Solution de la Série TD N° 3 :

Exo 1

$F_1 = xy + yz + xz$ (3 variables d'entrée = 8 combinaisons)

x	y	z	F_1	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	1	

$F_1 = xy(z+\bar{z}) + (x+\bar{x})yz + x(y+\bar{y})z$

$F_1 = xyz + xy\bar{z} + xyz + \bar{x}yz + xyz + x\bar{y}z$

$F_1 = xyz + xy\bar{z} + \bar{x}yz + x\bar{y}z$. $(F_1 = \Sigma\Pi)$ (1ère forme)

$F_1 = \Pi\Sigma = (x+y+\bar{z})(x+\bar{y}+z)(x+y+z)(\bar{x}+y+z)$ (2ème forme)

$F_2 = x + yz + \bar{y}\bar{z}t$ (4 variables d'entrée $\Rightarrow 2^4$ combinaisons)

x	y	z	t	F_2
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$F_2 = \Sigma\Pi = \bar{x}\bar{y}\bar{z}t + \bar{x}yz\bar{t} + \bar{x}yzt + x\bar{y}\bar{z}\bar{t} + x\bar{y}\bar{z}t + x\bar{y}z\bar{t}$
$+ x\bar{y}zt + xy\bar{z}\bar{t} + xy\bar{z}t + xyz\bar{t} + xyzt$

$F_2 = \Pi\Sigma$ (2ème forme canonique)

$F_2 = (x+y+z+t)(x+y+\bar{z}+t)(x+y+\bar{z}+\bar{t})(x+\bar{y}+z+t)(x+\bar{y}+z+\bar{t})$

$F_3 = (x+y)(\bar{x}+y+z) = (x+y+z)(x+y+\bar{z})(\bar{x}+y+z) = (\Pi\Sigma)$ 2ème forme

x	y	z	F_3
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$F_3 = \Sigma\Pi = \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$ (1ère forme canonique

Suite Exo 1 (TD N°: 3)

$F_4 = (\bar{x}+\bar{z})(x+\bar{t}+z)y\bar{z}$

$F_4 = (\cancel{\bar{x}x} + \bar{x}\bar{t} + \bar{x}z + x\bar{z} + \bar{z}\bar{t} + \cancel{\bar{z}z})y\bar{z}$

$F_4 = \bar{x}y\bar{z}\bar{t} + \cancel{\bar{x}z\bar{z}y} + x\bar{z}y\bar{z} + \bar{z}\bar{t}y\bar{z} + \bar{z}ty\bar{z}$

$F_4 = \bar{x}y\bar{z}\bar{t} + xy\bar{z} + y\bar{z}\bar{t} + \cancel{y\bar{z}t} = \bar{x}y\bar{z}\bar{t} + xy\bar{z}(t+\bar{t}) + (x+\bar{x})y\bar{z}\bar{t}$

$F_4 = \bar{x}y\bar{z}\bar{t} + xy\bar{z}t + xy\bar{z}\bar{t} + \cancel{xy\bar{z}\bar{t}} + \cancel{\bar{x}y\bar{z}\bar{t}}$

$\boxed{F_4 = \bar{x}y\bar{z}\bar{t} + xy\bar{z}t + xy\bar{z}\bar{t}}$ (1ière forme canonique) (3 mintermes)

x	y	z	t	F_4
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

$F_4 = \Pi\Sigma =$ (13 Maxtermes).

$F_4 = (x+y+z+t)(x+y+z+\bar{t})(x+y+\bar{z}+t)(x+y+\bar{z}+\bar{t})(x+\bar{y}+z+\bar{t})$
$(x+\bar{y}+\bar{z}+t)(x+\bar{y}+\bar{z}+\bar{t})(\bar{x}+y+z+t)(\bar{x}+y+z+\bar{t})(\bar{x}+y+\bar{z}+t)$
$(\bar{x}+y+\bar{z}+\bar{t})(\bar{x}+\bar{y}+\bar{z}+t)(\bar{x}+\bar{y}+\bar{z}+\bar{t})$

(2ième forme canonique)

$F_5 = (\bar{x}y + x\bar{y})\bar{z} + (\bar{x}\bar{y} + xy)z$
$= \bar{x}y\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + xyz = \Sigma\Pi$ (1ière forme canonique 4 mintermes)

x	y	z	F_5
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$F_5 = \Pi\Sigma$ (2ième forme canonique) 4 maxtermes

$F_5 = (x+y+z)(x+y+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)$

Suite de la Solution de la STN°: 3.

EXO 2

$F_1 = \Sigma(0,2,4,6)$, $F_2 = \Sigma(1,3,5,7)$

Dec	a	b	c	F_1	F_2
0	0	0	0	1	0
1	0	0	1	0	1
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	1	0
7	1	1	1	0	1

F_1 : c'est une fonction décimale pair sous la 1ère forme canonique

F_2 = c'est une fonction décimale impair // // // canonique

$F_3 = \Pi(3,7,9,11,12,15)$ (15 s'écrit ($15 = (1111)_2$ ⟹ 4 bits (4 variable d'entrée ⟹ $2^4 = 16$ combinaisons)

F_3 : c'est fonction décimale sous la deuxième forme canonique.

