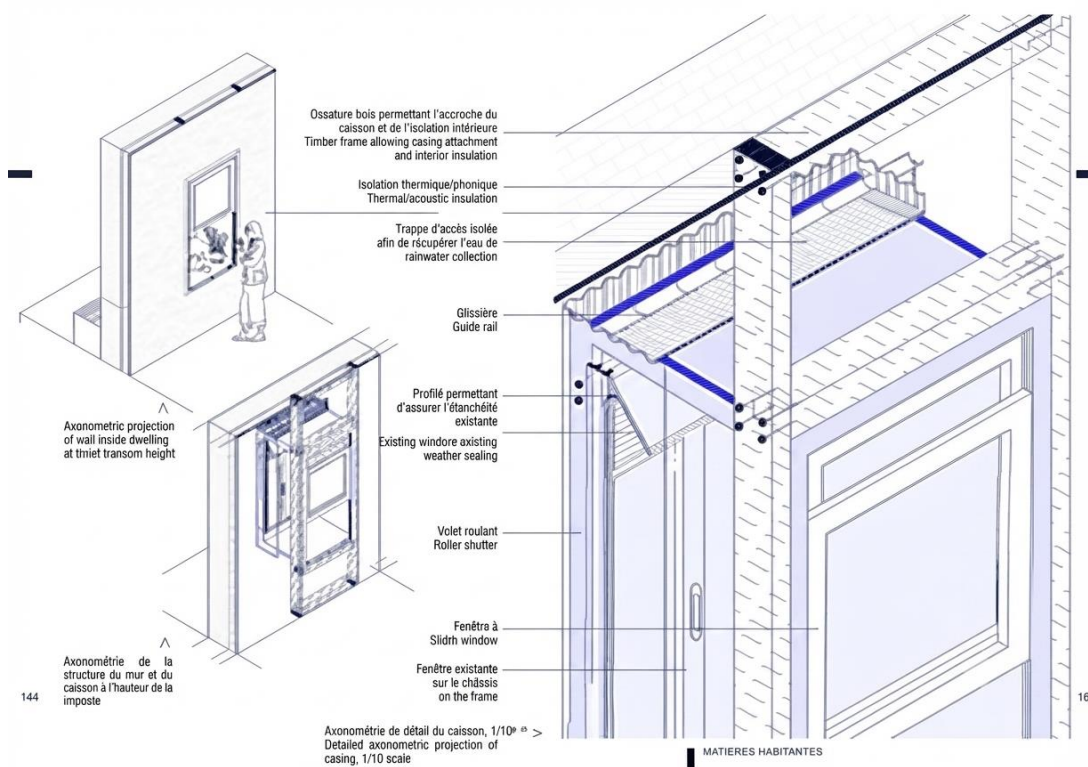


Figure VII. 2: Axonometry details

2.3. Ceiling Axonometry:

It is used to provide an analytical representation of historic buildings.

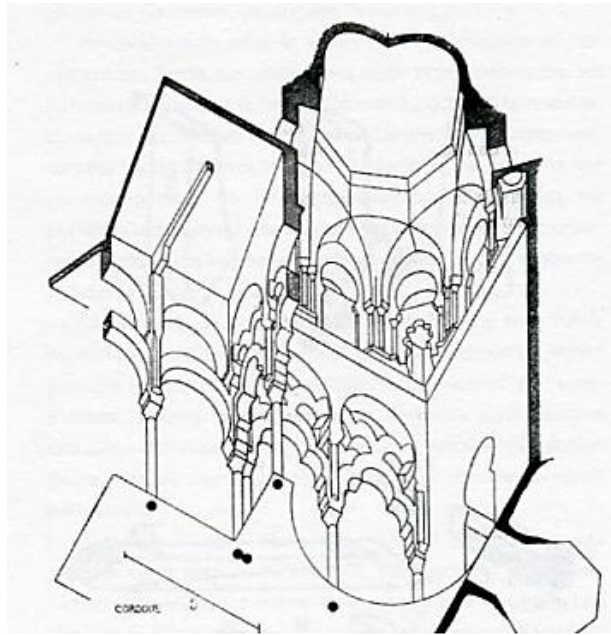


Figure VII. 3: Ceiling Axonometry

3. Types of axonometry

The main difference between types of axonometry is the arrangement of the angles between the projection axes.

3.1. Military axonometry

Axonometry of the plane: The plane retains its shape. The angle of view is arbitrary. Shortening the height to 3/4 improves the image.

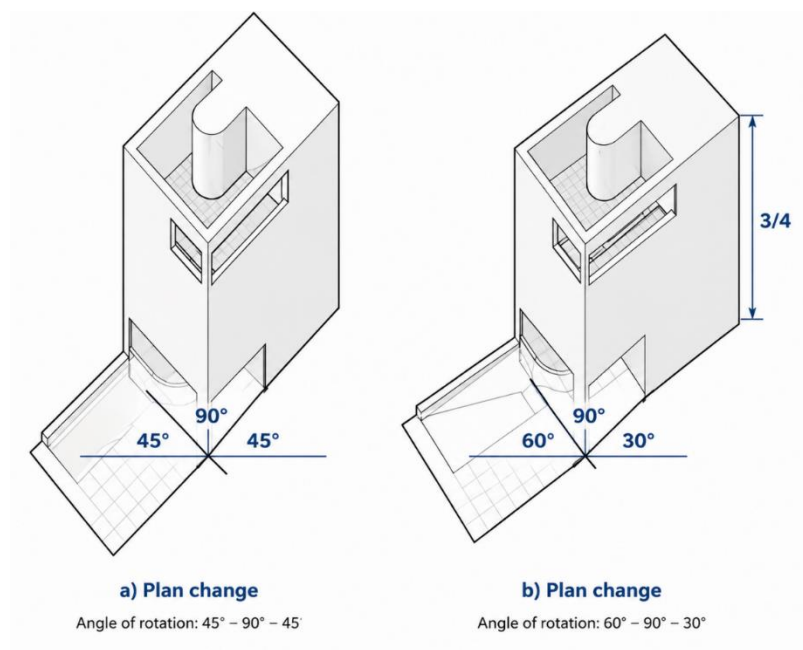


Figure VII. 4: Military axonometry (axonometry of plane)

3.2. Cavalier axonometry

Axonometry of the façade: The elevation retains its shape, and shortening the depths to 1/2 improves the image. Its advantages are that it preserves the full-scale images, i.e. the façade or elevation retains its shape and size.

Leaks are not shown to actual scale; their length is obtained by multiplying the actual length by a reduction coefficient.

