

:Exercises for Chapter Two

:Exercise 1

: A discrete random variable defined by a probability function $f(x) = \frac{1}{n}$

: Where $x = 1, 2, 3, \dots, n$, n is a natural number

Show that $f(x)$ is a probability distribution law

: It was shown that $E(X) = \frac{n+1}{2}$, $V(X) = \frac{n^2-1}{12}$

: Gives

: The sum of the terms $(1 + 2 + 3 + \dots + n) = \frac{n(n+1)}{2}$

: Total of the series $(1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{n(2n+1)(2n+2)}{12}$

:Exercise 2

Two snipers each fire one shot at a target; the first hits at a rate of 0.45, the second hits at a rate of 0.7. The random variable X represents the number of hits for the first sniper, and the random variable Y represents the number of hits for the second sniper

Find the probability distribution for each variable. 1. X and Y and find the mathematical expectancy and variance

Let the random variable $d = XY$. Find the probability distribution of the variable, mathematical expectation and variance of the variable. Calculate and then deduce the mathematical expectation and variance of the variable.

:Exercise 3

defective units was sampled with % A large shipment of a product containing 20 $n=10$ units. We call the random variable x the number of defective units in the sample

. Determine the shape of this distribution. 1

? What is the probability that the defective number is less than or equal to 2. 2

? What is the probability that the defective number is greater than 1. 3

: What is the probability that the defective number is $.41 \leq x \leq 3$

. Find the mathematical expectation and variance. 5

:Exercise 4

An authorized company sold six batteries of the same make; if this type of battery remains usable after two years of use. With a probability of 0.7, calculate the

: probability of

; The six batteries must be in good condition. 1

;That the batteries (6) are faulty_2
 ;That 3 batteries are faulty_3
 .At most, 2 batteries must be faulty_4
 Find .5E(X) and V(X) .

:Exercise 5

Let the probability function of the geometric : variable be $f(x) = \rho \cdot q^{(x-1)}$
 : where $x = 1, 2, 3, \dots \dots k$

$$\rho + q = 1 \quad , \quad 0 \leq \rho, q \leq 1$$

Show that $f(x)$ is a ; probability distribution law

:It was shown that $E(X) = \frac{1}{\rho} \quad , \quad V(X) = \frac{q}{\rho^2}$

: values Gives

$$\sum q^x = \frac{q}{\rho} \quad , \quad \sum x \cdot q^x = \frac{q}{\rho^2} \quad , \quad \sum x^2 \cdot q^x = \frac{q(1+q)}{\rho^3}$$

:Exercise 6

fires shots at a moving target until the % A sniper who usually hits at a rate of 90
 target is hit. We call the random variable x the number of shots with which the
 .target is hit

.Determine the shape of this distribution .1

?What is the probability of hitting the target with shot #2_2

?What is the probability of hitting the target after shot 2_3

.Find the mathematical expectation and variance_4

:Exercise 7

.workers, 3 of whom have seniority exceeding 10 years A production unit with 6
 We randomly selected 4 workers without return, and we call the attribute seniority
 .exceeding 10 years of success

What is it The probability that 2 factors in the sample have this characteristic _is
 $n=4$.

:Exercise 8

A shipment consisting of $N=50$ units of a product, containing 10 defective units, a
 sample of $n=3$ units was taken without returns. We call the random variable x the
 .number of defective units in the sample

.Determine the distribution pattern .1

?What is the probability that the defective number is equal to 2 .2

?What is the probability that the defective number is less than or equal to 1_3

.Find the mathematical expectation and variance_4

:Exercise 9

The average number of cars passing through a given section of road is 1.8 cars per minute. The number of cars passing through in a minute was taken randomly as a random variable x .

