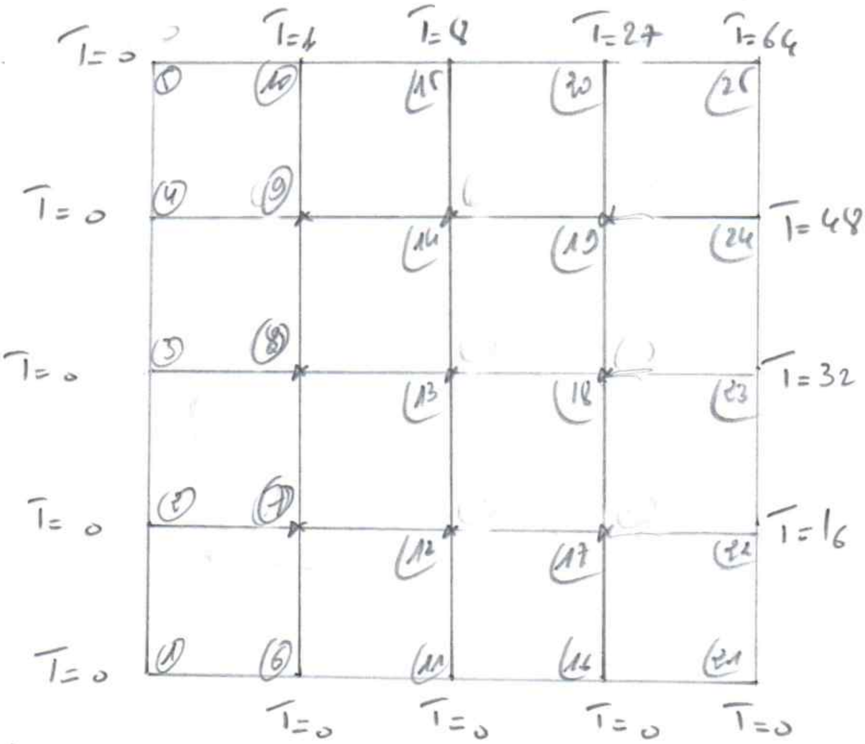
TD : N°3 :

Ex 1 :

Pour $\Delta x = \Delta y = 1$:

$$\lambda_y = \frac{1}{\Delta y^2} = 1$$

$$\lambda_x = \frac{1}{\Delta x^2} = 1.$$



$$\lambda_y T(i,j-1) - 2(\lambda_y + \lambda_x) T(i,j) + \lambda_y T(i,j+1) = -\lambda_x T(i-1,j) - \lambda_x T(i+1,j)$$

$$\Leftrightarrow T(i,j-1) - 4T(i,j) + T(i,j+1) = -T(i-1,j) - T(i+1,j)$$

Pour la colonne des noeuds : 6 à 10 :

$$\left.\begin{aligned} -4T_7 + T_8 &= -T_2 - T_{12} - T_6 \\ T_7 - 4T_8 + T_9 &= -T_3 - T_{13} \\ T_8 - 4T_9 \quad &= -T_4 - T_{14} - T_{10} \end{aligned}\right\} \Leftrightarrow \begin{array}{c} \begin{array}{cccc} T_7 & T_8 & T_9 & \end{array} \\ \begin{bmatrix} -4 & 1 & 0 & : 0 \\ 1 & -4 & 1 & : 0 \\ 0 & 1 & -4 & : -1 \end{bmatrix} \end{array}$$

• Calcul de $\beta_1^{(1)}$ et $g_1^{(1)}$:

$$\beta_1^{(1)} = \frac{\beta_1^{(0)}}{A_1^{(0)}} = \frac{1}{-4} = -0{,}25 \quad , \quad g_1^{(1)} = \frac{g_1^{(0)}}{A_1^{(0)}} = \frac{0}{-4} = 0.$$

• Calcul de $\beta_2^{(1)}$;

$$\beta_2^{(1)} = \frac{\beta_2^{(0)}}{A_2 - [\beta_1^{(1)} \times C_1^{(0)}]} = \frac{1}{-4 - [-0{,}25 \times 1]} = \frac{1}{-3{,}75} = -0{,}266$$