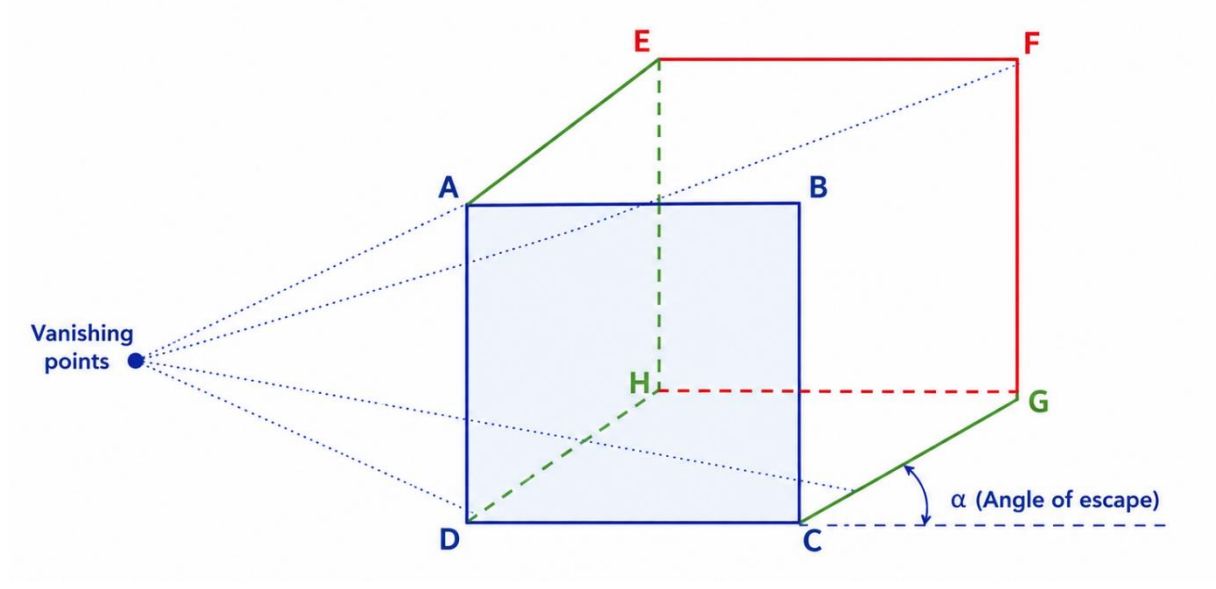


Figure VII. 5: Cavalier axonometry

3.3. Isometric axonometry

Isometry, as its name implies (equal measure), has the particularity of not deforming the object represented, since the equality of the 120° angles formed by the three axes results in the equality of the measures along these three axes. All edges are represented in true length. All surfaces are deformed.

Isometric representation is suitable when you want to quickly transfer all the edges to true size.

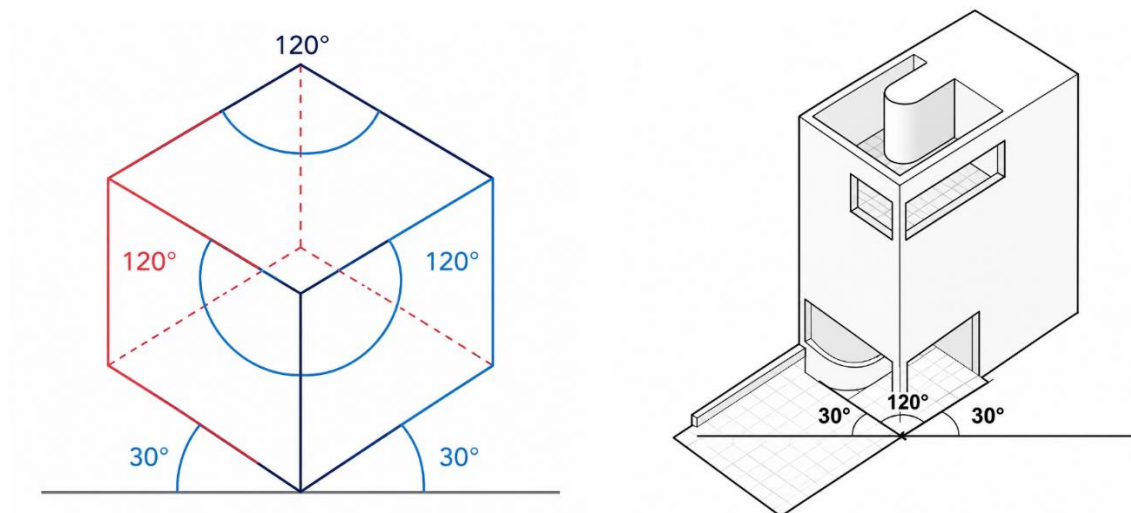


Figure VII. 6: Isometric axonometry

3.4. Dimetric axonometry

The elevation seen obliquely is shown full size, with the depths shortened to $\frac{3}{4}$. All surfaces are distorted.

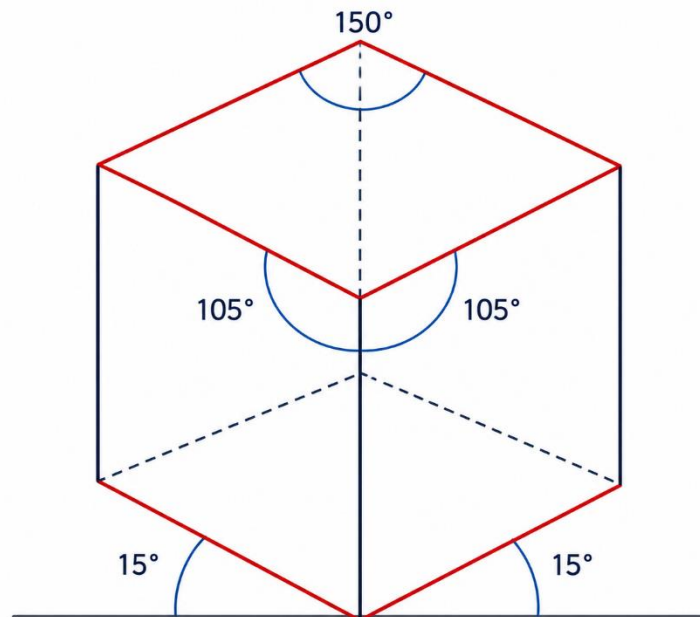


Figure VII. 7: Dimetric axonometry

3.5. Trimetric axonometry:

A trimetric projection is an axonometric projection in which the three principal axes are inclined at different angles to the plane of projection. Consequently, each axis and all lines parallel to it have a distinct foreshortening factor. The object is oriented so that no axis forms an angle smaller than 90° with the others, requiring the use of three different trimetric scales to represent dimensions along the three axes.

