Université de Jijel
Faculté des Sciences et Technologie
Département d'électronique

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2ieme Année
Matière : Vibration et Ondes

Série de TD N°1

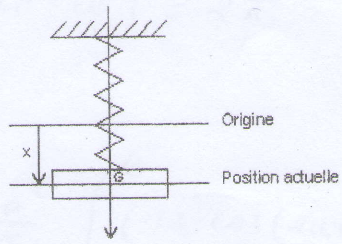
Exercice N°1 :

1) Représenter graphiquement les fonctions $x_1(t) = a\cos(\omega t + \varphi)$ et $x_2(\omega t) = a\cos(\omega t + \varphi)$ en précisant leurs périodes.
2) Calculer la valeur moyenne et efficace de $x_1(t)$

Exercice N°2 :

Le système masse-ressort est décrit par $x(t) = a\cos(\omega t + \varphi)$ si a = 5cm, φ = 60°, m = 10g et l'énergie totale est E = 3.1 10^{-5} J. Déterminer x sachant que $\omega = \sqrt{\frac{k}{m}}$.

Système masse-ressort



Exercice N°3 :

Soit $x_1(t) = a_1\cos(\omega t + \varphi_1)$ et $x_2(t) = a_2\cos(\omega t + \varphi_2)$, sachant que la résultante $x = x_1 + x_2$ est de la forme $x(t) = A\cos(\omega t + \varphi)$.

Calculer A et φ en utilisant la représentation complexe.

AN : $\boldsymbol{a_1 = 0.03,\ a_2 = 0.04,\ \varphi_1 = \frac{\pi}{4},\ \varphi_2 = \frac{\pi}{6}}$

Exercice N°4 :

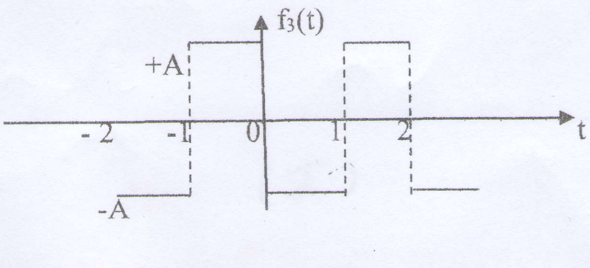
Représenter graphiquement le mouvement résultant de la composition $x_1(t) = 4\cos(34\pi t + \pi)$ et $x_2(t) = 4\cos(36\pi t)$.

Exercice N°5 :

Développer en séries de Fourier les fonctions suivantes :

1) $F_1(t) = \pi^2 - t^2$ pour $-\pi \leq t \leq \pi$, $F_2(t) = \begin{cases} 0 & -\pi \leq t \leq 0 \\ t & 0 \leq t \leq \pi \end{cases}$

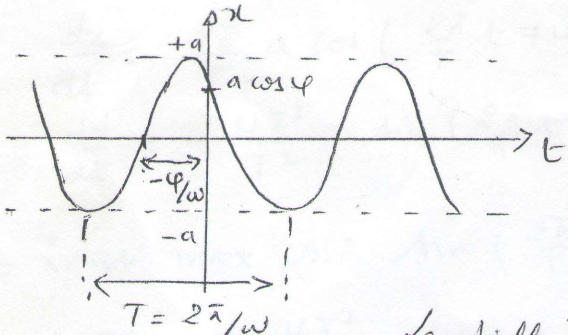
2) $F_3(t)$ est représenté graphiquement

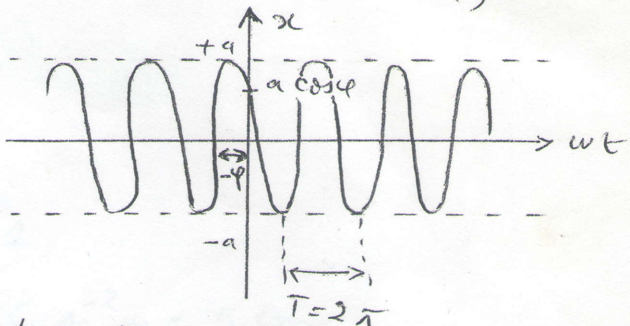


Correction série de TD N°=1 Phys 3 Zigha

Exo 1:

a) $x(t) = a\cos(\omega t+\varphi)$ $\qquad$ $x(\omega t) = a\cos(\omega t+\varphi)$





