

:Exercises for Chapter 1

:Exercise 1

A warehouse stores products sourced from three factories, with the following quotas: Factory 1 50% Factory 2 30% Factory 3 20% The percentage of defective products in these factories is, respectively 5%, 3%, 2% A product is taken randomly from the warehouse

1. What is the probability that it is undamaged?

2. If this product is not damaged, what is it? Is it possible that it's from the first factory?

:Exercise 2

1. A first sniper typically hits a target at a rate of $\frac{2}{10}$ a second sniper typically hits a target at a rate of $\frac{1}{10}$. They each fire a shot at a target simultaneously.

1. What is the probability of missing the target?

2. What is the probability of hitting the target once?

3. Hitting the target twice. What is the probability of hit?

4. Given that the target was hit once, what is the probability that it was shot by the first sniper?

:Exercise 3

probability distribution of the variable X be :

x	$f(x)$
1	$\frac{1}{4}$
2	a
3	$\frac{1}{4}$
4	$\frac{1}{4}$

Find the value of a ;

2. Find the distribution function.

3. Calculate $V(X)$, $E(X)$.

:Exercise 4

A player throws a die such that: if the number is 1, they score 2 points; if the number is 6, they score 3 points; if the number is from 2 to 5, they score 1 point.

Let X be a random variable representing the player's points.

Find the probability distribution table for the variable X calculate $V(X)$, $E(X)$.

Let's assume the player pays 3 dinars upfront, and when he scores a point he earns 2 dinars. Let Y be a random variable representing the player's profit

Find $V(Y)$ and $E(Y)$.

:Exercise 5

A continuous variable x : defined by the function

$$f(x) = \begin{cases} \frac{3}{64} * x^2, & 0 < x < 4 \\ 0, & x \leq 0, x \geq 4 \end{cases}$$

; Make sure that this function is a probability density function_1

;Find the distribution function .2

: Calculate the probabilities .3 $pr(x \leq 3)$, $pr(1 < x < 2)$

:Exercise 6

x be the : probability density function of a continuous variable

$$f(x) = \begin{cases} a \cdot x^2, & 0 \leq x \leq 1 \\ 0, & x < 0, x > 1 \end{cases}$$

Find the value of .1a ;

;Find the distribution function .2

: Calculate the probabilities .3 $pr(x \leq \frac{1}{2})$, $pr(x > \frac{3}{4})$

:Calculate_4 $E(X)$, $V(X)$

:Solution to Exercise 1

EventA1 represents the product from the first factory, EventA2 the product from the second factory, and EventA3 the product from the third factory. Event B represents the undamaged .product

:(Potential for each factory (quota

$$p(A_1) = 0.5, p(A_2) = 0.3, p(A_3) = 0.2$$

:(probabilities (relations

.tory 2: 0.03, Factory 3: 0.05Factory 1: 0.02, Fac

:Undamaged probabilities respectively

$$p(B/A_1) = 1 - 0.02 = 0.98, p(B/A_2) = 1 - 0.03 = 0.97$$

$$p(B/A_3) = 1 - 0.05 = 0.95$$

:The product may not be damaged_1

$$p(B) = p(A_1) * p(B/A_1) + p(A_2) * p(B/A_2) + p(A_3) * p(B/A_3)$$

$$= 0.5 * 0.98 + 0.3 * 0.97 + 0.2 * 0.95 = 0.97$$

Conditional probability: Probability from the first factory, provided that it is not _2

;damaged

$$p(A_1/B) = \frac{p(A_1 \cap B)}{p(B)} = \frac{p(A_1) * p(B/A_1)}{p(B)} = \frac{0.5 * 0.98}{0.97} = 0.505$$

:Solution to exercise 2

tWe denote the first sniper hit_{as A1} , and the first We denote the second . $\bar{A}$ missed as sniper hit_{as A2} , and the second We denote a single hit on the target as . $\bar{A}$ missed asB.

:Possibility of missing the target .1

$$p(\bar{A}_1 \cap \bar{A}_2) = p(\bar{A}_1 \cdot \bar{A}_2) = p(\bar{A}_1) * p(\bar{A}_2) = 0.8 * 0.9 = 0.72$$

:Probability of hitting the target once_2

$$p(B) = p(A_1 \cap \bar{A}_2) + p(\bar{A}_1 \cap A_2) = 0.2 * 0.9 + 0.1 * 0.8 = 0.26$$

:The possibility of hitting the target twice .3

$$p(A_1 \cdot A_2) = p(A_1) * p(A_2) = 0.2 * 0.1 = 0.02$$

;Conditional probability: The first sniper hit the target once_4

$$p(A_1/B) = \frac{p(A_1 \cap B)}{p(B)} = \frac{p(A_1) * p(B/A_1)}{p(B)} = \frac{p(A_1) * p(A_1 \cdot \bar{A}_2)}{p(B)} = \frac{p(A_1) * p(A_1) * p(\bar{A}_2)}{p(B)}$$

$$= \frac{0.2 * 0.2 * 0.9}{0.26} = \frac{0.036}{0.26} = 0.1384$$

:Solution to Exercise 3

x	f(x)	F(x)	xf (x)	x ² .f(x)
1	1/4	1/4	1/4	1/4
2	a	2/4	2/4	4/4
3	1/4	3/4	3/4	9/4
4	1/4	1	4/4	16/4
	1		E(x)=10/4	E(x ²)=30/4