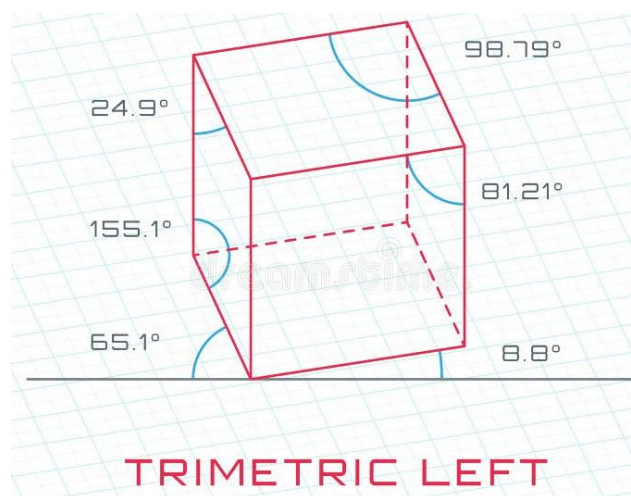
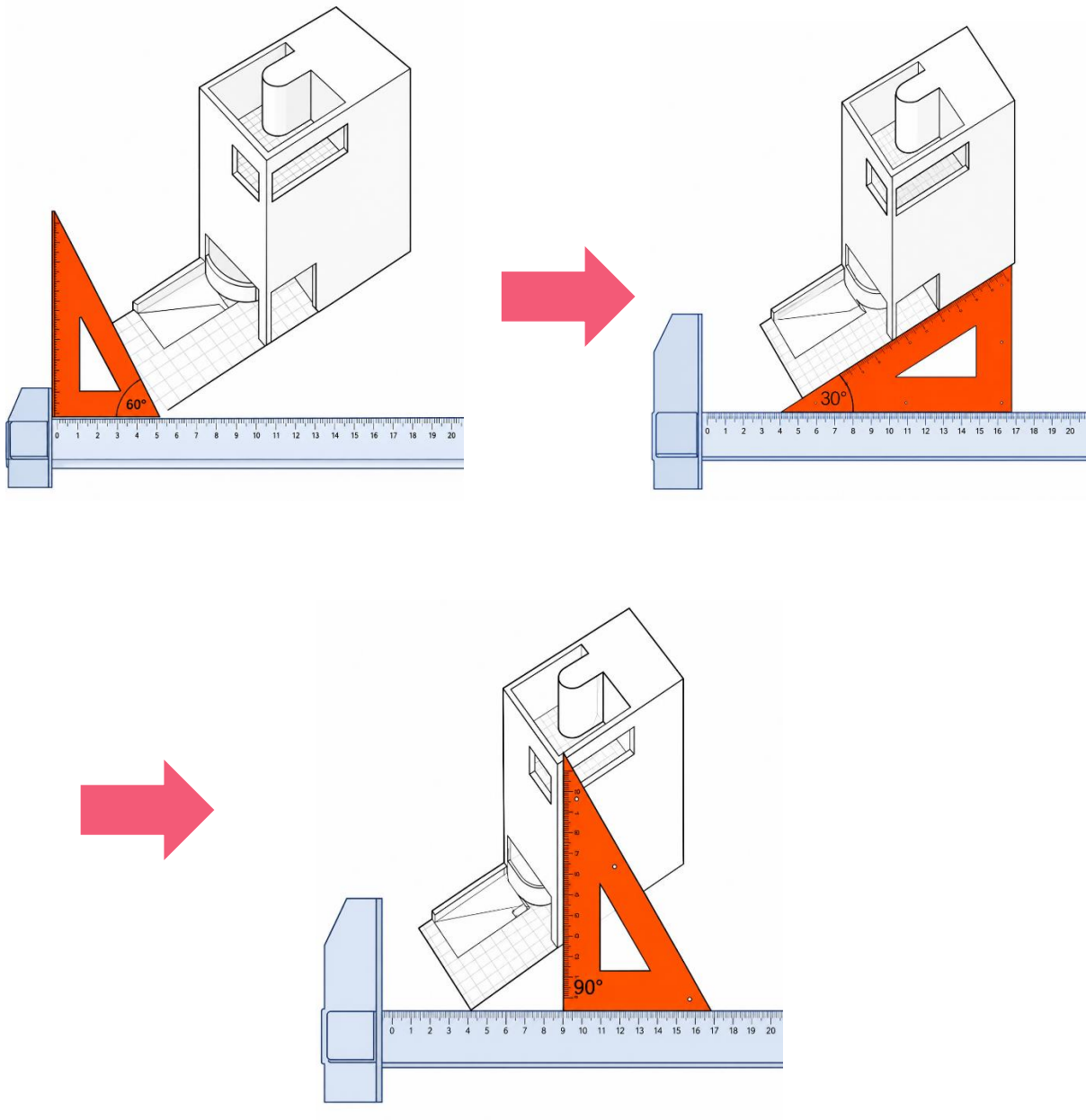
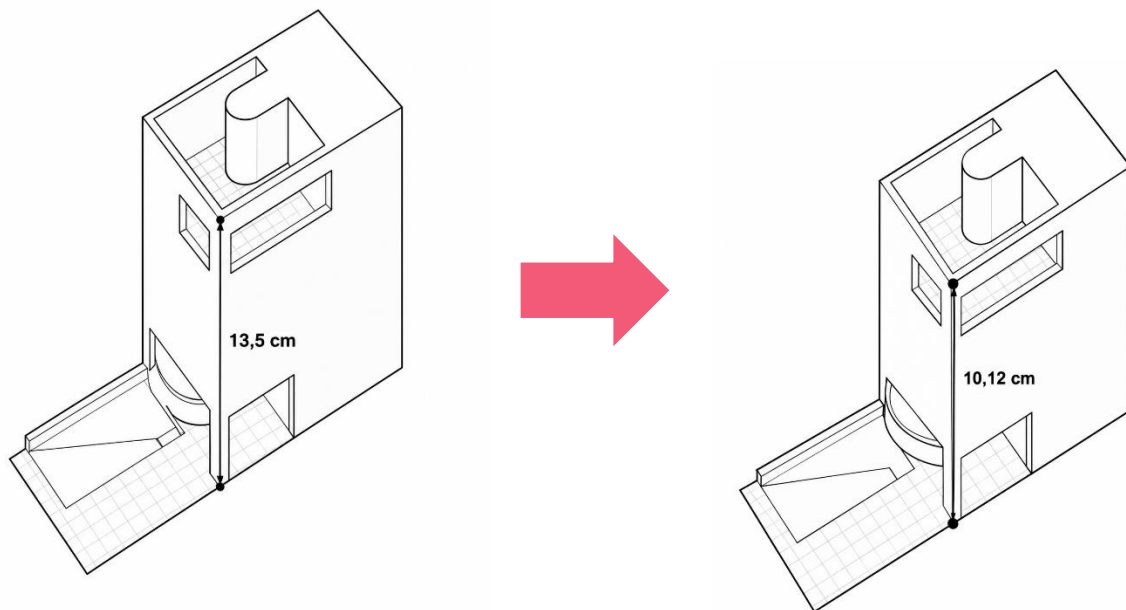


Figure VII. 8: Trimetric axonometry:

4. Using 30/60 military axonometry





Transfer of dimensions to scale 1cm/m with a correction factor of 0.75 to compensate for height deformation (here 13.5 metres $\times$ 1/100 = 10.125 cm).

Conclusion:

Axonometric representation constitutes a fundamental technique in descriptive geometry and graphical communication, providing a clear and measurable visualization of three-dimensional objects. By preserving the parallelism of lines and offering a comprehensive view of spatial forms, axonometric drawings facilitate the understanding, analysis, and presentation of architectural and engineering projects.

Throughout this chapter, the principles and applications of axonometry have been explored, from its role in architectural representation to its various forms, including military, cavalier, isometric, dimetric, and trimetric projections. Each method offers specific advantages depending on the intended purpose, level of accuracy, and visual effect required. The study of exploded, detailed, and ceiling axonometries has further demonstrated the versatility of this representation technique in illustrating both overall compositions and specific construction elements.

The practical application of the 30°/60° military axonometry has provided students with a reliable method for producing clear and efficient three-dimensional representations. Mastery of these techniques enhances spatial reasoning and graphical communication skills, which are essential for architects, engineers, designers, and urban planners.

In conclusion, axonometric representation serves as a bridge between technical accuracy and visual comprehension, making it an indispensable tool for the analysis, design, and communication of complex spatial configurations.

Chapter VIII Shadow

Introduction:

The representation of shadows is one of the most important applications of descriptive geometry, as it combines geometric principles with the visual perception of three-dimensional objects. Shadows provide valuable information about the shape, position, orientation, and spatial relationships of objects, allowing a more realistic and comprehensive understanding of architectural and engineering drawings.

In architecture, urban planning, design, and engineering, the study of shadows is essential not only for graphical representation but also for evaluating environmental conditions such as sunlight exposure, shading, and visual comfort. The accurate construction of shadows enables designers to predict the effects of natural and artificial lighting on buildings, public spaces, and technical objects.

This chapter introduces the fundamental concepts required for shadow construction. It begins with the definition of shadows and the general principles governing the propagation of light rays. Particular attention is given to the distinction between natural light sources, represented by the sun, and artificial light sources, represented by a point source such as a torch or lamp. The conventions used to define the direction of light and the methods for determining separating lines between illuminated and shaded regions are then presented.

The chapter continues with the study of shadows cast by plane surfaces and geometric solids, including cubes, pyramids, and curved volumes such as cylinders, cones, and spheres. Finally, the construction of shadows in axonometric representations is examined for both natural and artificial lighting conditions, illustrating the practical application of shadow theory in parallel perspective drawings.

By mastering the methods presented in this chapter, students will acquire the skills necessary to accurately represent illuminated objects, analyze the interaction between light and form, and enhance the visual and technical quality of their graphical representations.

1. Definition

Shadows are produced by the sun, which emits light formed by parallel rays.

The aim is not to obtain a realistic representation, but to give a coherent idea of the building's massing. In general, to draw the shadow, a graphic convention is adopted whereby the sun is at the top left of the sheet of paper, and the angle of incidence of the rays is then approximately 45°.

2. General concepts

A body of finite dimensions receives rays of light; it is illuminated by these rays and intercepts them; the various points located in the extension of these rays are not illuminated and are said to be in shadow. There are two types of shadow:

- **The core shadow (intrinsic shadow, form shadow):** This is the part of an object (solid) that is not exposed to the rays of light.