La différence réside dans la période.

b) * valeur moyenne de $x(t)$:

$$x_{moy} = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\int_0^T a\cos(\omega t+\varphi)\,dt = \frac{a}{T\omega}\Big[\sin(\omega t+\varphi)\Big]_0^T$$

$$= \frac{a}{T\omega}\Big[\sin(\underbrace{\omega T+\varphi}_{2\pi+\varphi}) - \sin\varphi\Big] = 0 \quad \text{car } \omega T = 2\pi$$

* valeur efficace de $x(t)$

$$x_{eff}^2 = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{a^2}{T}\int_0^T \cos^2(\omega t+\varphi)\,dt = \frac{a^2}{2T}\int_0^t (1+\cos(2\omega t+2\varphi))\,dt$$

$$= \frac{a^2}{2T}\Big[t\Big|_0^T + \frac{1}{2\omega}\sin(2\omega t+2\varphi)\Big|_0^T\Big] = \frac{a^2}{2T}\Big[T+\frac{1}{2\omega}(\sin(2\omega T+2\varphi) - \sin 2\varphi\Big]$$

$$= \frac{a^2}{2} \Rightarrow \boxed{x_{eff} = \frac{a}{\sqrt{2}}}$$

soit $x = a \sin(\omega t + \varphi) = a \sin\left(\frac{2\pi}{T} t + \varphi\right)$

accélération : $\gamma = \frac{d^2x}{dt} = \ddot{x}$

$\dot{x} = \frac{dx}{dt} = \frac{2\pi}{T} a \cos\left(\frac{2\pi}{T} t + \varphi\right)$

$\ddot{x} = \frac{d\dot{x}}{dt} = -\frac{4\pi^2}{T^2} a \sin\left(\frac{2\pi}{T} t + \varphi\right)$

$\gamma = \ddot{x}$ est max ssi $\sin\left(\frac{2\pi}{T} t + \varphi\right) = 1$

$\Rightarrow \quad \gamma_{max} = \frac{4\pi^2}{T^2} a = 49,3 \Rightarrow a = 5.10^{-2} m = 5\ cm$

à $t = 0 \quad x(0) = 25\ mm = 2,5\ cm = \frac{a}{2}$.

$\frac{a}{2} = a \sin\varphi \Rightarrow \sin\varphi = \frac{1}{2} \Rightarrow \varphi = 30^0 = \frac{\pi}{6}$.

donc : $x = 5 \sin\left(\pi t + \frac{\pi}{6}\right)$ cm

$\frac{1}{2} m v^2 + \frac{1}{2} K x^2$

$+ \frac{1}{2} \frac{m\omega_0^2 a^2}{n} \cancel{\cos}\sin^2(\omega t + \varphi) + \frac{1}{2} K a^2 \cos^2(\omega t + \varphi)$

$\frac{de}{dt} = 0$

Solution TD N° 1 Phys3

Exo 1

* $x_{moy} = \frac{1}{T}\int_0^T x(t)\,dt = \frac{a}{T}\int_0^T \cos(wt+\varphi)\,dt = \frac{a}{T}\left[\frac{1}{w}\sin(wt+\varphi)\right]_0^T$

$$= \frac{a}{T}\left[\frac{1}{w}\underbrace{\sin(wT+\varphi)}_{\sin\varphi} - \frac{1}{w}\sin\varphi\right] = 0 \quad \text{car } \omega T = 2\pi$$

