Exercises in Combinatorial Analysis

Exercise 01

A combination padlock has three dials, each marked with the digits 0 through 9. How many secret passcodes (“numbers”) are possible?

Exercise 02

Telephone area codes consist of three digits. The first digit is an integer from 2 to 9 (inclusive), the second digit is either 0 or 1, and the third digit is an integer from 1 to 9 (inclusive).

a. How many distinct area codes are possible?

b. How many possible area codes begin with 4?

Exercise 03

How many distinct 6-digit integers exist under each of the following conditions?

a. There are no restrictions.

b. The numbers must be divisible by 5.

c. Digit repetitions are excluded.

Exercise 04

In how many ways can 3 Physics books, 2 Mathematics books, and 1 Chemistry book be arranged on a shelf if:

a. there are no restrictions;

b. the Mathematics books must be placed together and the Physics books must also be placed together;

c. the Physics books are separated by the Mathematics books?

Exercise 05

From a group of 5 women and 7 men, how many distinct committees consisting of 2 women and 3 men can be formed? What if 2 of the men are on poor terms and refuse to serve together on the same committee?

Exercise 06

In how many ways can 4 boys and 3 girls be seated in a single row? What if:

a. all boys must sit together and all girls must also sit together;

b. only the boys must sit together;

c. no two individuals of the same sex may sit adjacent to one another?

Exercise 07

Determine the number of 10-digit telephone numbers such that:

a. the number is formed using two 1s, two 3s, and six 7s;

b. the number is formed using exactly two distinct digits;

c. the number contains exactly three 1s.

Exercise 08 (Quiz)

MCQ 1

A child has 5 coins of different denominations. How many different monetary sums can be formed?

A) 25 B) 20 C) 31 D) 32

MCQ 2

How many answer keys exist for a 12-question multiple-choice quiz with 4 choices per question, where each question has one and only one correct answer?

A) 11,880 B) 20,736 C) 16,777,216 D) 495

A hospital has 15 candidate patients for an experimental protocol. These patients are followed by a team of 15 physicians from the hospital department.

SCQ 3

Three distinguishable groups of 5 patients each must be formed (Treatment A, Treatment B, Treatment C). How many distinct allocations can be made?

A) $15! / (5! \times 5! \times 5!)$ B) $C_{15}^{5} \times C_{10}^{5}$ C) 15^3 D) $15! / ((5!)^3 \times 3!)$

SCQ 4

For a specific study, 7 patients are selected out of the 15, requiring that 2 immunocompromised patients, P1 and P2, be mandatorily included. How many choices are possible?

A) C_{15}^{7} B) C_{13}^{5} C) C_{13}^{7} D) C_{13}^{11}

SCQ 5

Among the 15 physicians in the department, a chair of the scientific committee and a pair of rapporteurs (2 rapporteurs) must be designated. How many distinct selections can be made?

A) $15 \times C_{14}^{2}$ B) C_{15}^{3} C) $15 \times 14 \times 13$ D) $15! / 12!$

SCQ 6

The 15 patients are divided into two groups: a Medication group (at least 1 patient) and a Placebo group (at least 1 patient). How many partitions are possible?

A) 2^{15} B) $2^{15} - 2$ C) C_{15}^{7} D) $15! / (7! \times 8!)$

The keypad shown below is located at the entrance of a building. To unlock it, a code consisting of three (03) digits followed by two letters must be entered. The digits must be distinct, whereas the letters need not be.

Numeric Keys			Letter Keys			
1	2	3	A	B	C	D
4	5	6				
7	8	9				

SCQ 7

What is the number of possible codes?

A) 6,048 B) 8,064 C) 3,024 D) 1,344 E) None of the above (ARNJ)

SCQ 8

What is the number of possible codes if the three digits have the same parity (all 3 are even or all 3 are odd)?

A) 1,008 B) 224 C) 84 D) 1,344 E) None of the above (ARNJ)