- **The cast shadow:** This is the oblique cylindrical projection of a body's separatrix onto one or more planes, obtained by the intersection of tangent (contact) light rays with the plane(s) casting the shadow.
- **Light rays :**
Natural (parallel): Light rays are parallel (shadow in the sun), and they are very far from the object. There are three types of ray: incident, lateral and tangent (boundary). These last rays (tangents) are of most interest to us because they determine the dividing line between light and shadow.

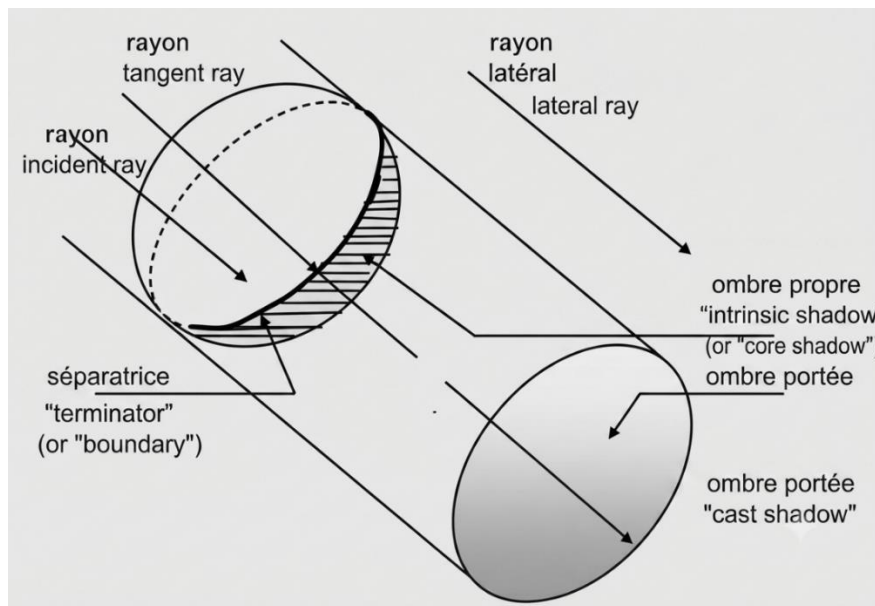


Figure VIII. 1: Shadow elements

Artificial (concentrated): The light rays are centred (candle, lantern... Etc), and they are not parallel. These rays emitted from an artificial light source meet in the same source, because the latter is very close to the object.

The length and direction of the shadow depend on the position of the sun in the sky. It is represented in the solar diagram.

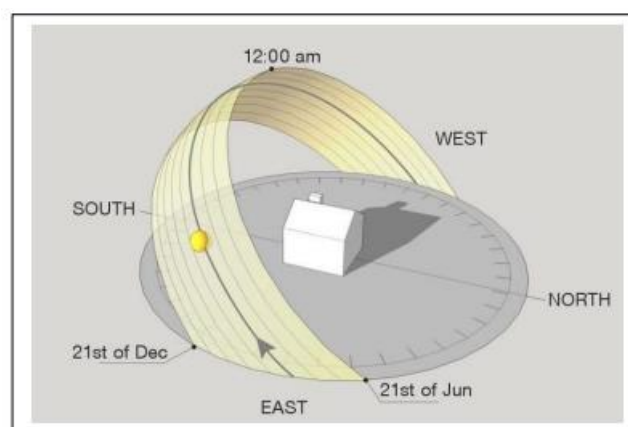


Figure VIII. 2: Solar diagram

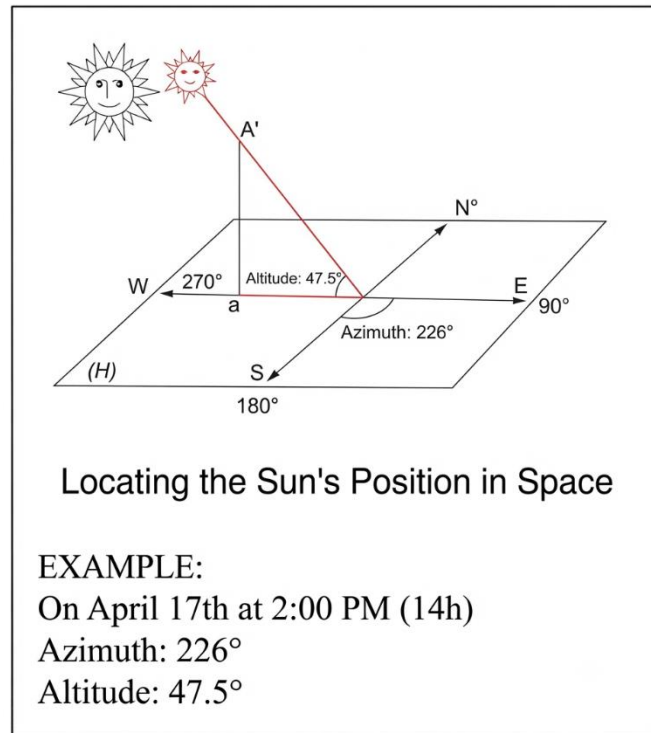


Figure VIII. 3: Projection of the solar rays on the plane

3. Direction of light rays

3.1. Convention

The direction of the light rays is determined by the diagonal of the reference cube (Diagonal AH).

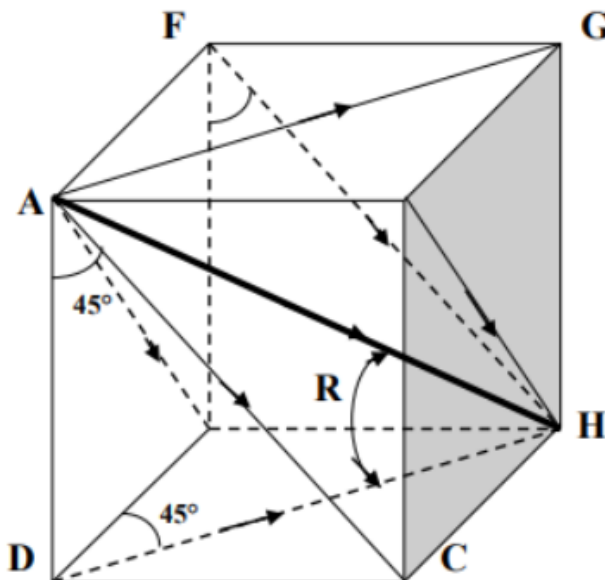


Figure VIII. 4: Convention

3.2. Position of the sun

The sun is located behind the observer at the top left. The different projections of the direction of the light rays (the AH diagonal) are given by the reference cube.

3.3. Determining the separating line

To draw the core and cast shadows of an object, we must first determine the dividing line between light and shadow (i.e. identify the illuminated sides from the non-illuminated sides).
Example: A cube is placed on the ground.

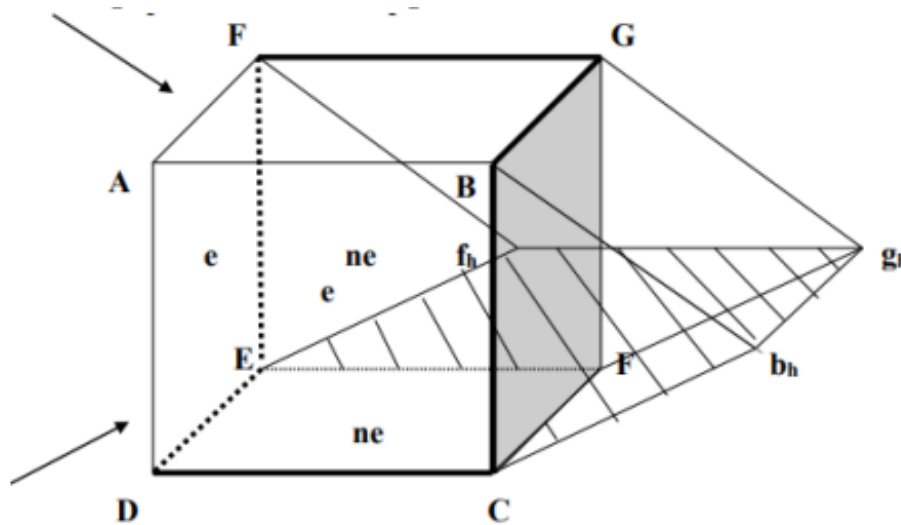


Figure VIII. 5: Determining the separating line

The illuminated faces are : ABCD , AFED , ABGF - The non-illuminated faces are : DCHE , BGFC , FGHE - The dividing line is composed of the succession and articulation of the following edges: DE , EF , FG , GB , BC , CD .