1. Determine the distribution pattern of this variable.

2. What is the probability that no car will pass?

3. What is the probability of two cars passing through?

4. What is the probability of at least 1 car passing through?

5. What is the probability of more than one car passing through?

6. What is the probability of 4 cars passing through?

Exercise 10

Let the probability function of the variable poisson be : $f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$

where $x = 0, 1, 2, 3, \dots$

Show that $f(x)$ is a probability distribution law

It was shown that $E(X) = \lambda$, $V(X) = \lambda$

sums values of the : Gives

$$\sum \frac{\lambda^x}{x!} = \sum \frac{\lambda^{(x-1)}}{(x-1)!} = \sum \frac{\lambda^{(x-2)}}{(x-2)!} = \dots = e^{\lambda}$$

Exercise 11

are defective, a sample of lamps % In a lamp factory where 1 of $n=30$ lamps is taken. The random variable x is the number of defective lamps in the sample

What is it The probability of the defective number being greater than 1. Use two distributions binomial and poisson.

Solution to Exercise 1

Probability distribution law is positive, therefore $f(x) = \frac{1}{n} \geq 0$

$$\sum f(x) = 1$$

$$\sum f(x) = \sum_1^n \left(\frac{1}{n}\right) = \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} = n \cdot \frac{1}{n} = 1$$

We will show that mathematical prediction 2

$$E(X) = \sum x f(x) = \sum_1^n x \cdot \frac{1}{n} = \frac{1}{n} \sum_1^n x$$

We substitute the sum of the terms of the sequence

$$E(X) = \frac{1}{n} (1 + 2 + 3 \dots + n) = \frac{1}{n} \left(\frac{n(n+1)}{2} \right) = \frac{n+1}{2}$$

Contrast

$$V(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum x^2 \cdot f(x) = \sum_1^n x^2 \cdot \frac{1}{n} = \frac{1}{n} \sum_1^n x^2$$

:We substitute the sum of the series limits

$$E(X^2) = \frac{1}{n}(1^2 + 2^2 + 3^2 \dots + n^2) = \frac{1}{n} \left(\frac{n(2n+1)(2n+2)}{12} \right) = \frac{(2n+1)(2n+2)}{12}$$

$$V(X) = \frac{(2n+1)(2n+2)}{12} - \left[\frac{n+1}{2} \right]^2 = \frac{4n^2+4n+2n+2}{12} - \frac{n^2+2n+1}{4}$$

$$V(X) = \frac{4n^2+6n+2}{12} - \frac{3n^2+6n+3}{12} = \frac{n^2-1}{12}$$

:cise 2Solution to exer

: Probability distribution_1

:Sniper 1x Bernoulli variable, number of hits $x = 1, 0$

:Probability of success $p_1 = 0.45$:Probability of failure $q_1 = 0.55$

: Probability distribution table

x	f(x)
1	0.45
0	0.55
	1

:Mathematical prediction $E(X) = p_1 = 0.45$

:Contrast $V(X) = p_1 * q_1 = 0.2475$

:The second snipery, Bernoulli variable, number of hits $y = 1, 0$

:Probability of success $p_2 = 0.7$ (Possibility of not getting sick (failure $q_2 = 0.3$:

: Probability distribution table

y	f(y)
1	0.7
0	0.3
	1

:Mathematical prediction $E(Y) = p_2 = 0.7$

:Contrast $V(Y) = p_2 * q_2 = 0.21$

:Variable_2d=xy

: Common probability distribution

YX	1 0.45	0 0.55
1	0 $f(1,1)=f(1).f(1)$ $0.45 * 0.7 = 0.315$	-1 $f(0,1)=f(0).f(1)$ $0.55 * 0.7 = 0.385$
0	1 $f(1,0)=f(1).f(0)$ $0.45 * 0.3 = 0.135$	0 $f(0,0)=f(0).f(0)$ $0.55 * 0.3 = 0.165$

The values of the variable : $-1, 0, 1$

: Possibilities 0.385 , 0.48 , 0.135

: Probability distribution table

d	f(d)	df (d)	d ² .f(d)
-1	0.385	-0.385	0.385
0	0.48	0	0
1	0.135	0.135	0.135
	1	E(d)=-0.25	E(d ²)=0.52

