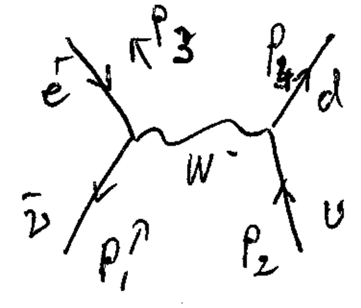
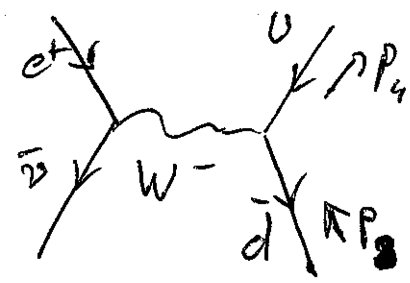
$\boxed{\bar{\nu} p \to e^+ X}$

1) 



M_1 $\qquad\qquad\qquad\qquad$ M_2

On a $q = p_1 - p_3 = p_4 - p_2$

2) $$M_1 = (-i g_W)^2 \, \bar{v}_1 \gamma_\mu (1-\gamma_5) v_3 \; -i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right) \frac{1}{q^2 - M_W^2}$$

$$\bar{u}_4 \gamma_\nu (1-\gamma_5) u_2$$

La combinaison $\bar{u}_4 \not{q} (1-\gamma_5) u_2 = 0$ car

$$\bar{u}_4 \not{q} (1-\gamma_5) u_2 = \bar{u}_4 (\not{p}_4 - \not{p}_2)(1-\gamma_5) u_2$$
$$= \bar{u}_4 \not{p}_4 (1-\gamma_5) u_2 - \bar{u}_4 (1+\gamma_5) \not{p}_2 u_2$$

et pour fermions de masse nulle, l'équation de Dirac est $\not{p}_2 u_2 = 0$ $\quad$ $\bar{u}_4 \not{p}_4 = 0$

Donc
$$M_1 = i g_W^2 \, \bar{v}_1 \gamma_\mu (1-\gamma_5) v_3 \, \bar{u}_4 \gamma^\mu (1-\gamma_5) u_2 / (q^2 - M_W^2)$$

Pour M_2, seule la partie $\bar{u}_4 \gamma^\mu (1-\gamma_5) u_2$ change et devient $\bar{v}_2 \gamma_\mu (1-\gamma_5) v_4$, c'est à dire

$u_2 \to v_i$ et l'annilation de $\bar{d}(p_2)$ est associée à $\bar{v}_2$.

Done

$$M_2 = i g_w^2 \; \bar{v}_1 \gamma_\mu (1-\gamma_5) v_3 \; \bar{v}_2 \gamma^\mu (1-\gamma_5) v_4 \Big/ (q^2 - M_w^2)$$

3) $$\bar{\sum} |M_2|^2 = \frac{g_w^4}{(q^2-M_w^2)^2} \frac{1}{4} \bar{v}_1 \gamma_\mu (1-\gamma_5) v_3 \; \bar{v}_3 (1+\gamma_5) \gamma_\nu v_1$$
$$\times \bar{v}_2 \gamma^\mu (1-\gamma_5) v_4 \; \bar{v}_4 (1+\gamma_5) \gamma^\nu v_2$$

$$= \frac{g^4}{(q^2-M_w^2)^2} \frac{1}{4} \; \mathrm{Tr}\, \not{p}_1 \gamma_\mu (1-\gamma_5) \not{p}_3 (1+\gamma_5) \gamma_\nu$$
$$\mathrm{Tr}\, \not{p}_2 \gamma^\mu (1-\gamma_5) \not{p}_4 (1+\gamma_5) \gamma^\nu$$

$$= \frac{g^4}{(q^2-M_w^2)^2} \mathrm{Tr}\left[\not{p}_1 \gamma_\mu \not{p}_3 \gamma_\nu (1-\gamma_5)\right]$$
$$\times \mathrm{Tr}\left[\not{p}_2 \gamma^\mu \not{p}_4 \gamma^\nu (1-\gamma_5)\right]$$

$$\bar{\sum} |M_1|^2 = \frac{g^4}{(q^2-M_w^2)^2} \mathrm{Tr}\left[\not{p}_1 \gamma_\mu \not{p}_3 \gamma_\nu (1-\gamma_5)\right]$$
$$\mathrm{Tr}\left[\not{p}_4 \gamma^\mu \not{p}_2 \gamma^\nu (1-\gamma_5)\right]$$