If the line is parallel to the projection plane, its projection - e.g. a shadow - is strictly parallel to it and of the same length.

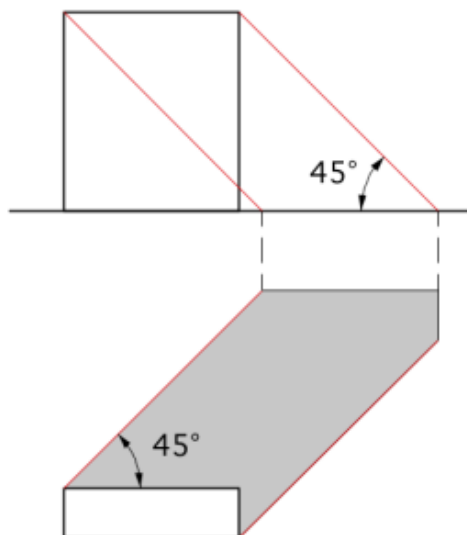


Figure VIII. 6: Shadow on HP and FP

4. Shadow of a surface

For a surface, the shadow will be closed and hatched.

- The illuminated element will always be in the foreground in relation to the shadow.

- The hatching of the shadow must not cover the illuminated element.

4.1. Surface illuminated by the sun (natural)

Example on the frontal plane:

A flat surface parallel to a projection plane, illuminated by the sun, will have a shadow that is equal and parallel to it. The crosshatch of the shadow passes behind the surface.

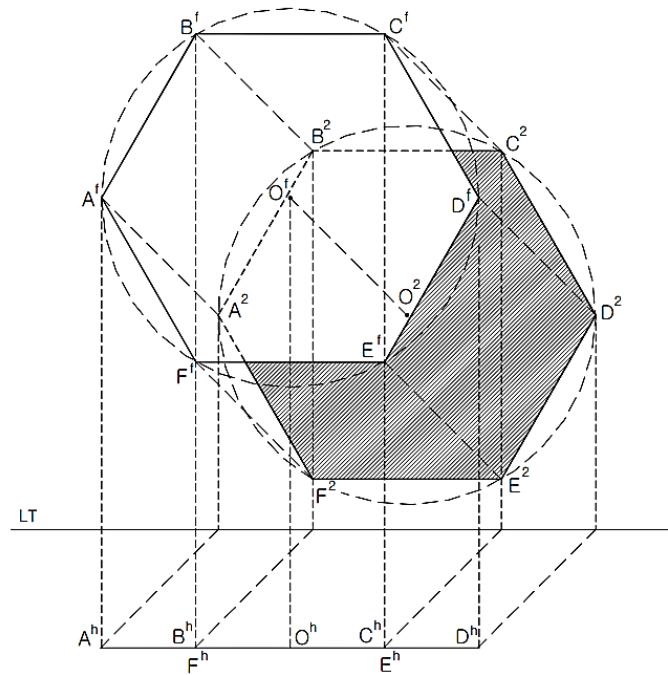


Figure VIII. 7: Shadow of a surface illuminated by natural source

4.2. Surface lit by a torch (artificial)

Example on the frontal plane:

Each point on a surface must be connected to the artificial source (the lines are not parallel). The crosshatch of the shadow passes behind the surface.

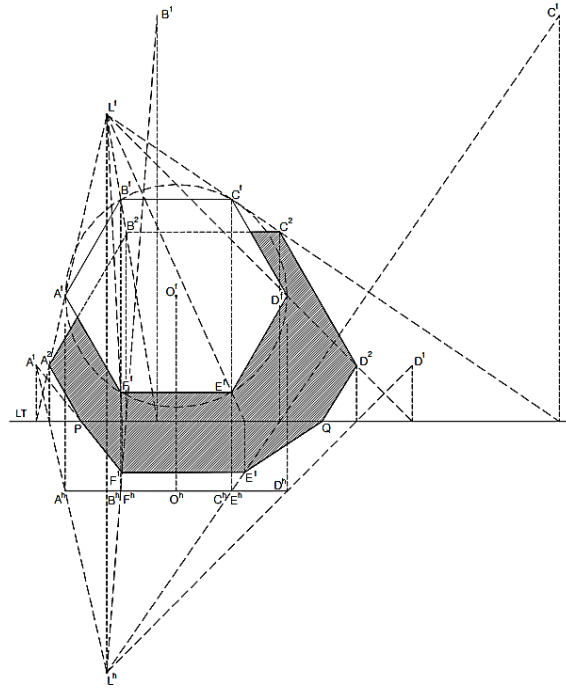


Figure VIII. 8: Shadow of a surface illuminated by artificial source

5. Shadow of a volume

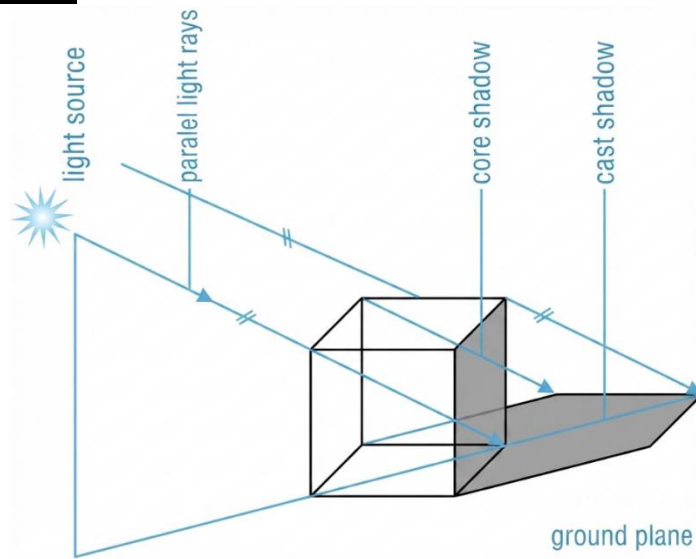


Figure VIII. 9: Shadow of a cube illuminated by natural source

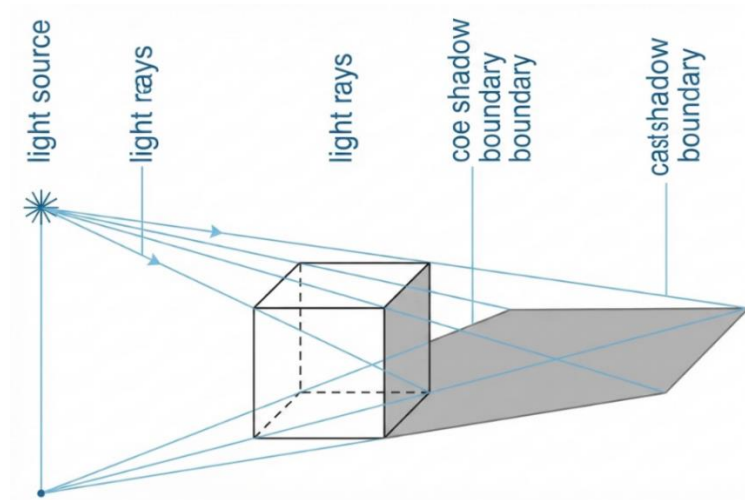


Figure VIII. 10: Shadow of a cube illuminated by artificial source

The solid is illuminated successively by a solar source and then by a point source. The shadow separator corresponds to the edges of the solid.

