

Solution de TD 01

Exercice 01 :

$$T_{01}=292K, P_{01}= 1\text{bar}, \eta_s=0.85, \gamma=1.4$$

1- Calcul la température finale :

$$\frac{P_{02}}{P_{01}} = \left(\frac{T_{02S}}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}} \Rightarrow T_{02S} = T_{01} \left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} = 555.92K$$

$$T_{02S} = 555.92K$$

$$\eta_s = \frac{T_{02S} - T_{01}}{T_{02} - T_{01}} \Rightarrow T_{02} = T_{01} + \frac{T_{02S} - T_{01}}{\eta_s}$$

$$\text{A.N. } T_{02} = 602.49K$$

Calcule le travail de la compression à la sortie :

$$w = Cp(T_{02} - T_{01}) = 1005(602.49 - 292) = 312kJ/kg$$

$$w = 312kJ/kg$$

Exercice 02 :

$$R_s = 1.22, T_{01} = 288K, \Delta T_{0S} = 21K$$

1- Calcule le rendement isentropique

$$R_s = \left(1 + \frac{\eta_s \Delta T_{0S}}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \eta_s = \frac{T_{01}}{\Delta T_{0S}} \left(R_s^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$\eta_s = 0.8026$$

$$U = \frac{\pi D N}{60} \Rightarrow D = \frac{60.200}{\pi(45000)}$$

$$D = 0.42m$$

Exercice 03 :

$$D_{sup} = 0.95m, \quad D_{inf} = 0.85m, \quad \alpha_1 = 28^\circ, \quad \beta_1 = 56^\circ, \quad \alpha_2 = 56^\circ, \quad \beta_2 = 28^\circ,$$

$$N = \frac{5000tr}{m}, \quad \rho = 1.2kg/m^3$$

1- La viresse de rotor :

$$U = \frac{\pi DN}{60} = \frac{\pi 0.95.5000}{60} = 249m/s$$

$$U = 249m/s$$

2- La vitesse axiale :

$$\frac{U}{Ca} = tg\alpha_1 + tg\beta_1 \Rightarrow Ca = \frac{U}{tg\alpha_1 + tg\beta_1} = 123.6m/s$$

$$Ca = 123.6m/s$$

3- Le débit massique :

$$\dot{m} = A\rho Ca = \pi(D_{sup}^2 - D_{inf}^2)/4\rho Ca$$

$$\dot{m} = \frac{\pi(0.95^2 - 0.85^2)}{4(1.2)(123.6)} = 20.98kg/s$$

$$\dot{m} = 20.98kg/s$$

4- Le travail echangé :

$$W = UCa (tg\beta_1 - tg\beta_2) \Rightarrow W = 29.86kJ/kg$$

5- La puissance:

$$P = \dot{m}W = 614kW$$

6- Les angles au diamètre inferieur:

$$A \text{ l'entrée : } C_{w1} = Ca \, tg\alpha_1 = 123.6tg28 = 65.72m/s$$

$$On \text{ à aussi : } C_{wi}r_i = cst \Rightarrow C_{w1inf}r_{inf} = C_{w1sup}r_{sup}$$

$$C_{W1inf} = C_{W1sup} \frac{r_{sup}}{r_{inf}} = \frac{(65.72)(0.475)}{0.425} = 73.45m/s$$

Donc : $C_{W1inf} = 73.45m/s$

A la sortie :

$$C_{W2sup} = Ca \operatorname{tg} \alpha_2 = 183.24 \operatorname{tg} 56 = 183.24m/s$$

On à aussi : $C_{W2inf} = C_{W2sup} \frac{r_{sup}}{r_{inf}} = \frac{(183.24)(0.475)}{0.425} = 204.8m/s$

Donc : $C_{W2inf} = 204.8m/s$

Finalement :

$$\operatorname{tg} \alpha_1 = \frac{C_{W1inf}}{Ca} = \frac{73.45}{123.6} \Rightarrow \alpha_1 = 30.72^\circ$$

$$\operatorname{tg} \beta_1 = \frac{u_{inf}}{Ca} - \operatorname{tg} \alpha_1 = \frac{223}{123.6} - \operatorname{tg} 30.72 \Rightarrow \beta_1 = 50.43^\circ$$

$$\operatorname{tg} \alpha_2 = \frac{C_{W2inf}}{Ca} = \frac{204}{123.6} \Rightarrow \alpha_2 = 58.69^\circ$$

$$\operatorname{tg} \beta_2 = \frac{u_{inf}}{Ca} - \operatorname{tg} \alpha_2 = \frac{2.4}{123.6} - \operatorname{tg} 8.69 \Rightarrow \beta_2 = 8.73^\circ$$

7- Le degré de réaction :

$$R = \frac{Ca}{2U} (\operatorname{tg} \beta_1 + \operatorname{tg} \beta_2) = \frac{123.6}{2(223)} (\operatorname{tg} 50.43 + \operatorname{tg} 8.73)$$

$$R = 37.61\%$$

Exercice 04 :