4) $$\bar{\sum}|M_1|^2 = \frac{g_w^4}{(q^2-M_w^2)^2}\Big\{\big[\mathrm{Tr}[\not p_1\gamma_\mu \not p_3 \gamma_\nu]\,\mathrm{Tr}[\not p_4\gamma^\mu \not p_2\gamma^\nu] + \mathrm{Tr}[\not p_1\gamma_\mu \not p_3\gamma_\nu\gamma_5]\,\mathrm{Tr}[\not p_4\gamma^\mu \not p_2\gamma^\nu\gamma_5]\big]\Big\}$$

Il n'y a pas de terme $\mathrm{Tr}[////]\,\mathrm{Tr}[////\gamma_5]$ car une trace est paire en permutation des indices et l'autre est impaire.

$$\bar{\sum}|M_1|^2 = \frac{g_w^4}{(q^2-M_w^2)^2}\,16\Big\{(p_{1\mu}p_{3\nu}+p_{3\mu}p_{1\nu}-p_1\cdot p_3\, g_{\mu\nu})(2p_4^\mu p_2^\nu - p_2\cdot p_4\, g^{\mu\nu}) + (i)^2\epsilon_{\alpha\mu\beta\nu}\,p_1^\alpha p_3^\beta\,\epsilon^{\gamma\mu\delta\nu}\,p_{4\gamma}\, p_{2\delta}\Big\}$$

$$= \frac{g_w^4}{(q^2-M_w^2)^2}\,16\Big\{2\,p_1\cdot p_4\; p_2\cdot p_3 + 2\,p_3\cdot p_4\; p_1\cdot p_2 - \cancel{2\,p_1\cdot p_3\; p_2\cdot p_4} - \cancel{2\,p_1\cdot p_3\; p_2\cdot p_4} + \cancel{4\,p_1\cdot p_3\; p_2\cdot p_4} - (-2)\big[p_1\cdot p_4\; p_2\cdot p_3 - p_1\cdot p_2\; p_3\cdot p_4\big]\Big\}$$

En rappelant
$$\hat s = 2p_1\cdot p_2 = 2p_3\cdot p_4$$
$$\hat t = q^2 = -Q^2 = -2p_1\cdot p_3 = -2p_2\cdot p_4$$
$$\hat u = -2\,p_1\cdot p_4 = -2p_2\cdot p_3,$$

on a

$$\bar{\sum}|M_1|^2 = \frac{8g_w^4}{(Q^2+M_w^2)^2}\Big\{\hat s^2+\hat u^2+(\hat u^2-\hat s^2)\Big\}$$

Pour cette dérivation on a utilisé

$$\mathrm{Tr}(\not{V}_1 \gamma^\mu \not{V}_2 \gamma^\nu \gamma_5) = -4i\,\epsilon^{\alpha\mu\beta\nu}\, V_{1\alpha} V_{2\beta}$$
$$= +4i\,\epsilon^{\alpha\beta\mu\nu}\, V_{1\alpha} V_{2\beta}$$

et $\quad \epsilon_{\gamma\delta\mu\nu}\,\epsilon^{\alpha\beta\mu\nu}\, V_3^\gamma V_4^\delta V_{1\alpha} V_{2\beta}$

$$= -2\left[\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right] V_3^\gamma V_4^\delta V_{1\alpha} V_{2\beta}$$
$$= -2\left[V_1\cdot V_3\, V_2\cdot V_4 - V_1\cdot V_4\, V_2\cdot V_3\right]$$

On obtient $\bar{\Sigma}|\mathcal{M}_2|^2$ en remarquant que $\bar{\Sigma}|\mathcal{M}_2|^2$ diffère de $\bar{\Sigma}|\mathcal{M}_1|^2$ par permutation $\not{p}_4 \leftrightarrow \not{p}_2$ dans les traces ce qui laisse invariant $\mathrm{Tr}(1\mu 3\nu)\,\mathrm{Tr}(4\mu 2\nu)$ mais change le signe de $\mathrm{Tr}(1\mu 3\nu\gamma_5)\,\mathrm{Tr}(4\mu 2\nu\gamma_5)$, donc le 2ᵉ terme ds le produit ds traces ~~deviendra~~ deviendra

$$2\left[p_1\cdot p_4\, p_2\cdot p_3 - p_1\cdot p_2\, p_3\cdot p_4\right] \to 2\left[p_1\cdot p_2\, p_3\cdot p_4 - p_1\cdot p_4\, p_2\cdot p_3\right]$$
$$= \frac{1}{2}\left[\hat{s}^2 - \hat{u}^2\right], \text{ d'où}$$

$$\bar{\Sigma}|\mathcal{M}_2|^2 = \frac{8 g_w^4}{(Q^2+M_W^2)^2}\left\{\hat{s}^2 + \hat{u}^2 + (\hat{s}^2 - \hat{u}^2)\right\}$$