:Mathematical prediction and variance, from the table_3

$$E(d) = \sum d \cdot f(d) = -0.25$$

$$V(d) = E(d^2) - [E(d)]^2 = 0.52 - (-0.25)^2 = 0.4575$$

:Inference of expectancy and variance x and y :are independent, therefore

$$E(d) = E(x - y) = E(x) - E(y) = 0.45 - 0.7 = -0.25$$

$$V(d) = V(x - y) = V(x) + V(y) = 0.2475 + 0.21 = 0.4575$$

:Solution to Exercise 3

Distribution pattern: Large charge_1N :Large, consistent (flawed) success rate

$$\rho = 0.2$$

:Binomial distribution $f(x) = c_n^x \cdot \rho^x \cdot q^{n-x}$

$$f(x) = c_{10}^x \cdot 0.2^x \cdot 0.8^{10-x}$$

$$x = 0, 1, 2, \dots, 10$$

:the defective number is less than or equal to 2 that The probability .2

$$\begin{aligned} pr(x \leq 2) &= f(0) + f(1) + f(2) = c_{10}^0 \cdot 0.2^0 \cdot 0.8^{10} + c_{10}^1 \cdot 0.2^1 \cdot 0.8^9 \\ &+ c_{10}^2 \cdot 0.2^2 \cdot 0.8^8 = 0.107374 + 0.268435 + 0.301989 = 0.67779 \end{aligned}$$

:The probability that the defective number is greater than 1 .3

$$pr(x > 1) = 1 - pr(x \leq 1) = 1 - f(0) - f(1) = 1 - 0.375809 = 0.62419$$

:The probability that the defective number is between 1 and 3_4

$$\begin{aligned} pr(1 \leq x \leq 3) &= f(1) + f(2) + f(3) = 0.570424 + c_{10}^3 \cdot 0.2^3 \cdot 0.8^7 \\ &= 0.570424 + 0.201326 = 0.77175 \end{aligned}$$

:Mathematical prediction and variance_5

$$E(X) = n \cdot \rho = 10 * 0.2 = 2$$

$$V(X) = n \cdot \rho \cdot q = 10 * 0.2 * 0.8 = 1.6$$

:Solution to exercise 4

The binomial distribution n=6 ρ :(with a probability of success (validity ,= 0.7

The number of usable batteries is a random variable x .

: Probability function

$$f(x) = c_6^x \cdot 0.7^x \cdot 0.3^{6-x}$$

$$x = 0, 1, 2, \dots, 6$$

:Probability of all six batteries being valid .1x=6

$$f(6) = c_6^6 \cdot 0.7^6 \cdot 0.3^0 = 0.117649$$

The probability that all 6 batteries are faulty: therefore, the number of faulty batteries is x=0

$$f(0) = c_6^0 \cdot 0.7^0 \cdot 0.3^6 = 0.000729$$

The probability of 3 batteries being faulty: therefore, the number of faulty batteries is n-3=6-3=3

$$f(3) = c_6^3 \cdot 0.7^3 \cdot 0.3^3 = 0.018522$$

:There is a possibility that at most 2 batteries are faulty_4

:At most 2 are invalid 0, 1, 2 =>> ≤ 2

:Validity: at least 4x = 6, 5, 4 =>> $x \geq 4$, (6 - 2 = 4)

$$\begin{aligned} pr(x \geq 4) &= f(4) + f(5) + f(6) = \\ &= c_6^4 \cdot 0.7^4 \cdot 0.3^2 + c_6^5 \cdot 0.7^5 \cdot 0.3^1 + c_6^6 \cdot 0.7^6 \cdot 0.3^0 = \\ &= 0.324135 + 0.302526 + 0.117649 = 0.74431 \end{aligned}$$

Mathematical prediction and variance_5E(X), V(X):

$$E(X) = n \cdot \rho = 6 * 0.7 = 4.2$$

$$V(X) = n \cdot \rho \cdot q = 6 * 0.7 * 0.3 = 1.26$$

:Solution to Exercise 5

: Probability distribution law $0 \leq \rho$, $q \leq 1$: permission $f(x) = \rho \cdot q^{x-1} \geq 0$

$$\sum f(x) = 1$$

$$\sum f(x) = \sum \rho \cdot q^{x-1} = \rho * \sum \frac{q^x}{q} = \frac{\rho}{q} * \sum q^x = \frac{\rho}{q} * \frac{q}{\rho} = 1$$