* $x_{eff}^2 = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{a^2}{T}\int_0^T \cos^2(wt+\varphi)\,dt = \frac{a^2}{T}\int_0^T \frac{1+\cos 2(wt+\varphi)}{2}\,dt$

$$= \frac{a^2}{2T}\left[\int_0^T dt + \int_0^T \cos(2wt+2\varphi)\,dt\right] = \frac{a^2}{2T}\left[t + \frac{1}{2w}\sin 2(wt+\varphi)\right]_0^T$$

$$= \frac{a^2}{2} + \frac{a^2}{4\omega T}\left[\underbrace{\sin 2(wT+\varphi)}_{\sin 2\varphi} - \sin 2\varphi\right] = \frac{a^2}{2}$$

$$\rightarrow \quad x_{eff} = \frac{a\sqrt{2}}{2}$$

Exo 3

Soient $x_1 = a_1\cos(wt+\varphi_1) \xrightarrow{R.C} \bar{x}_1 = a_1 e^{j(wt+\varphi_1)} = \bar{a}_1 e^{jwt}$ / $\bar{a}_1 = a_1 e^{j\varphi_1}$

$x_2 = a_2\cos(wt+\varphi_2) \xrightarrow{R.C} \bar{x}_2 = a_2 e^{j(wt+\varphi_2)} = \bar{a}_2 e^{jwt}$ / $\bar{a}_2 = a_2 e^{j\varphi_2}$

$x = A\cos(wt+\phi) \xrightarrow{R.C} \bar{x} = A e^{j(wt+\phi)} = \bar{A} e^{jwt}$ / $\bar{A} = A e^{j\phi}$

$x = x_1 + x_2 \rightarrow \bar{x} = \bar{x}_1 + \bar{x}_2$

$$\bar{A}e^{jwt} = \bar{a}_1 e^{jwt} + \bar{a}_2 e^{jwt} \Rightarrow \bar{A} = \bar{a}_1 + \bar{a}_2$$

$$\bar{A} = a_1 e^{j\varphi_1} + a_2 e^{j\varphi_2} = a_1(\cos\varphi_1 + j\sin\varphi_1) + a_2(\cos\varphi_2 + j\sin\varphi_2)$$

$$= (a_1\cos\varphi_1 + a_2\cos\varphi_2) + j(a_1\sin\varphi_1 + a_2\sin\varphi_2)$$

$$\bar{A}^2 = \bar{A}.\bar{A}^* = A^2 = (a_1\cos\varphi_1 + a_2\cos\varphi_2)^2 + (a_1\sin\varphi_1 + a_2\sin\varphi_2)^2$$

$$= a_1^2 + a_2^2 + 2a_1a_2(\cos\varphi_1\cos\varphi_2 + \sin\varphi_1\sin\varphi_2) = a_1^2 + a_2^2 + 2a_1a_2\cos(\varphi_1 - \varphi_2).$$

rgument de $\bar{A}$:

$$tg\,\phi = \frac{Im(\bar{A})}{Re(\bar{A})} = \cancel{\left[\frac{a_1\cos\varphi_1 + a_2\cos\varphi_2}{a_1\sin\varphi_1 + a_2\sin\varphi_2}\right]^{-1}} = \frac{a_1\sin\varphi_1 + a_2\sin\varphi_2}{a_1\cos\varphi_1 + a_2\cos\varphi_2}$$

A.N :

$$A = \sqrt{(0{,}04)^2 + (0{,}03)^2 + 2(0{,}04).(0{,}03)\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)} =$$

$$\phi = \text{arctg}\,\frac{0{,}03\sin\pi/4 + 0{,}04\sin\pi/6}{0{,}03\cos\pi/4 + 0{,}04\cos\pi/6} =$$

(III)

$\omega_1 = 34\pi \simeq \omega_2 = 36\pi \Rightarrow$ Phénomène de battement.

$x = x_1 + x_2 \Rightarrow x = 4\cos(34\pi t + \pi) + 4\cos(36\pi t)$.