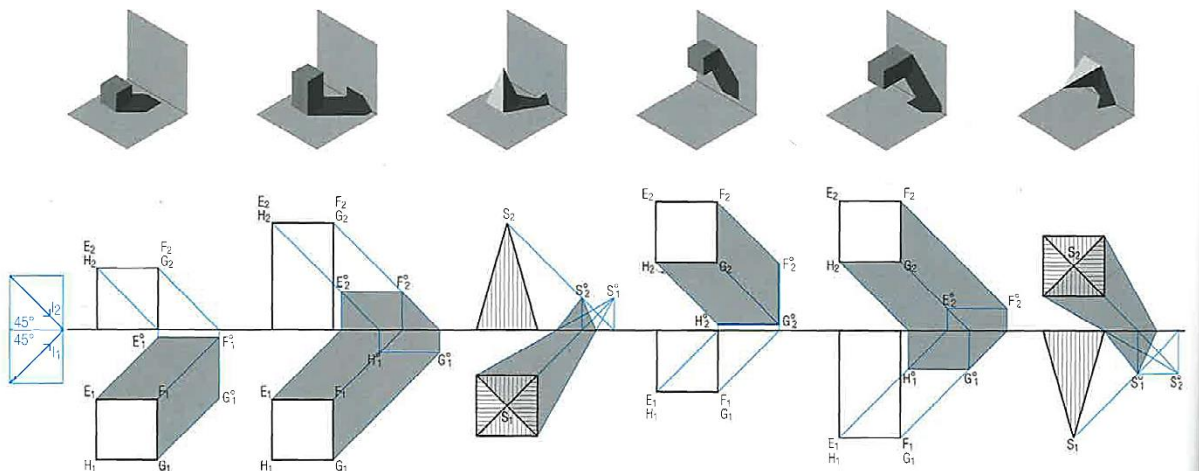


Figure VIII. 11: Shadow of a volumes illuminated by natural source on HP and FP

The solids are placed in the projection dihedron. The lighting is solar. The direction of light corresponds to the diagonal of a cube.

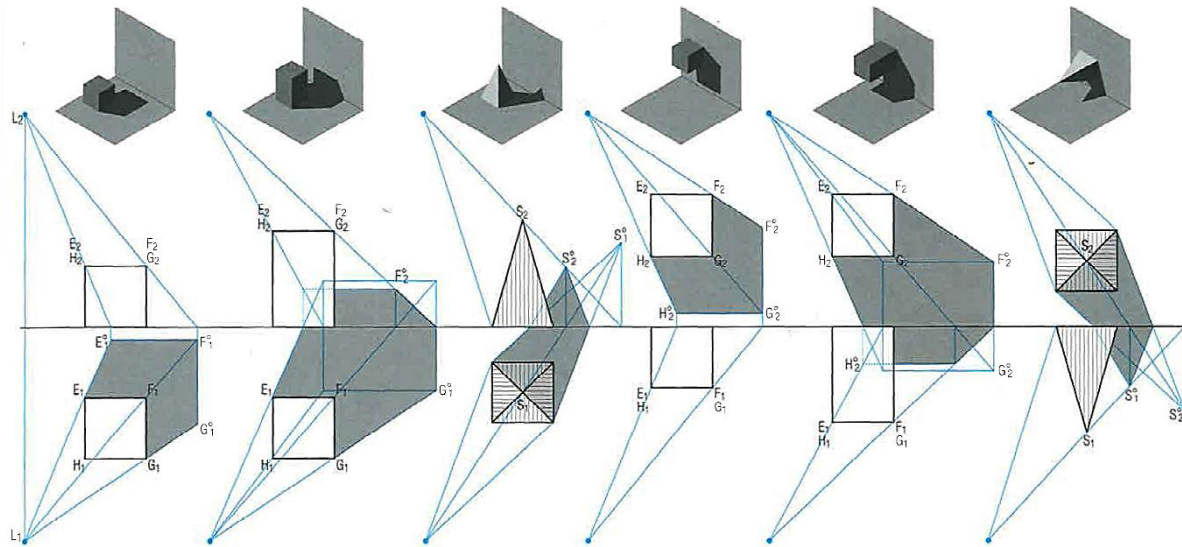


Figure VIII. 12: Shadow of a volumes illuminated by artificial source on HP and FP

The solids are placed in the projection dihedron. The lighting is punctual. Arbitrarily positioned.

5.1. Example of a sunlit cube

Choice of architectural elements: these elements are frequently used for classic shadow drawing exercises.

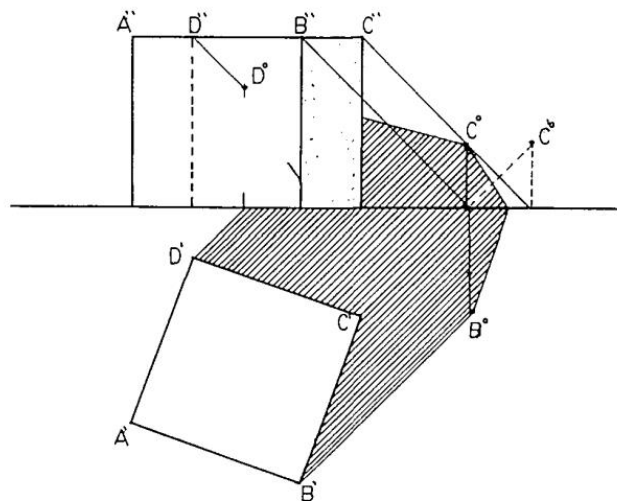


Figure VIII. 13: Example of a sunlit cube

5.2. Example of a sunlit pyramid

The divider passes through points A - S - C - D - A. It is the shadow of this separatrix that will give the cast shadow of the entire volume.

So all you have to do is draw the shadow of the line segments A-S, S-C, C-D and D-A, as explained above.

The crosshatch of the shadow passes behind the projections of the volume.

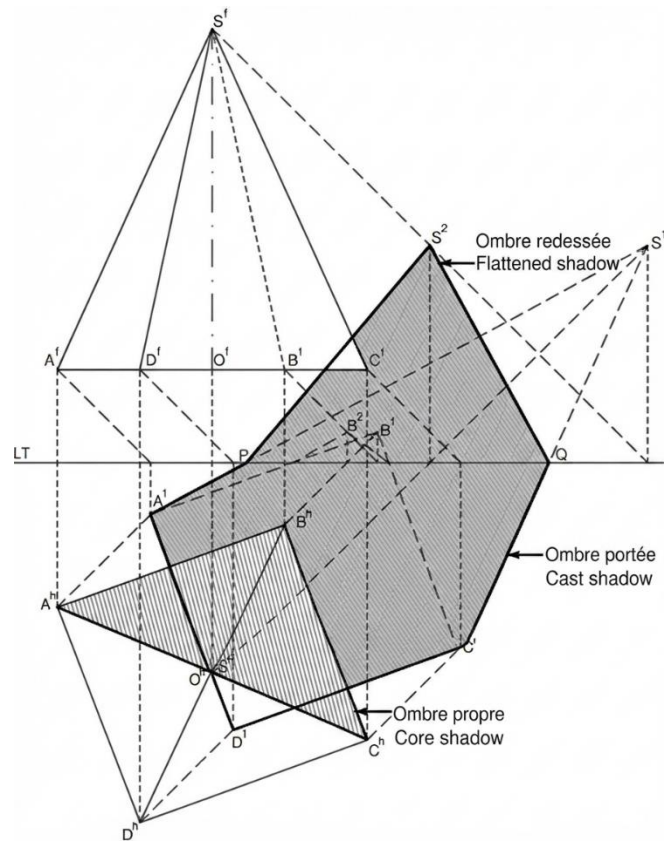


Figure VIII. 14: Example of a sunlit pyramid

5.3. Example of a torch-lit pyramid

The lines forming the shadow are not parallel to each other, but they are parallel to the lines directing the light in the horizontal and frontal planes.

The crosshatch of the shadow passes behind the projections of the volume.