:We will show that mathematical prediction .2

$$E(X) = \sum x f(x) = \sum x \cdot \rho \cdot q^{x-1} = \frac{\rho}{q} * \sum x \cdot q^x = \frac{\rho}{q} * \frac{q}{\rho^2} = \frac{1}{\rho}$$

:Contrast

$$V(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum x^2 \cdot f(x) = \sum x^2 \cdot \rho \cdot q^{x-1} = \frac{\rho}{q} * \sum x^2 \cdot q^x = \frac{\rho}{q} * \frac{q(1+q)}{\rho^3} = \frac{1+q}{\rho^2}$$

$$V(X) = \frac{1+q}{\rho^2} - \left(\frac{1}{\rho}\right)^2 = \frac{1+q-1}{\rho^2} = \frac{q}{\rho^2}$$

:Solution to exercise 6

the random ;The distribution pattern is geometric: each shot either hits or misses_1
variable x .The attempt that results in an injury

:on rate) is constantThe success rate (infecti $\rho = 0.9$

:Failure rate $q = 1 - \rho = 0.1$

:Geometric distribution function $f(x) = \rho \cdot (1 - \rho)^{x-1}$

$$f(x) = 0.9 * 0.1^{(x-1)}$$

$$x = 1, 2, \dots, k$$

:Probability of hitting the target with shot #2 .2

$$pr(x = 2) = f(2) = 0.9 * 0.1^1 = 0.09$$

:Possibility of injury after shot 2 .3

$$pr(x > 2) = 1 - pr(x \leq 2) = 1 - f(1) - f(2) = 1 - 0.9 - 0.09 = 0.01$$

:Mathematical prediction and variance _4

$$E(X) = \frac{1}{\rho} = \frac{1}{0.9} = 1.1111$$

$$V(X) = \frac{1-\rho}{\rho^2} = \frac{0.1}{0.9^2} = 0.1234$$

:Solution to exercise 7

The attribute of seniority exceeding 10 years of success: the number of workers corresponding to success $r=3$ the number of workers corresponding to failure , $Nr=6-3=3$.

:Number of workers $N=6$,i.e $\{ 1, 1, 1, 0, 0, 0 \}$

Sample $n=4$;ess rate changes without return: $\text{Succ} \frac{r}{N} = \frac{3}{6}, \frac{3}{5}, \frac{2}{5}, \dots$

: geometric distribution -Ultra

$$f(x) = \frac{c_r^x c_{(N-r)}^{(n-x)}}{c_N^n} = \frac{c_3^x c_3^{4-x}}{c_6^4}$$

$$x = 1, 2, 3$$

The probability that two factors have this property $x=2$:

$$f(2) = \frac{c_3^2 c_3^{4-2}}{c_6^4} = \frac{3*3}{15} = \frac{3}{5}$$

:Solution to exercise 8

Distribution format: Shipment of product_1 $N=50$ number of defective units , (successes) $r=10$ sample , $n=3$.without return

:The success rate (flawed) is not constant $\frac{r}{N} = \frac{10}{50}, \frac{10}{49}, \frac{9}{49}, \dots$

x Number of defective units taken from among 10

$(3-x)$ defective units is taken from-The number of non $50 - 10 = 40$

: geometric distribution -Ultra $f(x) = \frac{c_r^x c_{(N-r)}^{(n-x)}}{c_N^n}$

$$f(x) = \frac{c_{10}^x c_{40}^{3-x}}{c_{50}^3}$$

$$x = 0, 1, 2, 3$$

:the defective number is equal to 2 that The probability .2

$$pr(x = 2) = f(2) = \frac{c_{10}^2 c_{40}^1}{c_{50}^3} = \frac{10! / 2! 8! * 40! / 1! 39!}{50! / 3! 47!}$$

$$= \frac{1800}{19600} = 0.0918$$

:The probability that the defective number is less than or equal to 1 .3

$$\begin{aligned} pr(x \leq 1) &= f(0) + f(1) = \frac{c_{10}^0 c_{40}^3}{c_{50}^3} + \frac{c_{10}^1 c_{40}^2}{c_{50}^3} \\ &= \frac{10! / 0! 10! * 40! / 3! 37!}{50! / 3! 47!} + \frac{10! / 1! 9! * 40! / 2! 38!}{50! / 3! 47!} = \frac{17680}{19600} = 0.9020 \end{aligned}$$