$\cancel{\cos(a+b)} = \cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right)$ (Voir cours)

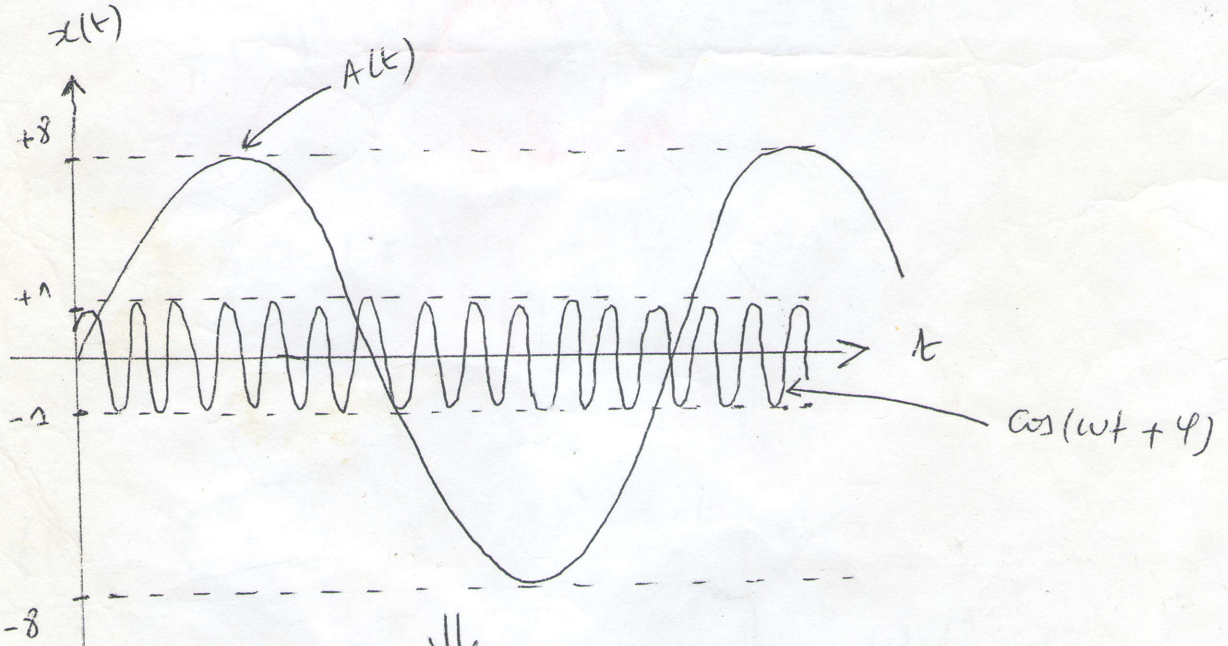
Donc : $x = 4 \times 2 \cos\left(\frac{34\pi t + \pi - 36\pi t}{2}\right) \cos\left(\frac{34\pi t + \pi + 36\pi t}{2}\right)$

$x = 8\cos\left(-\pi t + \frac{\pi}{2}\right)\cos\left(35\pi t + \frac{\pi}{2}\right)$.

$x = 8\cos(\pi t - \pi/2)\cos(35\pi t + \pi/2) = A(t)\cos(\omega t + \varphi)$.

Tels que : $A(t) = 8\cos(\pi t - \pi/2)$: Amplitude ou enveloppe de pulsation $\omega_A = \pi \Rightarrow T_A = \frac{2\pi}{\pi} = 2\,s$

$\cos(\omega t + \varphi) = \cos(35\pi t + \pi/2)$: Terme vibratoire de pulsation $\omega = 35\pi \Rightarrow T = \frac{2\pi}{\omega} = \frac{2}{35}\,s$.