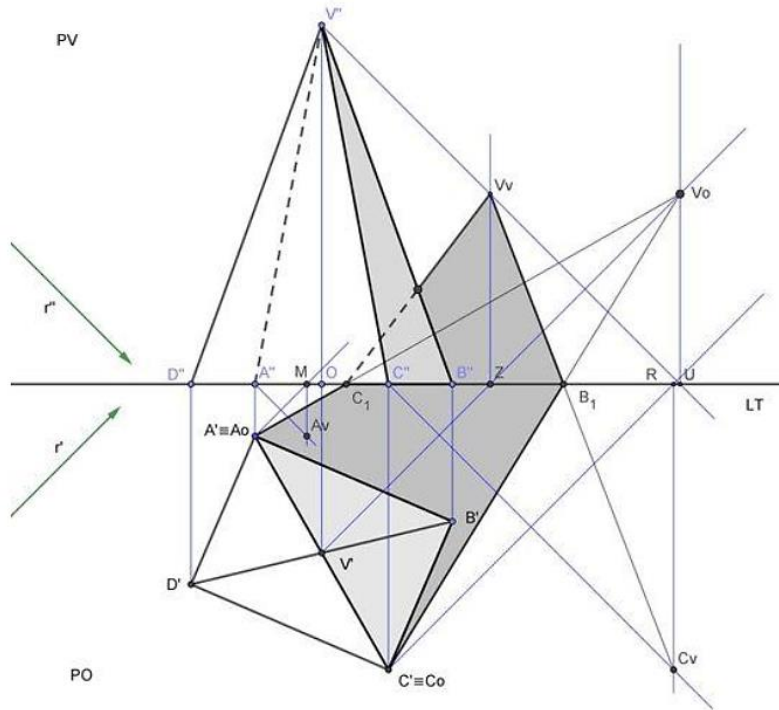


Figure VIII. 15: Example of a torch-lit pyramid

6. Shadow of a round volume

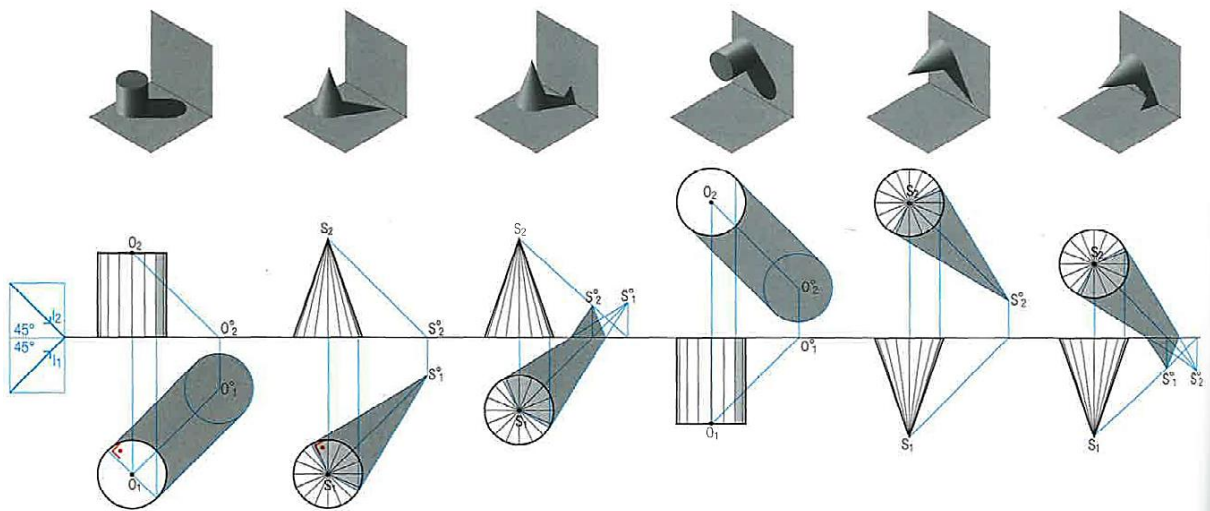


Figure VIII. 16: Shadow of a round volumes illuminated by natural source on HP and FP

The solids are placed in the projection dihedron. The lighting is solar. The direction of light corresponds to the diagonal of a cube.

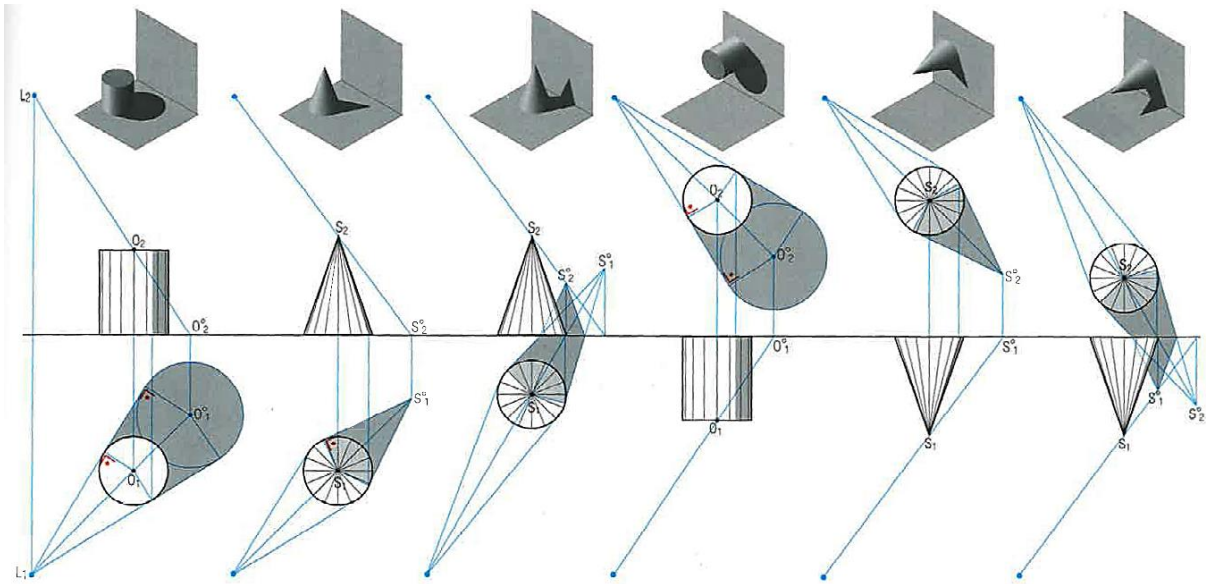


Figure VIII. 17: Shadow of a round volumes illuminated by artificial source on HP and FP

The solids are placed in the projection dihedron. The lighting is punctual. Arbitrarily positioned.

7. Shadow of a parallel perspective (axonometry)

In parallel perspectives, the principle remains the same as for orthogonal projections. For shadows in the sun, all the rays are always parallel to each other in a given direction.

suivant :