:Mathematical prediction and variance_5

$$\begin{aligned} E(X) &= n * \frac{r}{N} = 3 * \frac{10}{50} = 0.6 \\ V(X) &= n * \frac{r}{N} * \frac{(N-r)}{N} * \left(\frac{N-n}{N-1} \right) = 3 * \frac{10}{50} * \frac{40}{50} * \frac{47}{49} = 0.4604 \end{aligned}$$

:exercise 9 Solution to

:Distribution pattern _1

The number of cars per minutex is a discrete random variable that follows a Poisson distribution with a success rate of 1.8λ ,

: Probability function $f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$

$$\begin{aligned} f(x) &= \frac{(1.8)^x \cdot e^{-(1.8)}}{x!} \\ x &= 0, 1, 2, 3 \dots \end{aligned}$$

:The possibility that no cars will pass .2

$$f(0) = \frac{(1.8)^0 \cdot e^{-(1.8)}}{0!} = e^{-1.8} = 0.165298$$

3. Probability of two cars passing: 2

$$f(2) = \frac{(1.8)^2 \cdot e^{-(1.8)}}{2!} = \frac{3.24 * e^{-1.8}}{2} = 0.267784$$

4 :at least 1 car passing by Probability of -

$$pr(x \geq 1) = 1 - pr(x < 1) = 1 - f(0) = 1 - 0.165298 = 0.834701$$

5. :Possibility of more than one car passing through

$$\begin{aligned} pr(x > 1) &= 1 - pr(x \leq 1) = 1 - f(0) - f(1) = \\ &= 1 - 0.165298 - \frac{(1.8)^1 \cdot e^{-(1.8)}}{1!} = 1 - 0.165498 - 0.297537 = 0.537163 \end{aligned}$$

:ing throughPotential of 2 to 4 cars pass_6

$$\begin{aligned} pr(2 \leq x \leq 4) &= f(2) + f(3) + f(4) = 0.267784 + \frac{(1.8)^3 \cdot e^{-(1.8)}}{3!} + \frac{(1.8)^4 \cdot e^{-(1.8)}}{4!} \\ &= 0.267784 + 0.1606705 + 0.07230173 = 0.500756 \end{aligned}$$

:Solution to exercise 10

: Probability distribution law λ :is a positive constant, therefore $f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!} \geq$

0

$$\sum f(x) = 1$$

:We substitute the total $\sum \frac{\lambda^x}{x!} = e^{+\lambda}$

$$\sum f(x) = \sum \frac{\lambda^x \cdot e^{-\lambda}}{x!} = e^{-\lambda} \cdot \sum \frac{\lambda^x}{x!} = e^{-\lambda} \cdot e^{+\lambda} = e^0 = 1$$

:We will show that mathematical prediction .2

$$E(X) = \sum x f(x) = \sum \frac{x \cdot \lambda^x \cdot e^{-\lambda}}{x!} = e^{-\lambda} * \sum \frac{x \cdot \lambda^x}{x!} = e^{-\lambda} * \sum \frac{\lambda^x}{(x-1)!}$$

:We substitute the total

$$E(X) = e^{-\lambda} * \sum \frac{\lambda^{x+1} \cdot \lambda^{(x-1)}}{(x-1)!} = e^{-\lambda} * \lambda \cdot \sum \frac{\lambda^{(x-1)}}{(x-1)!} = e^{-\lambda} \cdot \lambda \cdot e^{+\lambda} = \lambda$$