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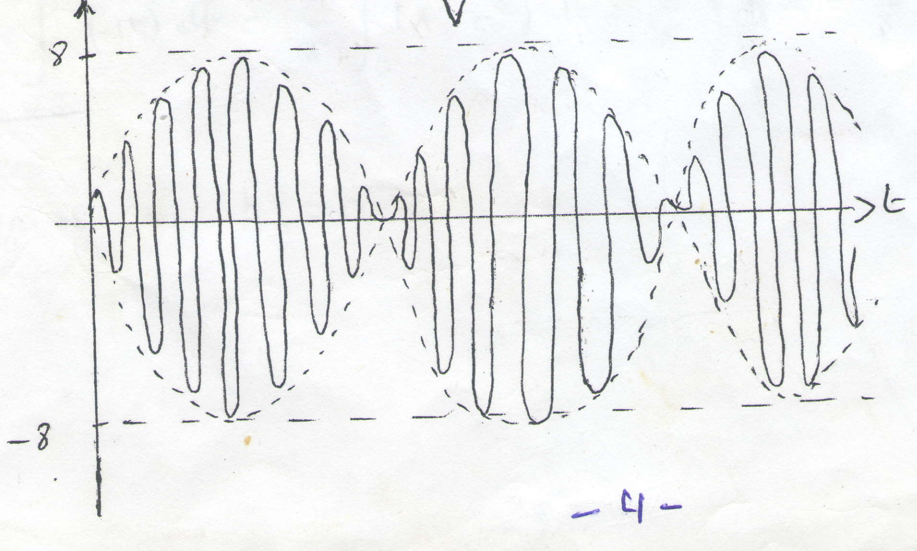


$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\, dt = \frac{2}{\pi}\int_0^{\pi} (\pi^2-t^2)\cos nt\, dt$$

Par partie : $\int_a^b u v'\,dt = uv\Big|_a^b - \int_a^b u'v\,dt$

Soit $u = \pi^2 - t^2$ et $v' = \cos nt$

$\downarrow \qquad\qquad \downarrow$

$u' = -2t \qquad v = \frac{1}{n}\sin nt$

$$\int_0^{\pi}(\pi^2-t^2)\cos nt\,dt = \overset{0}{\overbrace{(\pi^2-t^2)\frac{1}{n}\sin nt\Big|_0^{\pi}}} - \int_0^{\pi} -2t\,\frac{1}{n}\sin nt\,dt$$

$$= \frac{2}{n}\int_0^{\pi} t\sin nt\,dt$$

Soit $u = t$ et $v' = \sin nt$

$\downarrow \qquad\qquad \downarrow$

$u' = 1 \qquad v = -\frac{1}{n}\cos nt$

$$\int_0^{\pi} t\sin nt\,dt = -\frac{t}{n}\cos nt\Big|_0^{\pi} - \int_0^{\pi} -\frac{\cos nt}{n}\,dt$$

$$= -\frac{\pi}{n}(-1)^n + \frac{1}{n}\underbrace{\frac{\sin nt}{n}\Big|_0^{\pi}}_{0} = \frac{-1}{n}\pi(-1)^n$$

on remplace :

$$a_n = \frac{2}{n}\,\frac{2}{\pi}\,\frac{(-1)}{n}\cdot\pi\,(-1)^n = \frac{-4}{n^2}(-1)^n.$$

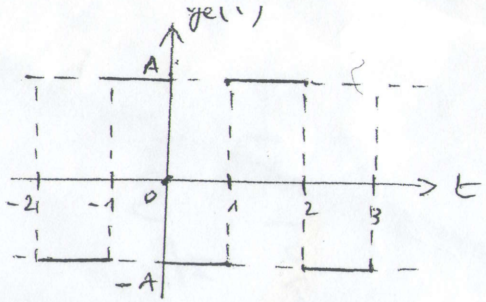
Finalement :

$$f(t) = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty}\frac{-4}{n^2}(-1)^n\cos nt$$

$$\frac{a_0}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\pi}\int_0^{\pi}(\pi^2-t^2)\,dt = \frac{1}{\pi}\left(\pi^2 t - \frac{t^3}{3}\right)\Big|_0^{\pi} = \frac{2\pi^2}{3}$$