1- Les angles au diamètre moyen :

$$\Delta T_{0s} = (T_{02} - T_{01}) = \frac{\lambda U Ca}{Cp} (\operatorname{tg} \beta_1 - \operatorname{tg} \beta_2) \Rightarrow 24 = \frac{(0.93)(205)(155.5)}{1005} (\operatorname{tg} \beta_1 - \operatorname{tg} \beta_2)$$

Où :

$$\operatorname{tg} \beta_1 - \operatorname{tg} \beta_2 = 0.814 \dots \dots (1)$$

D'autre coté :

$$R = \frac{Ca}{2U} (tg \beta_1 + tg \beta_2) \Rightarrow tg \beta_1 + tg \beta_2 = \frac{(0.5)2(205)}{155.5}$$

Où:

$$tg \beta_1 + tg \beta_2 = 1.318 \dots \dots (2)$$

De (1) et (2) on trouve $\beta_1 = 46.83 = \alpha_2$ et $\beta_2 = 14.14 = \alpha_1$

2- Le diamètre moyen :

$$r = \frac{U}{2\pi N} = \frac{205}{2\pi 152} = 0.215$$

Donc ; $Dm = 0.430m$

3- La surface de l'écoulement :

$$\dot{m} = A\rho Ca \Rightarrow A = \frac{\dot{m}}{\rho Ca}$$

$$\rho = \frac{P_1}{RT_1}$$

$$T_1 = T_{01} - C_1^2/2Cp,$$

$$C_1 = Ca/\cos\alpha_1 = 155.5/\cos 14.14 = 160.31 \text{ m/s}$$

$$\text{Donc: } T_1 = 290 - \frac{16.31^2}{2(1005)} = 277.21K.$$

De plus:

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_1 = 1 \left(\frac{277.21}{290}\right)^{3.5} = 0.854 \text{ bar}$$

$$\rho = \frac{P_1}{RT_1} = \frac{(0.854) 10^5}{287 (277.21)} = 1.073 \text{ kg/m}^3$$

Donc

$$A = \frac{22}{1.73 (155.5)} = 0.132$$

$$A = 0.132 \text{ m}^2$$

1- Les angles :

$$R = 0.5 \Rightarrow \beta_1 = \alpha_2 \text{ et } \beta_2 = \alpha_1$$

$$\cos \alpha_1 = \frac{Ca}{C_1} = \frac{180}{185} = 0.973 \Rightarrow \alpha_1 = \beta_2 = 13.35^\circ$$

$$C_{W1} = \sqrt{C_1^2 - Ca^2} = \sqrt{185^2 - 180^2} = 42.72 \text{ m/s}$$

$$\tan \beta_1 = \frac{U}{Ca} - \tan \alpha_1 = 0.874 \Rightarrow \beta_1 = \alpha_2 = 41.15^\circ$$

2- La température statique :

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{185^2}{2(1005)} = 273 \text{ K}$$

3- Le rapport de pression :

$$R_s = \left(1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Delta T_{0s} = \frac{\lambda U C_a}{C_p} (\tan \beta_1 - \tan \beta_2) = \frac{0.85(200)(180)}{1005} (0.874 - 0.237)$$

$$\Delta T_{0s} = 19.62 \text{ K}$$

$$R_s = \left(1 + \frac{0.86(19.62)}{290}\right)^{3.5} = 1.22$$

Exercice 06 :

$$R = 0.5 \Rightarrow \beta_1 = \alpha_2 \text{ et } \beta_2 = \alpha_1$$

$$\text{et } \beta_2 = \alpha_1 = 30^\circ, \beta_1 = !!$$

$$\cos \beta_1 = \frac{Ca}{V_1}, V_1 = !!!$$

$$R = 0.5 \Rightarrow V_1 = C_2$$

$$C_2^2 = C_a^2 + (U - V_{2x})^2 \Rightarrow V_1 = C_2 = \sqrt{C_a^2 + (U - V_{2x})^2}, \quad V_{2x} = !!!$$

$$\tan \beta_2 = \frac{V_{2x}}{C_a} \Rightarrow V_{2x} = 158 \tan 30 = 91.22 \text{ m/s}$$

donc:

$$V_1 = C_2 = \sqrt{158^2 + (245 - 91.22)^2} = 220.48 \text{ m/s}$$

$$\cos \beta_1 = \frac{C_a}{V_1} = \frac{185}{220.48} = 0.716 \Rightarrow \beta_1 = \alpha_2 = 44.23^\circ$$

$$\Delta T_{0s} = \frac{\lambda U C_a}{C_p} (\tan \beta_1 - \tan \beta_2) = \frac{(245) 185}{1005p} (\tan 44.23 - \tan 30) = 15.21 \text{ K}$$

$$R_T = \left(1 + \frac{N \Delta T_{0s}}{T_{01}}\right)^{\frac{n}{n-1}}$$

$$\text{On a } \frac{n}{n-1} = \eta_s \frac{\gamma}{\gamma-1} = 3.05$$

$$4.5 = \left(1 + \frac{N 15.21}{290}\right)^{3.05} \Rightarrow N = 12$$

Donc N=12 étage