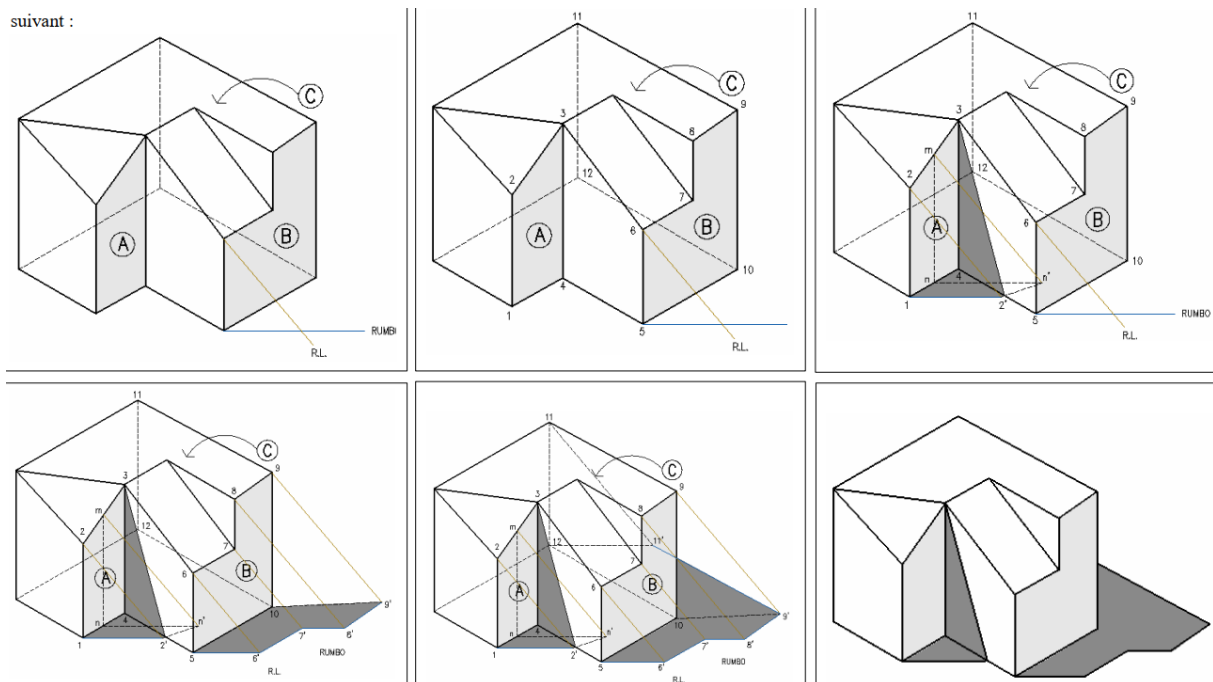


Figure VIII. 18: Shadow of axonometry

7.1. Natural light source

The direction of the shadow on the ground is horizontal, from left to right. The edges above will be struck by a light beam at 45° , also from left to right.

The intersection of the two gives the location of the shadow.

In this example, the sun is positioned laterally to the left (in plane) and at an angle of 45° (in height).

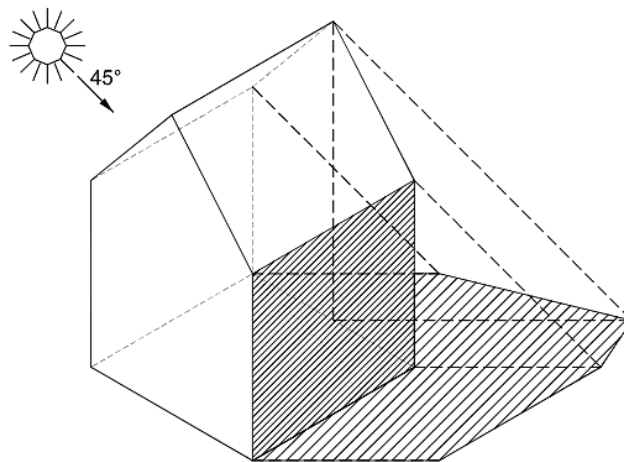


Figure VIII. 19: Shadow of the inclined roofs illuminated by natural source

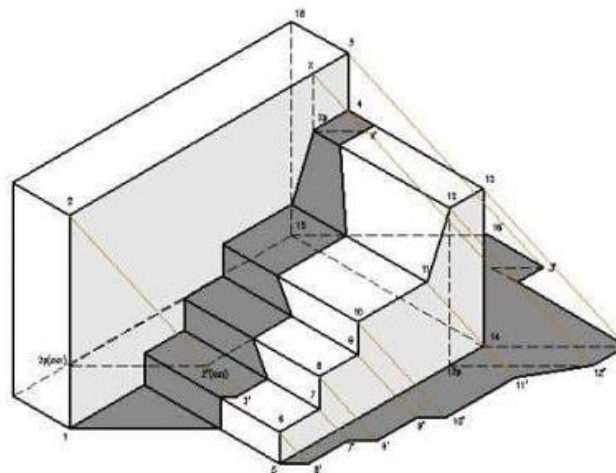


Figure VIII. 20: Shadow of the stairs illuminated by natural source

7.2. Artificial light source

In this example, the light source is a point in space.

The actual location of the torch in the perspective space is the source L of light.

Projecting this point onto the ground gives the L1 source of light.

In this example, the shadow is on the ground, so we only need the L1 projection on the ground (floor).

If the shadow lies on a plinth at a certain height, for example, we will need to project L2 onto the plinth.

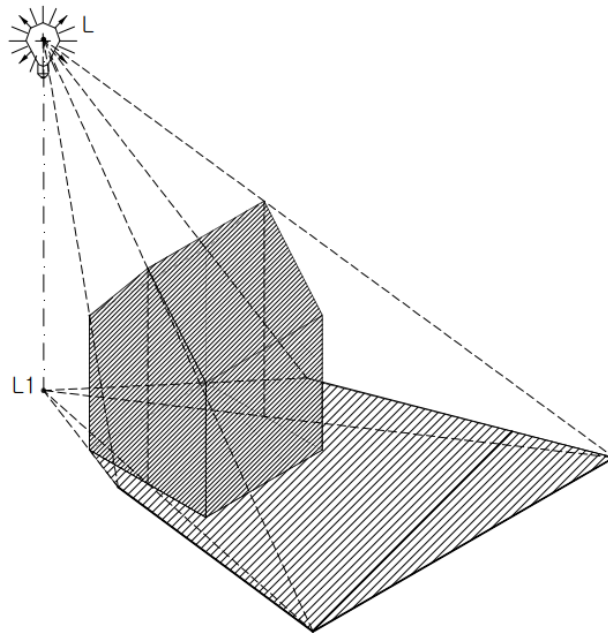


Figure VIII. 21: Shadow of the inclined roofs illuminated by artificial source

Conclusion:

The study of shadows constitutes an essential component of descriptive geometry, as it combines geometric rigor with visual perception. Throughout this chapter, the fundamental principles governing the formation of shadows have been presented, beginning with the concepts of light direction and the conventions used to represent natural and artificial light sources.

The methods for determining the shadows of surfaces and volumes have demonstrated how geometric constructions can accurately reproduce the interaction between light and objects. The analysis of simple solids such as cubes and pyramids has provided a practical understanding of shadow generation, while the treatment of curved volumes has extended these principles to more complex geometries. Furthermore, the application of shadow construction in axonometric representation has highlighted the importance of shadows in enhancing the readability and spatial comprehension of technical drawings.

Beyond their graphical function, shadows play a crucial role in architecture, engineering, design, and urban planning by providing information about form, depth, orientation, and environmental conditions. Mastering shadow construction therefore enables students not only to improve the accuracy of their drawings but also to better understand the relationship between objects, light, and space.

In conclusion, the techniques presented in this chapter provide the necessary foundations for the graphical analysis of illuminated objects and prepare students for more advanced applications in architectural representation, environmental studies, and three-dimensional visualization.

General conclusion:

Descriptive geometry is a fundamental discipline for understanding, analyzing, and accurately representing forms in space. Through the various chapters presented in this course manual, students have progressively acquired the knowledge and methods required to transform the three-dimensional perception of an object into its two-dimensional graphical representation while preserving its geometric accuracy.

The study of projection methods provided the foundations of spatial representation, while the representation of points, lines, and planes supplied the essential tools for solving geometric problems. The concepts of intersections between lines, planes, and solids further demonstrated how to precisely determine spatial relationships among different geometric elements.

The techniques of folding and affinity enriched these analytical methods by enabling the determination of true dimensions and simplifying certain geometric constructions. The representation of common geometric forms such as circles, cones, cylinders, and spheres illustrated the practical application of these principles to more complex objects. The chapters devoted to perspective and axonometric representations highlighted different approaches to visualization, ranging from realistic depiction to technical representation, thus meeting various objectives of graphical communication. Finally, the study of shadows introduced the concepts of light and visual perception, which are essential in the fields of architecture, design, and engineering.

Together, these topics provide an essential foundation for students in architecture, urban planning, civil engineering, and related technical disciplines. Beyond the acquisition of graphical construction techniques, descriptive geometry develops spatial reasoning, analytical rigor, and problem-solving skills for complex representation challenges. Therefore, this course manual serves as a fundamental methodological resource for approaching advanced studies in design, modeling, and graphical communication throughout the students' academic and professional careers.

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