$f(t)$ paire $\Rightarrow b_n = 0$

$y_e(t)$ st impaire (symétrie / o) $\Rightarrow a_n = 0$



$$b_n = \frac{2}{T}\int_0^T y_e(t)\sin n\omega t \,.\, dt .$$

$T = 2s \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \ rad/s$.

$$\Rightarrow b_n = A\left[\int_0^1 -\sin n\pi t \ dt + \int_1^2 \sin n\pi t \,.\, dt\right].$$

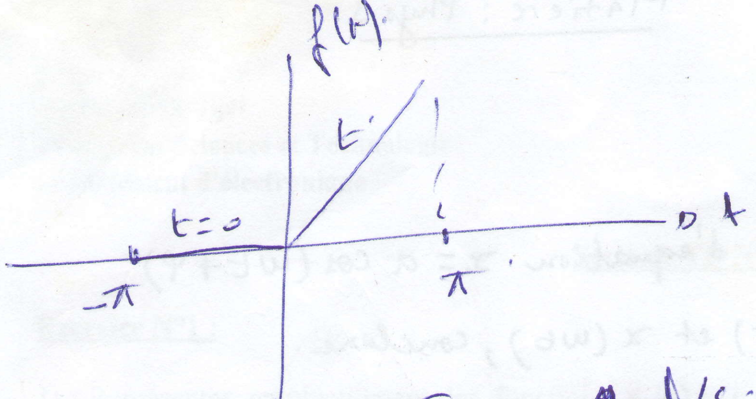
$$b_n = \frac{A}{n\pi}\left[\cos n\pi t\Big|_0^1 - \cos n\pi t\Big|_1^2\right] = \frac{A}{n\pi}\left[\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cos 0}_{1} - \underbrace{\cos 2n\pi}_{1} + \underbrace{\cos n\pi}_{(-1)^n}\right].$$

$$b_n = \frac{2A}{n\pi}\left[(-1)^n - 1\right] = \begin{cases} 0 & \text{si } n \text{ st pair} \\ \frac{-4A}{n\pi} & \text{si } n \text{ st impair} \Rightarrow n = 2N+1. \end{cases}$$

Donc : $$\boxed{y_e(t) = \frac{-4A}{\pi}\sum_{N=0}^{\infty} \frac{\sin(2N+1)\pi t}{2N+1}}$$

Exo5

$$f(t) = \begin{cases} 0 & -\pi \leq t \leq 0 \\ t & 0 \leq t \leq \pi \end{cases}$$



$f(t)$ est Impaire

$a_n = 0 \quad a_0 = 0$

$$\boxed{f(t) = \pi^2 - \frac{\pi^2}{3} - \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n}$$

$T = 2\pi \qquad \omega = \frac{2\pi}{T} = 1 \text{ rad/s}$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt$$

$$b_n = 2\left[\int_{-\pi}^{0} 0 \sin nt\,dt + \int_0^{\pi} t \sin nt\,dt\right]$$

$u = t \to du = dt$

$dv = \sin nt \qquad v = -\frac{1}{n}\cos nt$

$$\int_0^{\pi} t\sin nt\,dt = -\frac{t}{n}\cos nt\Big|_0^{\pi} - \int_0^{\pi}\left(-\frac{1}{n}\right)\cos nt\,dt$$

$$= -\frac{\pi}{n}\cos n\pi + \frac{1}{n}\,\frac{\sin nt}{n}\Big|_0^{\pi}$$

$$= -\frac{\pi}{n}\cos n\pi + \frac{1}{n^2}\overset{0}{\sin n\pi} = -\frac{\pi}{n}\cos n\pi$$

$$= -\frac{\pi}{n}(-1)^n$$

$$b_n = -\frac{2\pi}{n}(-1)^n \qquad \sin n\omega t$$

$$f(t) = -2\pi\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nt$$