• Calcul de $g_2^{(1)}$ et $g_3^{(1)}$:

$$g_2^{(1)} = \frac{g_2^{(0)} - [g_1^{(1)} \times C_1^{(0)}]}{A_2 - [\beta_1^{(1)} \times C_1^{(0)}]} = \frac{0 - [0 \times 1]}{-3{,}75} = \frac{0}{-3{,}75} = 0.$$

$$g_3^{(1)} = \frac{g_3^{(0)} - [g_2^{(1)} \times C_1^{(0)}]}{A_3 - [\beta_2^{(1)} \times C_2^{(0)}]} = \frac{-1 - [0 \times 1]}{-4 - [-0{,}266 \times 1]} = \frac{-1}{-3{,}734} = 0{,}268$$

$$T_9 = g_3^{(1)} = 0{,}268$$

$$T_8 = g_2^{(1)} - \beta_2^{(1)} \times T_9 = 0{,}071$$

$$T_7 = g_1^{(1)} - \beta_1^{(1)} \times T_8 = 0{,}018$$

Pour la colonne des noeuds : 11 à 15 :

$$\left.\begin{array}{l} -4T_{12} + T_{13} = -T_7 - \cancel{T_{17}} - \cancel{T_{11}} \\ T_{12} - 4T_{13} + T_{14} = -T_8 - \cancel{T_{18}} \\ T_{13} - 4T_{14} = -T_9 - \cancel{T_{19}} - T_{15} \end{array}\right\} \Leftrightarrow \begin{array}{cccc} T_{12} & T_{13} & T_{14} & \\ \end{array} \begin{bmatrix} -4 & 1 & 0 & : -0{,}018 \\ 1 & -4 & 1 & : -0{,}071 \\ 0 & 1 & -4 & : -8{,}268 \end{bmatrix}$$

· Calcul de $g_1^{(1)}$, $g_2^{(1)}$ et $g_3^{(1)}$:

$$g_1^{(1)} = \frac{g_1^{(0)}}{A_1^{(0)}} = \frac{-0{,}018}{-4} = \boxed{0{,}0045}$$

$$g_2^{(1)} = \frac{g_2^{(0)} - \left[g_1^{(1)} \times c_1^{0}\right]}{A_2 - \left[\beta_1^{(1)} \times c_1^{(0)}\right]} = \frac{-0{,}071 - \left[0{,}0045 \times 1\right]}{-3{,}75} = \frac{-0{,}0755}{-3{,}75} = \boxed{0{,}02}$$

$$g_3^{(1)} = \frac{g_3^{(0)} - \left[g_2^{(1)} \times c_2^{(0)}\right]}{A_3 - \left[\beta_2 \times c_2^{(0)}\right]} = \frac{-8{,}268 - \left[0{,}02 \times 1\right]}{-3{,}734} = \boxed{2{,}22}$$

$$T_{14} = g_3^{(1)} = 2{,}22$$

$$T_{13} = g_2^{(1)} - \beta_2^{(1)} \times T_{14} = 0{,}02 + 0{,}266 \times 2{,}22 = \boxed{0{,}61}$$

$$T_{12} = g_1^{(1)} - \beta_1 \times T_{13} = 0{,}0045 + 0{,}25 \times 0{,}61 = \boxed{0{,}157}$$

Pour la colonne des noeuds : 16 à 20 :

$$\left.\begin{array}{l} -4T_{17} + T_{18} = -T_{12} - T_{22} - T_{16} \\ T_{17} - 4T_{18} + T_{19} = -T_{13} - T_{23} \\ T_{18} - 4T_{19} = -T_{14} - T_{24} - T_{20} \end{array}\right\} \Rightarrow \begin{array}{cccc} T_{17} & T_{18} & T_{19} & \\ \end{array} \begin{bmatrix} -4 & 1 & 0 & : -16{,}157 \\ 1 & -4 & 1 & : -32{,}61 \\ 0 & 1 & -4 & : -77{,}22 \end{bmatrix}$$

· Calcul de $g_1^{(1)}$, $g_2^{(1)}$ et $g_3^{(1)}$:

$$g_1^{(1)} = \frac{g_1^{(0)}}{A_1^{(0)}} = \frac{-16{,}157}{-4} = 4{,}04$$

$$g_2^{(1)} = \frac{g_2^{(0)} - \left[g_1^{(1)} \times c_1^{(0)}\right]}{A_2 - \left[\beta_1^{(1)} \times c_1^{(0)}\right]} = \frac{-32{,}61 - \left[4{,}04 \times 1\right]}{-3{,}75} = \boxed{9{,}77}$$

$$g_3^{(1)} = \frac{g_3^{(0)} - \left[g_2^{(1)} \times c_2^{(0)} \right]}{A_3 - \left[\beta_2^{(1)} \times c_2^{(0)} \right]} = \frac{-77,22 - \left[9,77 \times 1 \right]}{-3,734} = \boxed{23,3}$$

$$T_{19} = \boxed{23,3}$$

$$T_{18} = g_2^{(1)} - \beta_2^{(1)} \times T_{19} = 9,77 + 0,266 \times 23,3 = 15,967 = \boxed{15,97}$$

$$T_{17} = g_1^{(1)} - \beta_1^{(1)} \times T_{18} = 4,04 + 0,25 \times 15,97 = \boxed{8,032}$$