Dec	a	b	c	d	F_3
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	0

EXO 3

$F_1 = \bar{a}\bar{b}\bar{c} + a\bar{b}\bar{c} + ab\bar{c}$

a	b	c	F_1
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$F_2 = \bar{a}c + ab$
$= \bar{a}(b+\bar{b})c + ab(c+\bar{c})$
$= \bar{a}bc + \bar{a}\bar{b}c + abc + ab\bar{c}$

a	b	c	F_2
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$F_3 = \bar{a}cd + bcd + ab$
$= \bar{a}bcd + \bar{a}\bar{b}cd + abcd + \bar{a}bcd + \ldots$

a	b	c	d	F_3
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Suite de la Série de TD N°: 3

Exo 4

1) $f_1 = a\bar{b}\bar{c}d + \bar{a}bc + ac\bar{d} + acd$

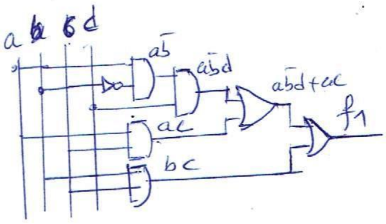
$= a\bar{b}\bar{c}d + \bar{a}bc(d+\bar{d}) + a(b+\bar{b})c\bar{d} + a(b+\bar{b})cd$

$= a\bar{b}\bar{c}d + \bar{a}bcd + \bar{a}bc\bar{d} + abc\bar{d} + a\bar{b}c\bar{d} + abcd + a\bar{b}cd$

$= \underbrace{a\bar{b}\bar{c}d}_{1001} + \underbrace{\bar{a}bcd}_{0111} + \underbrace{\bar{a}bc\bar{d}}_{0110} + \underbrace{abc\bar{d}}_{1110} + \underbrace{a\bar{b}c\bar{d}}_{1010} + \underbrace{abcd}_{1111} + \underbrace{a\bar{b}cd}_{1011}$

cd \ ab	00	01	11	10
00	0	0	0	0
01	0	0	0	1
11	0	1	1	1
10	0	1	1	1

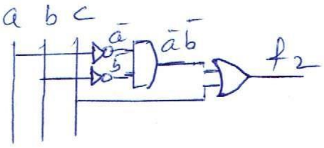
$f_1 = a\bar{b}d + ac + bc$



2) $f_2 = \sum(0,1,3,5,7)$ $(7 = 111$ (3 bits))

c \ ab	00	01	11	10
0	1	0	0	0
1	1	1	1	1

$f_2 = \bar{a}\bar{b} + c$

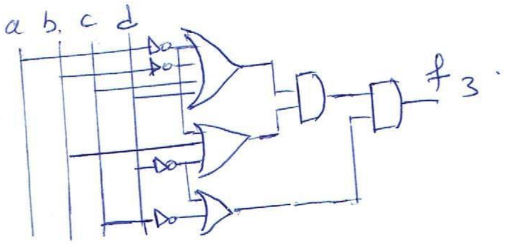


3) $f_3 = \Pi(3,7,9,11,12,15) \rightarrow (15 = 1111$ (4 bits))

0011 0111 1001 1011 1100

cd \ ab	00	01	11	10
00	1	1	0	1
01	1	1	1	0
11	0	0	0	0
10	1	1	1	1

$f_3 = (\bar{a}+\bar{b}+c+d)(\bar{a}+b+\bar{d})(\bar{c}+\bar{d})$ $(\Pi\Sigma)$



Exo 5 Suite la Série TD N°: 3

1) $f = \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + \bar{a}bc\bar{d}$

cd \ ab	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	0	1	0	0
10	0	1	0	0

$f = \bar{a}b$

2) $f = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + a\bar{b}\bar{c}d + a\bar{b}cd$

cd \ ab	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$f = \bar{b}d$

3) $f = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d} + a\bar{b}c\bar{d}$

cd \ ab	00	01	11	10
00	1	0	1	1
01	0	1	0	0
11	0	1	0	0
10	1	0	0	1

$f = \bar{b}\bar{d} + a\bar{c}\bar{d} + \bar{a}bd$

4) $f = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + a\bar{b}\bar{c}d + \bar{a}bc\bar{d} + \bar{a}\bar{b}\bar{c}d + a\bar{b}c\bar{d}$

cd \ ab	00	01	11	10
00	1	0	0	0
01	1	0	0	1
11	0	0	0	0
10	1	1	0	1

$f = \bar{a}\bar{b}\bar{c} + \bar{b}\bar{c}d + \bar{b}c\bar{d} + \bar{a}c\bar{d}$