On a donc

$$\bar{\Sigma}|M_1|^2 = \frac{16 g_w^4}{(Q^2+M_w^2)^2}\,\hat{u}^2$$

$$\bar{\Sigma}|M_2|^2 = \frac{16 g_w^4}{(Q^2+M_w^2)^2}\,\hat{s}^2$$

5) $$\hat{\sigma}_i = \frac{1}{2\hat{s}}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4)\,\bar{\Sigma}|M_i|^2$$

$$\int \frac{d^4p_4}{2E_4}\,\delta^{(4)}(p_1+p_2-p_3-p_4) = \int d^4p_4\,\delta^{(4)}(p_1+p_2-p_3-p_4)\,\delta(p_4^2)\,\theta(E_4)$$

$$= \delta\left((p_1+p_2-p_3)^2\right) = \delta\left((p_1-p_3)^2+2p_2(p_1-p_3)\right)$$

$$= \delta(q^2+2p_2\cdot q)$$

$$= \delta(Q^2-2p_2\cdot q)$$

$$\hat{\sigma}_i = \frac{1}{2\hat{s}}\frac{1}{(2\pi)^2}\frac{d^3p_3}{2E_3}\,\bar{\Sigma}|M_i|^2\,\delta(Q^2-2p_2\cdot q)$$

$$\boxed{\frac{E_3\,d\hat{\sigma}_i}{dp_3^3} = \frac{1}{4\hat{s}}\frac{1}{(2\pi)^2}\,\bar{\Sigma}|M_i|^2\,\delta(Q^2-2p_2\cdot q)}$$

6) Dans le modèle des partons on ajoute des sections efficaces invariantes telles que $E\dfrac{d\hat{\sigma}}{dp^3}$

$$\frac{E_3\, d\sigma^{\bar{\nu}P}}{dp_3^3} = \int_0^1 dz \left\{ f_u(z)\, \frac{E_3\, d\sigma}{dp_3^3} + f_{\bar{d}}(z)\, E_3 \frac{d\sigma^2}{dp_3^3} \right\}$$

$$= 4\, g_W^4 \frac{1}{(2\pi)^2} \int dz\, \frac{1}{(Q^2+M_W^2)^2}\, \frac{1}{\hat{s}}\, \delta(Q^2 - 2p_2\cdot q)$$

$$\left\{ f_u(z)\, \hat{u}^2 + f_{\bar{d}}(z)\, \hat{s}^2 \right\}$$

Mais $p_2 = zP \Rightarrow \hat{s} = z\,s$

$$\delta(Q^2 - 2p_2\cdot q) = \delta(Q^2 - 2z\,P\cdot q)$$

$$= \frac{1}{2P\cdot q}\, \delta\left(\frac{Q^2}{2P\cdot q} - z\right)$$

$\dfrac{Q^2}{2P\cdot q} = x$

$$= \frac{1}{2P\cdot q}\, \delta(x - z)$$

$$= \frac{x}{Q^2}\, \delta(z - x)$$

~~Dans le repère du laboratoire = proton = $(M, \vec{0})$~~

~~$\bar{\nu} = (E_1, 0, 0, E_1)$~~

~~$S = 2ME_1$~~ ~~$e^+ = (E_3, E_3\sin\theta_3, 0, E_3\cos\theta_3)$~~

~~$t = -2\,p_1\cdot p_3 = -2E_1E_3(1-\cos\theta_3)$~~

~~$u = -2\,p_1\cdot p_4 \simeq -2p_2\cdot p_3 = -2ME_3$~~

Pour la diffusion $\bar{\nu}\, P$ on a

$$p_2 = z\, P \quad \Rightarrow \quad \begin{aligned} \hat{s} &= 2 p_1 \cdot p_2 = z\, 2 p_1 \cdot P = z S \\ \hat{t} &= -2 p_1 \cdot p_3 = -Q^2 \\ \hat{u} &= -2 p_2 \cdot p_3 = z(-2 P \cdot p_3) = z U \end{aligned}$$

$$E_3 \frac{d\sigma^{\bar{\nu} P}}{d p_3^3} = \frac{g_W^4}{M_W^4} \frac{1}{\pi^2} \frac{1}{S} \int \frac{dz}{z} \frac{x}{Q^2} \delta(z - x) \left\{ f_u(z)\, z^2 U^2 + f_{\bar{d}}(z)\, z^2 S^2 \right\}$$

$$= \frac{g_W^4}{M_W^4} \frac{1}{\pi^2} \frac{x^2}{S Q^2} \left\{ f_u(x)\, U^2 + f_{\bar{d}}(x)\, S^2 \right\}$$