:We have_1 $\sum f(x) = 1 \Rightarrow \frac{3}{4} + a = 1 \Rightarrow a = \frac{1}{4}$

:Continue the distribution_2 $F(x) = 0, x < 1$

$$F(x) = \begin{cases} 0.25, & 1 \leq x < 2 \\ 0.5, & 2 \leq x < 3 \\ 0.75, & 3 \leq x < 4 \\ F(x) = 1, & x \geq 4 \end{cases}$$

:Mathematical prediction_3 $E(X) = \mu = \sum x \cdot f(x) = \frac{10}{4} = 2.5$

:Contrast $V(X) = \sigma^2 = E(X^2) - \mu^2 = \frac{30}{4} - \left(\frac{10}{4}\right)^2 = \frac{20}{16} = 1.25$

:Solution to exercise 4

Probability distribution table for variable (1X :

Number 1: 2 points, probability1/6

Number 6: 3 points, probability1/6

er (2, 3, 4, 5): 1 point, probabilityNumb4/6

x	F(x)	xf (x)	X ² .f(x)
1	4/6	4/6	4/6
2	1/6	2/6	4/6
3	1/6	3/6	9/6
	1	E(x)=9/6	E(x ²)=17/6

:Mathematical prediction $E(X) = \sum x \cdot f(x) = \frac{9}{6}$

:Contrast $V(X) = E(X^2) - (E(X))^2 = \frac{17}{6} - \left(\frac{9}{6}\right)^2 = \frac{7}{12}$

Player (2Y 's winnings :change with the number of points $Y = 2 \cdot X - 3$

:Mathematical prediction

$$\begin{aligned} E(Y) &= E(2 \cdot X - 3) = E(2X) - E(3) = 2E(X) - 3 \\ &= 2 \cdot \frac{9}{6} - 3 = 3 - 3 = 0 \end{aligned}$$

:Contrast

$$\begin{aligned} V(Y) &= V(2 \cdot X - 3) = V(2X) + V(3) = 4V(X) + 0 \\ &= 4 \cdot \frac{7}{12} = \frac{7}{3} \end{aligned}$$

:Solution to Exercise 5

: Probability density function_1

$$f(x) = \frac{3}{64} \cdot x^2 \geq 0, \quad 0 < x < 4$$

$$\int_0^4 f(x) \cdot dx = 1$$

$$\int_0^4 \frac{3}{64} x^2 \cdot dx = \left[\frac{1}{64} \cdot x^3 \right]_0^4 = \frac{1}{64} \cdot 4^3 - 0 = 1$$

:Distribution function_2

$$F(x) = \int_0^x f(X) \cdot dX = \int_0^x \frac{3}{64} X^2 \cdot dX = \left[\frac{1}{64} \cdot X^3 \right]_0^x = \frac{1}{64} \cdot x^3$$

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{64} \cdot x^3, & 0 < x < 4 \\ 1, & x \geq 4 \end{cases}$$

:Probability_3

$$pr(x \leq 3) = F(3) = \frac{1}{64} \cdot 3^3 = \frac{27}{64}$$

$$pr(1 < x < 2) = F(2) - F(1) = \frac{1}{64} \cdot 2^3 - \frac{1}{64} \cdot 1^3 = \frac{7}{64}$$

:Solution to exercise 6

:probability density function , meaning

$$\int_0^1 f(x) \cdot dx = 1$$

$$\int_0^1 ax^2 \cdot dx = \left[\frac{a}{3} \cdot x^3 \right]_0^1 = \frac{a}{3} \cdot 1^3 - 0 = 1$$

$$\Rightarrow \frac{a}{3} = 1 \Rightarrow a = 3$$

:She writes

$$f(x) = \begin{cases} 3x^2 & , \ 0 \leq x \leq 1 \\ 0 & , \ x < 0, \ x > 1 \end{cases}$$

:Distribution function_2

$$F(x) = \int_0^x f(X).dX = \int_0^x 3X^2.dX = [X^3]_0^x = x^3$$

$$F(x) = \begin{cases} 0 & , \ x < 0 \\ x^3 & , \ 0 \leq x \leq 1 \\ 1 & , \ x > 1 \end{cases}$$

: Calculating probabilities .3

$$pr\left(x \leq \frac{1}{2}\right) = F\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$pr\left(x > \frac{3}{4}\right) = 1 - F\left(\frac{3}{4}\right) = 1 - \frac{27}{64} = \frac{37}{64}$$

:and variance Calculating mathematical expectation_4

$$E(X) = \int_0^1 x.f(x)dx = \int_0^1 3x^3.dx$$

$$= \left[\frac{3}{4} * x^4\right]_0^1 = \frac{3}{4}$$

$$E(X^2) = \int_0^1 x^2.f(x)dx = \int_0^1 3x^4.dx$$

$$= \left[\frac{3}{5} * x^5\right]_0^1 = \frac{3}{5}$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \frac{3}{80}$$