:Contrast

$$V(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum x^2 \cdot f(x) = \sum \frac{x^2 \cdot \lambda^x \cdot e^{-\lambda}}{x!} = e^{-\lambda} * \sum \frac{x^2 \cdot \lambda^x}{x!} = e^{-\lambda} * \sum \frac{x \cdot \lambda^x}{(x-1)!}$$

$$E(X^2) = e^{-\lambda} * \sum \frac{x \cdot \lambda^{(x-1)} \cdot \lambda^{+1}}{(x-1)!} = e^{-\lambda} * \lambda \cdot \sum \frac{x \cdot \lambda^{(x-1)}}{(x-1)!} = e^{-\lambda} * \lambda \cdot \sum \frac{(x-1+1) \cdot \lambda^{(x-1)}}{(x-1)!}$$

$$E(X^2) = e^{-\lambda} * \lambda \cdot \sum \frac{(x-1) \cdot \lambda^{(x-1)} + \lambda^{(x-1)}}{(x-1)!} = e^{-\lambda} * \lambda \left[\sum \frac{(x-1) \cdot \lambda^{(x-1)}}{(x-1)!} + \sum \frac{\lambda^{(x-1)}}{(x-1)!} \right]$$

$$E(X^2) = e^{-\lambda} * \lambda \left[\sum \frac{\lambda^{(x-1)}}{(x-2)!} + \sum \frac{\lambda^{(x-1)}}{(x-1)!} \right] = e^{-\lambda} * \lambda \left[\sum \frac{\lambda^{(x-2)} \cdot \lambda^{+1}}{(x-2)!} + \sum \frac{\lambda^{(x-1)}}{(x-1)!} \right]$$

$$E(X^2) = e^{-\lambda} * \lambda \left[\lambda \cdot \sum \frac{\lambda^{(x-2)}}{(x-2)!} + \sum \frac{\lambda^{(x-1)}}{(x-1)!} \right]$$

:We compensate for the totals

$$E(X^2) = e^{-\lambda} * \lambda [\lambda \cdot e^{+\lambda} + e^{+\lambda}] = \lambda(\lambda + 1) = \lambda^2 + \lambda$$

$$V(X) = (\lambda^2 + \lambda) - (\lambda)^2 = \lambda$$

:Solution to exercise 11

The number of defective lamps (successes) in the sample is a random variable following a binomial distribution, with a success rate of 0.01 . ρ

Sample $n=30$: $x = 0, 1, 2, \dots, 30$

Distribution of Binomial :

: Probability function $f(x) = c_n^x \cdot \rho^x \cdot q^{n-x}$

$$f(x) = c_{30}^x \cdot 0.01^x \cdot 0.99^{30-x}$$

$$x = 0, 1, 2, \dots, 30$$

:the defective number is greater than 1 that The probability

$$\begin{aligned} pr(x > 1) &= 1 - pr(x \leq 1) = 1 - f(0) - f(1) = \\ &= 1 - c_{30}^0 \cdot 0.01^0 \cdot 0.99^{30} - c_{30}^1 \cdot 0.01^1 \cdot 0.99^{29} = \\ &= 1 - 0.7397003 - 0.224151 = 0.036149 \end{aligned}$$

Poisson : distribution

:Approximation of a binomial to Poisson , condition for approximation satisfied

$$\begin{cases} n \geq 30 \\ n \cdot \rho < 5 \end{cases}$$

$$n \cdot \rho = 30 * 0.01 = 0.3$$

$$\text{: Probability function Poisson } f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$\lambda = n \cdot \rho = 0.3$$

$$f(x) = \frac{0.3^x \cdot e^{-0.3}}{x!}$$

$$x = 0, 1, 2, \dots, 30$$

:the defective number is greater than 1 that The probability

$$\begin{aligned} pr(x > 1) &= 1 - pr(x \leq 1) = 1 - f(0) - f(1) = \\ &= 1 - \frac{0.3^0 \cdot e^{-0.3}}{0!} - \frac{0.3^1 \cdot e^{-0.3}}{1!} = 1 - e^{-0.3} - 0.3 * e^{-0.3} = \\ &= 1 - 0.740818 - 0.222245 = 0.036937 \end{aligned}$$